9. Planetary Orbits

The orbits of the planets are not co-planar with the plane of the Earth's orbit nor with the planes of each other's orbits. Their orbital planes make small angles, not usually more than a few degrees, with that of the Earth's orbit. The orbits of the planets projected from the Earth onto the celestial sphere results in paths which lie near to the ecliptic. It follows that the planets lie in or near to the zodiacal belt.

The plane of a planet's orbit around the Sun cuts the plane of the ecliptic at two points on the celestial sphere which are known as the Nodes of the planet. The straight line joining the two nodes of a planet is known as a Line of Nodes or as a Nodal line.

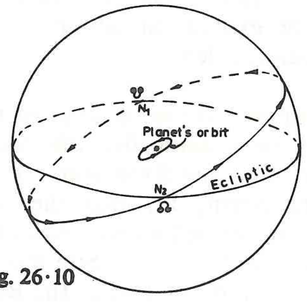


Fig. 26·10

When the planet whose orbit is illustrated in fig. 26·10 appears to be at the ecliptic at position N_1, it is said to be at its Descending Node. When at position N_2 it is at an Ascending Node. At its descending node a planet crosses the ecliptic from north to south: at an ascending node it crosses the ecliptic from south to north.

Should an inferior planet lie at a node at the same time as it is at inferior conjunction it may be observed as a small disc crossing the face of the Sun. Such a phenomenon is known as a Transit of the planet

10. Direct and Retrograde Motion

The motion of the Sun relative to the fixed stars is known as Direct Motion. Other celestial bodies which move relative to the fixed stars in this direction are said to move with Direct Motion. Celestial bodies which move relative to the fixed stars in the opposite direction are said to move with Retrograde Motion.

The apparent motion of a planet relative to the fixed stars is the resultant of its own orbital motion around the Sun and that which is imparted to it by virtue of the Earth's orbital motion. If the path of a planet is plotted on a star map, it will be found that, in some cases, the motion relative to the stars is sometimes direct and sometimes retrograde. When the motion changes from direct to retrograde and back again, the planet traces out a loop known as a Retrogressive Loop.

Exercises on Chapter 26

1. Describe the motions of the Earth. What natural units of time are derived from these motions?
2. Define: Solar Day; Sidereal Day. Compare the lengths of these units of time.
3. Draw a diagram to illustrate the Ecliptic and the Celestial Equator. Name the points of intersection of these circles.
4. Explain why the Ecliptic and the Earth's Orbit are co-planar.
5. What are the causes of the seasons? State the dates of the commencements of the seasons.
6. Describe the causes of unequal lengths of daylight and darkness during the year.
7. Explain the meaning of: Equinox; Solstice.

8. Describe the Earth's climatic zones.
9. Describe the Zodiacal Belt. Why are the planets normally found in this belt?
10. What is meant by: Ascending and Descending Nodes?
11. Show the greatest daily altitude of the Sun on the days of the equinoxes is equal to the complement of the Latitude of the observer.
12. Describe: Direct and Retrograde Motion as these terms apply to the apparent motions of certain planets.

CHAPTER 27

DEFINING CELESTIAL POSITIONS

1. The Cartesian System of Co-ordinates

The common system of describing the position of a point in a plane is the Cartesian System, named after the French philosopher René Descartes. In this system a position is described relative to two axes of reference—usually called the x- and y-axes, respectively, which are mutually perpendicular straight lines in the plane. The axes of reference intersect each other at a point called the Origin, and distances measured x-wards from the y axis and y-wards from the x axis are Abscissae and Ordinates respectively. The abscissa and ordinate of a given point in the plane are called the Co-ordinates of the point.

An extension of the Cartesian System is used for describing positions on a spherical surface. In this case the position is described relative to two great circles the plane of which are perpendicular to one another, and the two co-ordinates are the respective arcs of these great circles of reference. To describe a position on the surface of the spherical Earth, for example, the two great circles of reference are the Equator, from which the co-ordinate Latitude is measured; and the Greenwich Meridian from which the co-ordinates Longitude is measured. In other words the Latitude and Longitude of a point on the Earth are the co-ordinates of the points in terms of the commonly used system of defining terrestrial positions.

2. The Ecliptic System of Defining Celestial Positions

In the Ecliptic System of defining a celestial position the two co-ordinates are known, respectively, as Celestial Latitude and Celestial Longitude. The two reference great circles, in this case, are the Ecliptic, from whch Celestial Latitude is measured; and the Secondary to the Ecliptic through the First Point of Aries from which Celestial Longitude is measured.

Secondaries to the Ecliptic are known as Circles of Latitude. All circles of Latitude converge to the Pole of the Ecliptic. In other words, a circle of Latitude is a semi-great circle which connects the poles of the ecliptic.

The Celestial Latitude of a point is defined as the arc of the circle of Latitude on which the point is located, measured from the ecliptic to the point. It is named North or South according as the point lies north or south of the ecliptic, respectively.

The Celestial Longitude of a point is defined as the arc of the ecliptic measured eastwards from the First Point of Aries to the circle of Latitude on which the point is located. In other words, it is the angle at the pole of the ecliptic contained between the circles of Latitude through the First Point of Aries and the point.

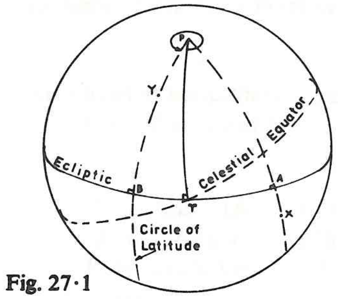


Fig. 27·1

In fig. 27·1:

Arc Ax = Celestial Latitude of X
Arc $\gamma\ A$ = Celestial Longitude of X
Arc BY = Celestial Latitude of Y
Reflex Arc $\gamma\ PB$ = Celestial Longitude of Y

3. The Horizon System of Defining Celestial Positions

Only a half of the celestial sphere is visible to an observer at any instant of time. This follows because the opaque Earth itself obscures the observer's view. The great circle on the celestial sphere which divides the celestial sphere into the Visible and Invisible Hemispheres is known as the Celestial Horizon. The poles of the celestial horizon are the Zenith for the visible hemisphere, and the Nadir for the invisble hemisphere.

Because they cross the celestial horizon at 90° secondaries to the celestial horizon are called Vertical Circles. A vertical circle is defined as a semi-great circle which connects the zenith and nadir of an observer.

The co-ordinates used in the horizon system of defining celestial positions are Altitude and Bearing or Azimuth. The Altitude of a celestial point is the angle at the centre of the celestial sphere contained between the point and the horizon measured in the plane of the vertical circle on which the point lies. The altitude of the zenith is 90°, and the altitude of every point on the celestial horizon is 0°.

All points on the celestial sphere having the same altitude are located on a small circle which is parallel to the celestial horizon. Such a small circle is called a Parallel of Altitude.

The Bearing of a celestial point is the arc of the celestial horizon, or the angle at the zenith of an observer, contained between the vertical circle which lies in the North-South plane, and the vertical circle on which the point is located. The bearing of a point is usually given as an acute angle contained between the North or South point of the horizon and the direction of the point.

Because celestial meridians converge towards the celestial pole the direction of either of the Earth's Poles from any observer is in the plane of the meridian on which the observer is located. The celestial meridian which lies in the same plane as that of the observer's terrestrial meridian is known as the Observer's Celestial Meridian; or, more simply as the Observer's Meridian. The zenith of an observer and also the celestial poles are located on the observer's celestial meridian.

The observer's celestial meridian is divided into two semi-great circles each terminating at the celestial poles. That part of the observer's celestial meridian which contains the observer's zenith is called the Observer's upper, or Superior, Celestial Meridian. The other part, which contains the observer's nadir, is called the Observer's Lower, or Inferior, Celestial Meridian.

The observer's celestial meridian coincides with the vertical circle passing through the North and South points of his horizon. The bearing of a celestial point may, therefore, be defined as the acute angle at the zenith contained between the observer's celestial meridian and the vertical circle passing through the point.

The Azimuth of a celestial point is the angle at an observer's zenith contained between the observer's upper celestial meridian and the vertical circle on which the point is located.

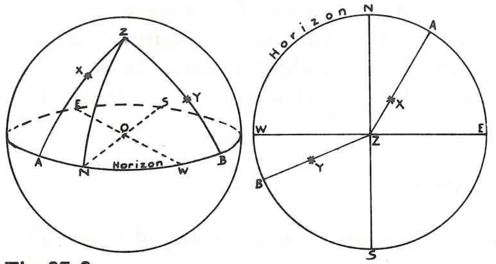


Fig. 27·2

In fig. 27·2:

AX or XOA = Altitude of X
NA or NOA = Bearing of X
NA or NZA = Azimuth of X

BY or YOB = Altitude of Y
SB or SOB = Bearing of Y
NB or NZB = Azimuth of Y

The bearing of a celestial point is named from North or South, whichever direction is nearer to that of the point itself, to East or West according as the point is to the east or the west, respectively, of the observer's celestial meridian.

4. The Celestial Equatorial System of Defining Celestial Positions

In the celestial equatorial system of defining celestial positions the co-ordinates are known as Declination and Hour Angle.

The celestial equator or equinoctial divides the celestial sphere into two celestial hemispheres. Every point in the Northern Celestial Hemisphere, the pole of which is the North Celestial Pole, is said to have North Declination. Every point in the South Celestial Hemisphere has South Declination.

The declination of a celestial position or point is the spherical distance of a great circle arc secondary to the celestial equator, measured from the celestial equator to the position or point. Secondaries to the celestial equator are known as Celestial Meridians. The declination of a point may, therefore, be defined as the arc of a celestial meridian intercepted between the celestial equator and the point.

All points having the same declination in either the northern or the southern celestial hemisphere lie on a small circle which is parallel to the celestial equator. Such a small circle is known as a Parallel of Declination. Parallels of Declination on the celestial sphere are analogous to parallels of Latitude on the Earth.

All celestial meridians converge towards two points on the celestial sphere known as the Celestial Poles. The North Celestial Pole is the point on the celestial sphere which is vertically above the Earth's North Pole, that is to say, it is at the zenith of the North Pole. The South Celestial Pole is at the zenith of the Earth's South Pole. Celestial meridians on the celestial sphere are analogous to meridians on the Earth.

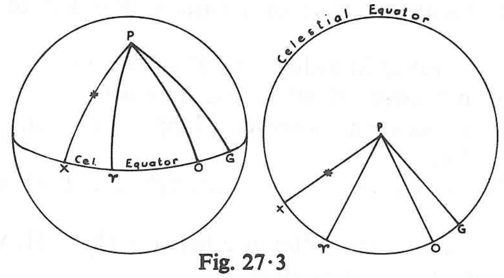
Celestial meridians are sometimes known as Hour Circles. This follows because the angle at the celestial pole at any instant, measure westwards from the observer's upper celestial

meridian, to the celestial meridian of a heavenly body, is the measure of the time that has elapsed since the body was on the observer's upper celestial meridian. That is to say, it is a measure of the time since the body was at Meridian Passage.

The angle at the celestial pole measured westwards from the observer's upper celestial meridian to the Hour Circle of a celestial body is known as the Local Hour Angle (L.H.A.) of the body.

The angle at the celestial pole between the Greenwich upper celestial meridian and the hour circle of a given celestial body, measured westwards from the Greenwich upper celestial meridian, is known as the Greenwich Hour Angle (G.H.A.) of the body.

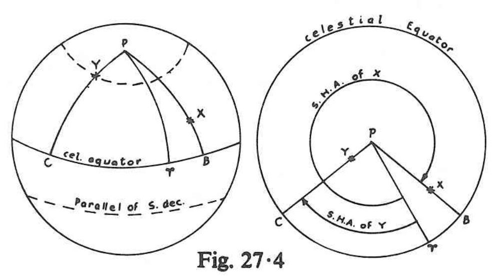
The angle at the celestial pole measured westwards from the celestial meridian of the First Point of Aries to the celestial meridian of a celestial body is known as the Sidereal Hour Angle (S.H.A.) of the body.



In fig. 27·3:

Fig. 27·3

L.H.A. of star ★ = OX
G.H.A. of star ★ = GX
S.H.A. of star ★ = ΥX



In fig. 27·4:

Fig. 27·4

Declination of star X = XB
Declination of star Y = YC
S.H.A. of star Y = γ C

The nautical astronomical problem of finding Longitude is one in which the navigator relates the position of an observed heavenly body using the co-ordinates of the Horizon System, with the body's position at the instant of the observation, using co-ordinates of the Celestial Equatorial System. A detailed discussion on this is given in Chapter 34 under the heading "The Astronomical Triangle".

Exercises on Chapter 27

1. Describe the Cartesian System of defining the position of a point in a plane, and explain how this system is adapted for describing terrestrial positions.
2. Define each of the co-ordinates used in each of the three systems employed for describing celestial positions.
3. Define: Sidereal Hour Angle; Local Hour Angle; Greenwich Hour Angle.
4. Define the position of the First Point of Aries using the Ecliptic and Celestial Equatorial Systems.
5. Compute the Celestial Latitude and the Celestial Longitude of a star whose declination is 30° 00′ N. and whose S.H.A. is 311° 15′.
6. What is the Sun's celestial position using (*a*) Ecliptical co-ordinates, and (*b*) Celestial Equatorial co-ordinates, when it is at the Summer Solstice?
7. What is the S.H.A. and the declination of a star whose Celestial Latitude is 30° 00′ N. and whose Celestial Longitude is 50° 00′ E.?
8. Define: Parallel of Altitude; Circle of Latitude; Parallel of Declination; Parallel of Latitude.
9. Distinguish between Celestial Meridian and Hour Circle.
10. Explain the distinction between Bearing and Azimuth.
11. Explain the difference between Observer's Upper Celestial Meridian and Observer's Lower Celestial Meridian.
12. Show that the celestial equator passes through the East and West points of any observer's celestial horizon.
13. Prove that for a given instant the difference between the L.H.A. of a star and its G.H.A. is a measure of the observer's Longitude.
14. Prove that if at a certain instant the G.H.A. of γ is 120° 50′ and the S.H.A. of a star is 15° 18′, the Longitude of an observer is 60° 00′ W. if the L.H.A. of the star is 76° 08′.

CHAPTER 28

THE APPARENT DIURNAL MOTION OF CELESTIAL BODIES

1. Diurnal Circles

Because of the axial rotation of the Earth towards East, the celestial sphere appears to revolve around the Earth towards the West. This causes the celestial bodies to perform daily circular paths around the Earth with the celestial pole as centre. These apparent daily paths are called Diurnal Circles.

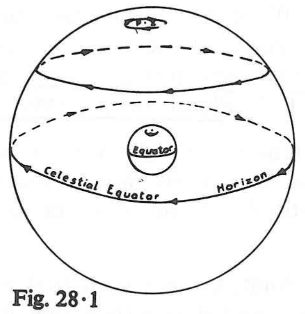


Fig. 28·1

In fig. 28·1, which illustrates diurnal circles as they appear to an observer in latitude 90°, the celestial horizon of the observer is coincident with the celestial equator.

A heavenly body whose declination is 0° travels around the celestial horizon of an observer at either of the Earth's poles, completing its diurnal circle in the time taken for the Earth to rotate once on its axis. Other celestial bodies to the same observer maintain constant altitudes which equal their respective declinations. All stars alter their bearings or azimuths to the extent of 15° 02′·5 per hour, and all objects in the visible hemisphere are above the horizon for the whole day.

If the time of the year is northern Spring or Summer, the Sun would be above the horizon of an observer at the Earth's North Pole for the whole day; but, unlike the stars, its altitude would change at a slow rate equivalent to the rate of change of its declination.

Celestial bodies which are above the horizon throughout the day are known as Circumpolar Bodies. At either pole of the Earth, all bodies in the visible hemisphere are circumpolar bodies.

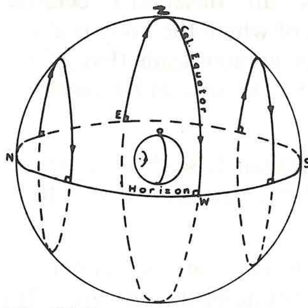


Fig. 28·2

In fig. 28·2, which illustrates diurnal circles as they appear to an observer in latitude 0°, the celestial horizon of the observer is coincident with a celestial meridian. The diurnal circles of all celestial bodies are bisected by the observer's celestial horizon. This means that all celestial bodies to an observer on the equator are above the celestial horizon for exactly half the day and below it for the other half. At the equator all celestial bodies rise and set, so that there are no circumpolar bodies.

When a heavenly body crosses an observer's celestial meridian it is said to Culminate; to transit; or to be at Meridian Passage. A celestial body at the point of culmination on an observers upper celestial meridian reaches its greatest altitude for the day.

From figs. 28·3 and 28·4 it will be noticed that, to an observer in latitude 0°, a star having a declination of 0° culminates with an altitude of 90°. The point of culmination is the zenith of the observer. Such a body crosses the sky in its diurnal path such that its bearings is always due East or due West. Its diurnal circle is a great circle on the celestial sphere which passes through the East and West points of the observer's celestial horizon. This great circle is known as the Prime Vertical Circle, or more commonly as the P.V.

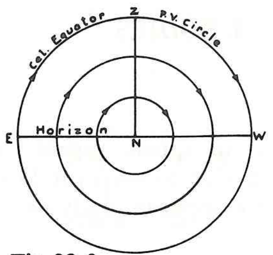


Fig. 28·3

The celestial meridian which is coincident with the horizon of an observer on the equator, cuts the observer's celestial meridian at an angle of 90°. Any celestial body on this meridian, which is known as the Six o-Clock Hour Circle, is 6 hours from the observer's celestial meridian.

It will be seen in fig. 28·4 that the interval between the instants at which a body rises and of its culmination is 6 hours. The interval from its culmination to its setting is also 6 hours.

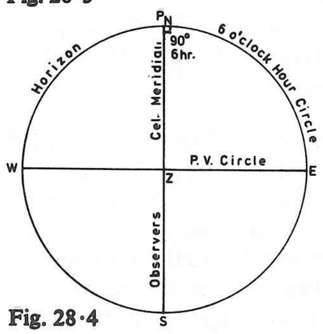


Fig. 28·4

Fig. 28·5 illustrates diurnal circles as they appear to an observer in any latitude other than 0° or 90°. In this case, the celestial pole lies between the horizon and the zenith of an observer. The horizon, therefore, cuts the celestial equator at an angle which depends upon the Latitude of the observer.

A celestial body having a declination of 0° rises at the East point of the horizon and sets at the West point, and is above the horizon, regardless of the Latitude of the observer, for exactly half a day.

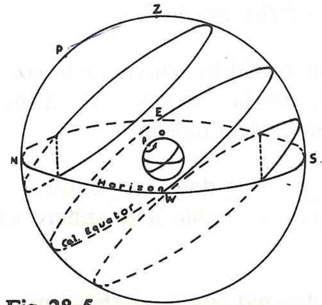


Fig. 28·5

A body which has a north declination rises such that its bearing is between North and East, and it sets such that its bearing is between North and West. Such objects are above the horizon of any observer in the nothern hemisphere for more than half the day.

It will be noticed in fig. 28·5 that some celestial bodies are above the horizon throughout the day. These are circumpolar bodies.

A circumpolar body crosses an observer's celestial meridian on two occasions, at both of which the body is above the horizon, during the day. The transit at which its altitude is greater than that of the celestial pole is known as its upper or Superior Transit: the other is known as its Lower or Inferior Transit.

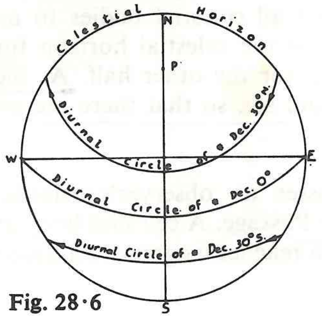


Fig. 28·6

A circumpolar body at upper transit is said to be on the meridian above the Pole: when at lower transit it is said to be on the meridian Below the Pole.

The number of stars above the horizon at both upper and lower transits depends upon the Latitude of the observer. The greater the Latitude, the greater the number of circumpolar stars. It will be remembered that to an observer at either of the Earth's poles all celestial bodies above the horizon are circumpolar.

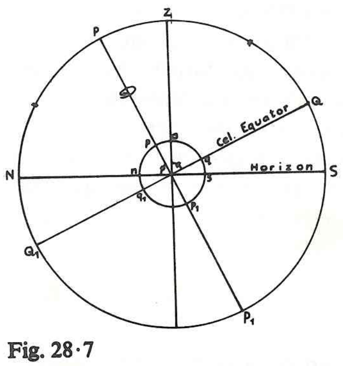


Fig. 28·7

Fig. 28·6 illustrates diurnal circles projected onto the plane of the horizon of an observer in Latitude 50°N.

It will be noticed from figs. 28·5 and 28·6 that some celestial bodies cross the prime vertical circle during their diurnal paths, whereas others do not. The conditions necessary for a celestial body to be circumpolar, and the conditions necessary for a body to cross the prime vertical circle of an observer, will now be considered.

Fig. 28·7 serves to demonstrate the very important relationship between the altitude of the celestial pole and the latitude of the observer.

In fig. 28·7.

$$oq = QZ = \text{Latitude of observer } O$$

$$PQ = 90°$$

Thus: $PZ = (90° - QZ)$

$$ZN = 90°$$

Thus: $PN = (90° - PZ)$

$$= 90° - (90° - QZ)$$

$$= QZ$$

But: $QZ = \text{Latitude of observer } O$

and: $PN = \text{Altitude of Celestial Pole}$

Therefore:

Latitude of Observer = Altitude of Celestial Pole

The arc of a celestial meridian contained between a celestial body and the celestial pole is known as the Polar Distance of the body. The polar distance of a heavenly body is equal to 90° ± declination of the body.

For a body to be circumpolar its polar distance must be less than the observer's latitude. Thus, in Latitude 30°N., all stars whose declinations are greater than 60° (the complement of 30°) will be circumpolar. It follows, therefore, that for a star to be circumpolar its declination must be greater than the co-latitude of the observer. Moreover, the Latitude and the declination must have the same name.

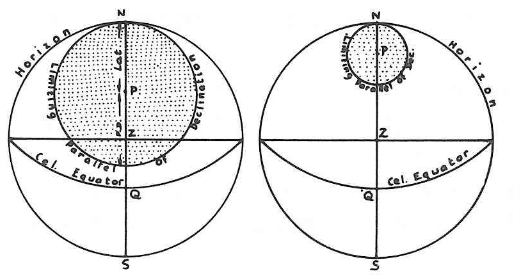


Fig. 28·8

Fig. 28·8 shows projections of the celestial sphere onto the planes, respectively, of the horizons of observers situated in the northern hemisphere. The arc NP is equal to the altitude of the celestial pole: it is, therefore, equal to the latitude of the observer.

$$PQ = 90°$$

$$NZ = 90°$$

Therefore: $ZQ = NP = \text{Latitude of Observer}$

For a celestial body to be circumpolar its polar distance must be less than the Latitude of the observer.

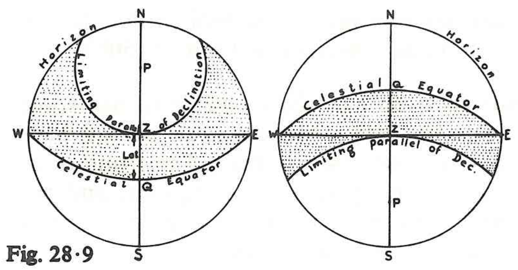


Fig. 28·9

Fig. 28·9 shows projections of the celestial sphere onto the planes of the horizons of observers in North and South Latitudes, respectively. It serves to illustrate that for a celestial body to cross the prime vertical circle of an observer its declination must be less than, but of the same name as, the latitude of the observer.

Exercises on Chapter 28

1. Describe the apparent daily motions of a fixed star as viewed by a stationary observer located (*a*) in Latitude 90°, (*b*) at the South Pole, (*c*) on the equator, and (*d*) in Latitude 40° N.
2. What is the Latitude of an observer whose celestial horizon coincides with the ecliptic?
3. Explain why celestial bodies having North declination are above the horizon of an observer in the northern hemisphere for more than 12 hours during each day.
4. What is meant by the term Culmination? What other terms are used to denote the same phenomenon?
5. Prove that all stars having declinations greater than 36° N. are circumpolar to an observer in Latitude 54° N.
6. What are the conditions necessary for a star not to rise above the horizon of any given observer?
7. What are the conditions necessary for a star to cross the prime vertical circle of an observer?
8. Prove that the altitude of the celestial pole is equal to the latitude of the observer.
9. Explain how an observer may find his latitude from an observation of a star on his meridian above the pole.
10. Explain how the latitude of an observer may be found from the altitudes of a star above and below the pole, repectively, when the declination of the star is not known.
11. What is meant by the Six o-Clock Hour Circle? What are the conditions necessary for a celestial body to cross the Six o-Clock Hour Circle?
12. Explain why it is that all celestial bodies rise out of and set into the horizon of an observer on the equator at an angle of 90°.

CHAPTER 29

TIME

1. The Units of Time

Time, in the astronomical sense, denotes that which persists while events take place. Events are contained in time as objects are contained in space. Time exists before an event and after an event, and it measures the event as it occurs. Time is, therefore, measurable duration.

Time is normally measured by a clock the mechanism of which is adjusted so that the clock registers, in hours, minutes and seconds, the interval that has elapsed since a certain astronomical event took place.

The Earth, itself, because of its relatively uniform rate of rotation, affords the means of establishing a suitable unit of time. The Earth's real axial motion is made manifest by the apparent diurnal motion of the objects on the celestial sphere. The rotation of the Earth on its axis is almost perfectly uniform, and the time taken for it to perform one rotation is a natural unit of time known as a Day. Depending upon what celestial body is used to establish this unit of time, determines the type of day and its length. The unit derived from the apparent motion of a fixed celestial body is referred to as a Star- or a Sidereal-Day. The unit derived from the apparent diurnal motion of the Sun is known as a Sun- or Solar-day. The unit derived from the apparent diurnal motion of the Moon is known as a Moon- or Lunar-day.

Because of the Earth's and Moon's orbital motions the lengths of the different time units are not equal. Moreover, because of the irregularities of the Earth's and Moon's orbital motions, the lengths of the Solar and Lunar Days are not uniform.

The day commences at the instant at which the celestial body used in its determination is on the observer's lower celestial meridian.

The Solar Day starts when the Sun is on the observer's lower celestial meridian: a solar clock set to solar time indicates 00 h. 00 m. 00 s. at this instant.

The fixed point on the celestial sphere used for determining the Sidereal Day is the point at which the Sun, in its apparent annual path, crosses the celestial equator from the southern into the northern celestial hemisphere. This point is the Spring Equinox, more usually known as the First Point of Aries when dealing with time.

The Sidereal Day commences at the instant at which the First Point of Aries is on the observer's upper celestial meridian: a clock set to sidereal time registers 00 h 00 m. 00 s. at this instant.

A Sidereal Day is defined as the interval which elapses between two successive transits of the First Point of Aries across the UPPER meridian of a stationary observer.

A Solar Day is defined as the interval between successive transits of the Sun across the LOWER meridian of an observer.

Although the Earth is moving in her orbit with a speed of about 18·5 miles per second, the direction of the First Point of Aries, or that of any other fixed point in space, does not change because the radius of the celestial sphere is infinite. The sidereal day, therefore, is the interval of time taken for the Earth to make exactly one rotation on its axis. Thus, for each angle of 15° through which the Earth rotates one sidereal hour passes.

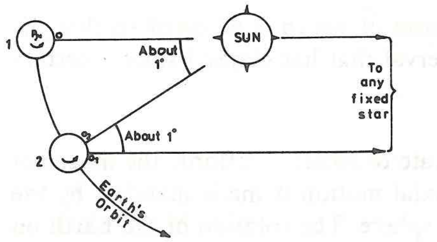


Fig. 29·1

As the Earth moves in its orbit around the Sun, relative to the fixed stars, appears to move eastwards across the sky at the rate of 360/365°, or approximately 1°, per day. It follows that, in the interval between two successive transits of the Sun across the upper meridian of a stationary observer, the Earth rotates through an angle of about 361° on its axis.

Referring to fig. 29·1 imagine the Sun and any fixed star to be on the upper meridian of the observer denoted by O. The next time that the star occupies the upper meridian of the observer occurs after the observer has been carried with the Earth to position 2, and when he is at O_1. In travelling from O to O_1 the observer has been carried around the Earth's axis through an angle of exactly 360°. Before the Sun is again on the observer's upper meridian, the Earth has to rotate through a further angle of about 1° in order to carry the observer to position O_2. A Solar Day, therefore, is a longer period of time than a Sidereal Day.

The sidereal day, assuming a uniform rate of rotation for the Earth, is a constant unit of time: but, because of the Earth's erratic orbital speed—being fastest when the Earth is at perihelion, and slowest when at aphelion—the length of the Solar Day is not uniform. In January for example, when the Earth is near perihelion, the daily angle swept out by the Earth in its orbit around the sun is more than 1°: In July, when the Earth is near aphelion, the angle is less than 1° per day. The Solar Day, therefore, is longer in January than it is in July.

It is not convenient to use Sidereal Time in everyday affairs because the Sun governs these to a large extent. But the Sun is not perfectly suitable because the length of the Solar Day is not constant.

In order to overcome the inconvenience of the inconstancy of the Solar Day, and yet use the Sun as the basis of time-keeping, an artificial point known as the Mean Sun is used. The

Mean Sun moves at the average speed of the True Sun but, instead of moving along the ecliptic, which is the path traced out by the True Sun, the Mean Sun moves in the Earth's rotation; that is to say, it moves along the celestial equator.

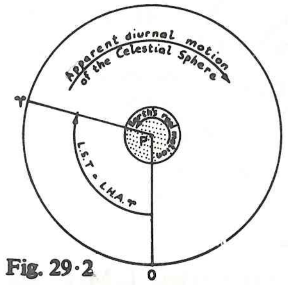


Fig. 29·2

The unit of time derived from the apparent diurnal motion of the Mean Sun is known as the Mean Solar Day. The True Solar Day is usually known as the Apparent Solar Day because it is derived from the diurnal motion of the Sun which is an "apparent" motion.

Fortunately the celestial meridian of the Mean Sun is never very far from that of the True Sun. Time by the Mean Sun, therefore, is very nearly the same as time by the True Sun.

2. Time at an Instant

As explained in Chapter 28, celestial meridians are sometimes called hour circles. The angle contained between an observer's celestial meridian and the meridian, or hour circle, passing through a given celestial point is known as the Hour Angle of that point.

The Sidereal Time at any instant for any place may be defined as the Hour Angle of the First Point of Aries. This is illustrated in fig. 29·2, in which it is seen that the Hour Angle of the First Point of Aries is a measure of the time that has elapsed since the meridian passage of the First Point of Aries. Thus, at any instant.

Local Sidereal Time (L.S.T.) = L.H.A. of ♈

The Apparent Solar Day commences when the True Sun crosses an observer's lower celestial meridian. Thus, the Apparent Solar Time at any instant is defined as the angle at the celestial pole measured westwards from the observer's lower celestial meridian to the meridian, or hour circle, of the True Sun at that instant. It follows that because Solar Time is measured from the observer's lower meridian, whereas Hour Angle is always measured from an observer's upper meridian, the Hour Angle of the True Sun differs from the Solar Time by 12 hours.

The Local Mean Time at any instant is defined as the angle at the celestial pole, or the arc of the celestial equator, measured westwards from an observer's lower meridian to the meridian of the Mean Sun at the instant. It is the time that has elapsed since the Mean Sun occupied the observer's lower celestial meridian. Thus, at any instant:

Local Mean Time (L.M.T.) = Hour Angle of the Mean Sun ± 12 hours
Local Apparent Time (L.A.T.) = Hour Angle of the True Sun ± 12 hours

Or:

L.M.T. = H.A.M.S. ± 12 hr.
L.A.T. = H.A.T.S. ± 12 hr.

In fig. 29·3:

L.M.T. = H.A.M.S. − 12 hr.
L.A.T. = H.A.T.S. − 12 hr.

In fig. 29·4:

L.M.T. = H.A.M.S. + 12 hr.
L.A.T. = H.A.T.S. + 12 hr.

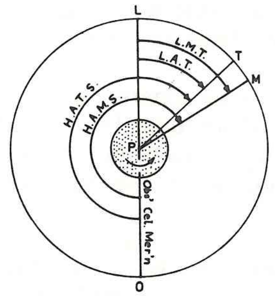


Fig. 29·3

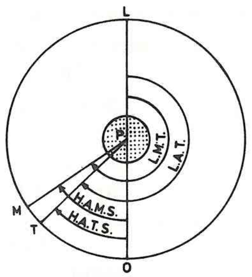


Fig. 29·4

Because the True and Mean Suns usually occupy different hour circles, L.M.T. and L.A.T. at any given instant are not the same. The L.M.T. differs from the L.A.T. by a quantity which is equivalent to the angle at the celestial pole, or the corresponding arc of the celestial equator, contained between the hour circles of the True and Mean Suns, respectively. This quantity, expressed in units of time, is called the Equation of Time. The Equation of Time is sometimes defined as the excess of L.M.T. over L.A.T. It is the amount of time that must be applied to the L.M.T. (or L.A.T.) to get the corresponding L.A.T. (or L.M.T.). If, for example, the L.M.T. is 11 h. 45 m. at the instant the L.A.T. is 11 h. 49 m., the excess of L.M.T. over L.A.T. is a negative amount, and the equation of time in this circumstance takes a minus sign and is described as − 4 minutes. If, on the other hand L.M.T. is 11 h. 49 m. and the corresponding L.A.T. is 11 h. 45 m. the equation of time is + 4 minutes.

$$\text{Equation of Time } (e) = \text{L.M.T.} - \text{L.A.T.}$$

or:

$$e = \text{H.A.M.S.} - \text{H.A.T.S.}$$

The equation of time is positive when the Mean Sun lies to the west of the True Sun, and it is negative when the Mean Sun lies to the east of the True Sun.

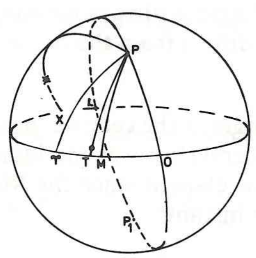


Fig. 29·5

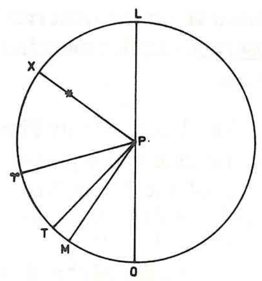


Fig. 29·6

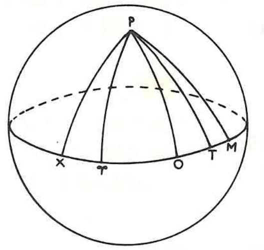


Fig. 29·7

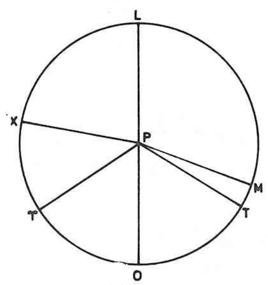


Fig. 29·8

Figs. 29·5, 29·6, 29·7, and 29·8, serve to illustrate the relationship between Time and Hour Angle. In these figures.

P denotes the North celestial pole.
PO denotes the observer's upper celestial meridian.
PL denotes the observer's lower celestial meridian
PM and *PT* denote, respectively, the meridians of the Mean and True Suns
PX denotes the celestial meridian of any celestial body other than the Sun
*P*ϒ denotes the celestial meridian of the First Point of Aries

$$\begin{aligned} \Upsilon O &= \text{L.S.T.} \\ LOM &= \text{L.M.T.} \\ LOT &= \text{L.A.T.} \\ OM &= \text{H.A.M.S.} \\ OT &= \text{H.A.T.S.} \\ MT &= e \\ \Upsilon X &= \text{S.H.A. of } X \end{aligned}$$

Note carefully that:

$$\begin{aligned} \text{L.M.T.} &= \text{H.A.M.S.} \pm 12 \text{ hr.} \\ \text{L.A.T.} &= \text{H.A.T.S.} \pm 12 \text{ hr.} \\ e &= \text{L.M.T.} - \text{L.A.T.} \ (-\text{ve}) \\ \text{L.S.T.} &= \text{H.A. of } X - \text{S.H.A. of } X \end{aligned}$$

In figs. 29·7 and 29·8, the True Sun and the Mean Sun are East of the observer's upper meridian. In this case:

$$\begin{aligned} \text{L.M.T.} &= \text{H.A.M.S.} - 12 \text{ hr.} \\ \text{L.A.T.} &= \text{H.A.T.S.} - 12 \text{ hr.} \end{aligned}$$

3. The Equation of Time

The True Sun is not a perfect timekeeper for two reasons, viz:

1. Its motion in its apparent annual orbit around the Earth is irregular.

2. It moves in the ecliptic whereas the Earth rotates in the plane of the celestial equator.

The equation of time, therefore, is considered to be composed of two parts. The first is due to (1) above, and is known as the component due to Eccentricity; and the second, due to (2) above is known as the component due to Obliquity.

The True Sun moves irregularly in the ecliptic. Consider a point moving in the ecliptic at a uniform rate equal to the average rate of the True Sun. This point is known as the Dynamical Mean Sun (D.M.S.). The component of the equation of time due to eccentricity is a measure of the difference between the Hour Angles of the true Sun and the Dynamical Mean Sun.

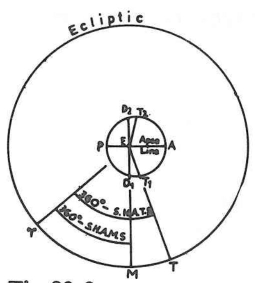


Fig. 29·9

It will be remembered that the Sun's apparent orbit around the Earth is an ellipse having the Earth at one of its foci; and that the speed of the Sun in the celestial sphere is greater when it is near perihelion than when it is near aphelion.

In fig. 29·9, the ellipse represents the apparent annual orbit of the Sun around the Earth. *T* and *D* represent, respectively, the True Sun and the D.M.S. *P* is the point in the Sun's apparent annual orbit at which it is closest to the Earth, and *A* represents the point at which its distance is greatest. These points are perihelion and aphelion respectively

The line of apsides bisects the Sun's apparent annual orbit. The time taken by the True Sun to move from *P* to *A* is, therefore, the same as that taken for it to move from *A* to *P*. This follows, because, according to Kepler's Second Law, the times taken for the Sun's radius vector to sweep out equal areas are equal.

The D.M.S. is coincident with the True Sun at perihelion. Now the D.M.S. moves such that it sweeps out equal ANGLES in equal intervals. The D.M.S. moves from *A* to *P*, therefore, in the same time as that taken for the True Sun to move from *P* to *A*. Thus, the True Sun and the D.M.S. are again in coincidence at aphelion.

In travelling from *P* to *A* the True Sun, which moves fastest when at perihelion moves ahead of the D.M.S., and their greatest angular separation occurs half way between perihelion and aphelion. This maximum separation, which amounts to about 2°, occurs in early April. From April until July—which is the time of aphelion—the angular separation decreases until the time when the True Sun and the D.M.S. are in coincidence at aphelion.

At aphelion the True sun moves slowest. Therefore, the D.M.S. moves ahead of the True Sun. The maximum separation occurs in early Ocotober. After this date the separation decreases until the True Sun and the D.M.S. are again in coincidence at the next perihelion.

The maximum difference of time between that of the True Sun and the D.M.S. amounts to about 8 minutes. From perihelion to aphelion the True Sun is ahead of the D.M.S. The component of the equation of time due to eccentricity, therefore, is positive. From aphelion to perihelion the D.M.S. is ahead of the True Sun and this component, accordingly, is negative.

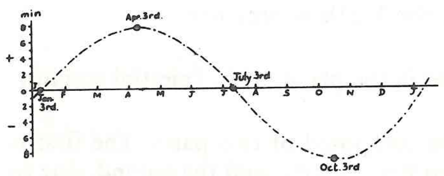


Fig. 29·10

The component of the equation of time due to eccentricity is represented graphically in fig. 29·10.

The D.M.S. increases its Celestial Longitude at a uniform rate. If the planes of the Earth's rotation and the ecliptic were coincident, the D.M.S. would be a perfect timekeeper. But because of the obliquity of the ecliptic, the point which is to afford the means of measuring time must move at a constant rate in the celestial equator. Consider a point on the celestial equator which is coincident with the D.M.S. at the Spring Equinox. Imagine this point to move such that its S.H.A. decreases uniformly at the same rate as the changing Celestial Longitude of the D.M.S. This point is known as the Astronomical Mean Sun (AMS) or, more commonly, as simply the Mean Sun.

The component of the equation of time due to obliquity is a measure of the difference between the Hour Angles of the D.M.S. and the A.M.S.

Component due to Obliquity = H.A. of the A.M.S. − H.A. of the D.M.S.

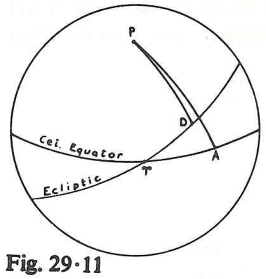


Fig. 29·11

In fig. 29·11 *D* represents the D.M.S., and *A* represents the A.M.S. The arc ♈ *D* is equal to the arc ♈ *A*; in other words, the Celestial Longitude of the D.M.S. is equal to 360° − S.H.A. of the A.M.S. It is evident from fig. 29·11 that in the northern Spring the S.H.A. of the A.M.S is less than the S.H.A. of the D.M.S. In Spring, therefore, the components of the equation of time due to obliquity is a negative quantity.

By the time of the Summer Solstice the D.M.S. will have increased its Celestial Longitude by 90°. The A.M.S. will have decreased its S.H.A. and increased its Celestial Longitude by the same amount. At the time of the Summer Solstice the component due to obliquity is zero because the D.M.S. is coincident with the A.M.S. The maximum value of the difference in the Hour Angles of the D.M.S. and the A.M.S. occurs half way between the times of the Spring Equinox and the Summer Solstice. At that instant the difference in times by the A.M.S. and the D.M.S. is about 10 minutes.

From the date of the Summer Solstice to that of the Autumnal Equinox, the D.M.S. is ahead of the A.M.S., and the component due to obliquity is a positive quantity. From the time of the Autumnal Equinox to that of the Winter Solstice, the D.M.S. is to the East, of the A.M.S. During this part of the year the component due to obliquity is a negative quantity. From the date of the Winter Solstice to that of the Spring Equinox the component due to the obliquity is positive; and from the time of the Spring Equinox to that of the Summer Solstice it is negative.

The component of the equation of time due to obliquity is shown graphically in fig. 29·12.

By combining the graphs of the two components, as shown in fig. 29·13, the equation of time for any time in the year may be found.

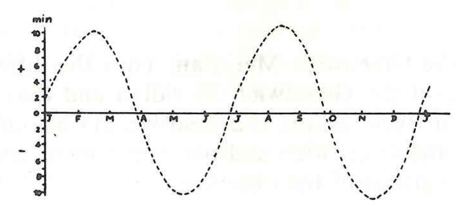


Fig. 29·12

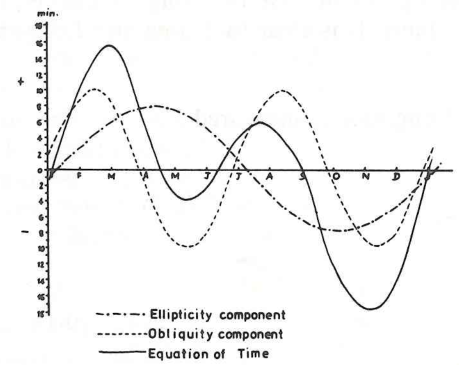


Fig. 29·13

It will be noticed in fig. 29·13 that the equation of time is zero on four occasions each year. The dates of these times may be verified by inspection in the *Nautical Almanac*.

4. Comparison of Solar and Sidereal Time Units

The Earth makes one revolution in its orbit in the time it takes to make 365¼ rotations on its axis. During this interval the Sun revolves once in the ecliptic and, therefore, makes one apparent revoltion with respect to the fixed stars. Thus, in relation to the stars, the Earth makes 366¼ rotations in the time it takes to make 365¼ rotations with respect to the Sun. Thus:

365¼ Solar Days = 366¼ Sidereal Days

From this relationship we see that:

1 Solar Day = 1 d. 00 h. 03 m. 56·5 s. of Sidereal Time
1 Sidereal Day = 0 d. 23 h. 04 m. 04·1 s. of Solar Time

5. Time and Longitude

The angle at the celestial pole contained between the celestial meridians of any two observers is a measure of the difference in Longitude between the positions of the observers.

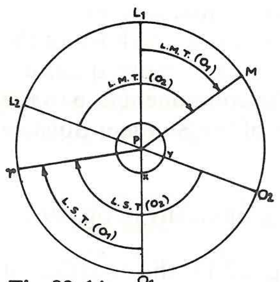


Fig. 29·14

in fig. 29·14 x and y denote two observers whose upper celestial meridians are PO_1 and PO_2, and whose lower celestial meridians are PL_1 and PL_2, respectively.

The L.M.T. at x is equal to the arc of the celestial equator L_1M. The L.M.T. at y is equal to the arc L_2M. The difference between these arcs is arc L_1L_2, which is equal to the arc O_1O_2 or arc xy. This, in turn, is equal to the difference of Longitude between x and y. Thus, the D. Long. between two observers is equal to the difference between their L.M.T.s. It may be seen from fig. 29·14 that D. Long. is also equal to the difference between L.S.T.s at the two meridians. It is clear that time and Longitude are closely related.

Longitude is measured East or West from the Greenwich Meridian. Thus the difference between the local time at the Greenwich Meridian and that at an observer's meridian at a given instant, is a measure, in time units, of the D. Long. between the Greenwich and observer's meridian, and this is equal to the Longitude of the observer.

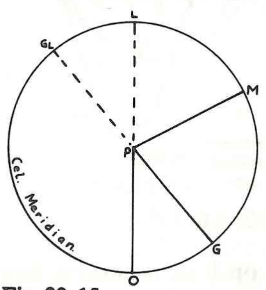


Fig. 29·15

In fig. 29·15, at the instant when the Mean Sun is at position M, the G.M.T. is equal to the arc G_LM; and the L.M.T. for an observer whose meridian is TO is equal to the arc LM. The arc G_LM is greater than the arc LM by an amount equal to arc G_LL which is equal to the arc GO, which is equal to the Longitude of the observer.

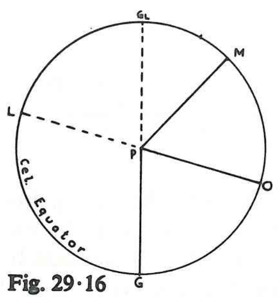


Fig. 29·16

In fig. 29·16 the difference between G.M.T. and L.M.T. is also equal to the arc GO; but, in this case, the Greenwich Meridian lies to the West of the observer, whereas in fig. 29·15 the observer's meridian lies to the West of Greenwich Meridian.

From figs. 29·14, 29·15 and 29·16, it is seen that when the G.M.T. is greater than the L.M.T. the Longitude of the observer is named West, and that when the G.M.T. is less than the L.M.T. the Longitude of the observer is named East.

Hence the mnemonical rule:

Longitude West Greenwich time best
Longitude East Greenwich time least.

6. Time at Sea

The respective local times at two places which lie on different meridians differ to the extent of the D. Long. between the two places. It follows that, if it is desired to keep L.M.T. on a moving vessel it will be necessary to adjust the clock steadily as the vessel moves eastwards or westwards. For a change of each 15° of Longitude the clock will have to be altered 1 hour; for each 15′ of change of Longitude it will have to be altered by 1 minute; and for each 15″ of change of Longitude it will have to be altered by 1 second. It is not convenient, neither is it necessary, to continually change the clock this time in this way.

In days gone by it was customary to set the clock, usually in the late evening, so that it registered the Apparent Time (A.T.S.) corresponding to the meridian at which the vessel was expected to reach at noon. (L.A.T.) on the following day. It will be remembered that at noon (L.A.T.) the Sun attains its greatest daily altitude. If this altitude is measured it is comparatively easy for the navigator to ascertain the Latitude of his vessel. It was, therefore, convenient to have the clock set such that the navigator could be warned of the time to make his noon observation.

In many vessels a system of timekeeping known as Zone Time is used. In this system the clock is always an exact number of hours different from G.M.T. to facilitate the use of Zone Time, the Earth's surface is divided into Time Zones which are depicted on a Time Zone Chart. Time Zones are regions bounded by meridians whose Longitudes differ by 15°. The region known as Zone O is bounded by the meridians of 7½° E. and 7½° W. All vessels keeping Zone Time in this region have their clocks set to G.M.T., which is the correct Mean Time for the central meridian, which is the Greenwich Meridian. The Time Zone immediately to the east of Zone O extends from Longitude 7½° E. to Longitude 22½° E. In this region, which is known as Zone − 1, vessels keeping Zone Time have their clocks set to one hour ahead of G.M.T. Thus, to find G.M.T. one hour is deducted from the clock time: hence the minus sign in front of the Zone number. Zone − 2 extends from Longitude 22½° E. to Longitude 37½ E., and so on. Time Zones which lie to the west of Zone O, have positive numbers: the zone extending from 7½° W. to 22½° W. is zone+ 1; that extending from 22½° W. to 37½° W.is Zone+ 2, and so on.

Time Zone 12 lies diametrically opposite to Zone O. It is divided into two parts by a line known as the Date Line. Ideally, the Date Line should coincide with the meridian of 180°, but its position is modified so as to avoid differences in time and date in certain island groups which straddle the 180th meridian. The part of Zone 12 which lies to the west of Date Line is known as Zone − 12, and that part which lies to the east is known as Zone+ 12.

Consider a vessel travelling westwards in the course of circumnavigating the Earth. For each 15° of Longitude made good to the West the clock will be retarded 1 hour. It is obvious that when the vessel arrives on her initial meridian, she will have changed her Longitude by 360° and her clock time will have been retarded by 24 hours. A whole day, therefore, would have been "lost" when the vessel's time is compared with the local calendar. To overcome this the date is altered by one day when crossing the Date Line, and this is the reason why this line is so named. When crossing the Date Line from West to East the date is advanced one day, and when crossing it from East to West the date is retarded one day.

7. Standard Time

When a vessel arrives in port it is often necessary for the clock to be re-set so that it corresponds to the time used by the people on shore. The times used by civil authorities are known as Standard time, a list of which is given in the *Nautical Almanac.*

8. The Years

The time taken by the Earth to make exactly one revolution around the Sun relative to the fixed stars is known as a Sidereal Year. The sidereal year in the Mean Solar units of time is 365 days 06 hours 09 minutes 09 seconds. The sidereal year is defined as the interval between successive instants when the Sun occupies the same position in the celestial sphere relative to a fixed point in space.

The time taken by the Earth to make one revolution with reference to the First Point of Aries is slighly shorter than a sidereal year. This is due to the precession of the Earth's axis which causes the equinoctial points to move westwards across the celestial sphere at an average rate of about 50″ of arc per year. This retrograde motion of the equinoxes gives rise to another unit of time known as the Tropical Year. The tropical year is defined as the interval between two successive Spring equinoxes. In Mean Solar units of time it is 365 days 05 hours 48 minutes 46 seconds.

The apse line of the Earth's orbit moves eastwards around the orbit at an average rate of about 11¼″ of arc per year. The interval between two successive perihelions, therefore, is slightly longer than a sidereal year. In Mean Solar units of time it is 365 days 06 hours 13 minutes 48 seconds. This period is known as an Anomalistic year.

9. Co-ordinated Universal Time

The bases of time-keeping described above is the rotation of the Earth. Solar and Sidereal Times are therefore, described as Rotational Times. Now the speed of rotation of the Earth is not perfectly uniform: it is affected by atmospheric and oceanic phenomena, as well as by movements of material within the body of the solid Earth. So that for highly accurate time measuring the rotation of the Earth is not satisfactory.

The determination of the units of rotational times is a highly specialized branch of astronomy. Astonomical observations at a given observatory led to the determination of what astronomers call Universal Time (U.T.). Because of a short-term irregularity in the Earth's rotation, known as Polar Variation, U.T. varies slightly as between observatories, so that a correction is applied to U.T. to give a standard form of time known as U.T. 1. An empirical correction to allow for annual changes in the speed of the Earth's rotation is applied to U.T. 1 to give U.T. 2. This system of time-keeping was adopted internationally in 1956.

But even U.T. 2 is not sufficiently accurate for certain scientific purposes, so that an alternative system of time-measuring was sought.

Now the unit of time is a fundamental physical quantity, so that in 1956, the International Committee of Weights and Measures adopted as the unit of time the "Second of Ephemeris Time". Up to 1956 the unit was a second of time determined by the irregular rotation of the Earth—the unit being 1/86,400 of the Mean Solar Day. Ephemeris Time (E.T.) obtained from the orbital motion of the Earth, corresponds to U.T. 2 over a long period of time. The problem of making E.T. available led to the use of an Atomic Clock, the principle of which in no way depends on the rotation speed of the Earth.

An Atomic Clock employs energy changes within atoms to produce extremely uniform waves of electro-magnetic radiation which can be counted. In 1967 the unit of time adopted was the second of the International System of Units (S.I.) This is defined as the duration of a stated number of periods of radiation of a "caesium atom-133".

A stystem of time-keeping, known as Co-ordinated Universal Time (U.T.C.), has been in use since 1972. This system developed from the relating of Ephemeris Time (E.T.) to Atomic Time (A.T.), and by adjusting the atomic clock so as to remain close to U.T. 2.

In 1972 the practice was adopted of keeping U.T.C. to within about 0·5 sec. of U.T. 2 by resetting a clock on U.T.C. by precisely one second when necessary. These one-second jumps, known as Leap Seconds, are inserted at the ends of solar days; and, of course, notice well in advance is given whenever a leap second is to be introduced. Time Signals are broadcast on this system so that U.T.C. is now the reference time for common, as well as for nautical astronomical, use.

10. The Calendar

The systematic arrangement of units of time constitutes an Almanac or Calendar. The prime function of a calendar is to provide the means of recording dates of important events for the benefit of posterity. The earliest calendars were devised mainly for the purpose of determining the dates of religious feasts, a fact which resulted in a profusion of styles. The present system in general use was prescribed by Pope Gregory XIII in 1582. The Old-Style calendar in use in Britain prior to the introduction of the Gregorian Calendar in 1752, was the Julian Calendar which was invented by an Alexandrian astronomer named Sosigenes and introduced by Julius Caesar after whom it is named.

The Julian year contains 365¼ days. The 365 days were divided into 12 months each having an integral number of days. The extra quarter-days were allowed to accumulate to form a whole day which was intercalated into the month of February at four-year intervals. Thus, every fourth year contained 366 days compared with 365 days which formed the so-called Common years. Years containing 366 days were called Leap Years or Bissextile Years, and the intercalated day was called the Bissextus. Leap years were so called because, in such a year, any given date does not advance one day on the date of the corresponding day of the preceding year, as in Common Years, but "leaps" over the additional day. Leap Years are those whose numbers are exactly divisible by four.

It is important that a calendar should be such that the seasons recur on the same dates of successive years. That this was not the case with ancient calendars resulted in them falling into disuse.

The ideal calendar year is the time taken by the Earth to revolve once in its orbit with respect to the equinoxes. This interval is the Tropical Year. The Julian Calendar was devised on the assumption that the year contained exactly 365 days 06 hours. It was, therefore, in error to the extent of about 11 minutes per year. The accumulation of this error over a period of 400 years amounts to about 72 hours or 3 days. The Gregorian Calendar took into account this error; and, accordingly, dropped 3 days every 400 years. Thus, leap years in our present calendar are those whose numbers are divisible by 4, *except* certain initial years of centuries. The initial years of centuries are not leap years unless the first two numbers of the year number are exactly divisible by 4. Thus, the years 1800 and 1900 were not leap years, but the year 2000 will be a leap year. The Gregorian Calendar is not perfect, although its error amounts to only about 2 days in 6000 years.

The first year of the Julian Calendar was made unduly long and it was known as the Year of Confusion. But the Roman writer Macrobius referred to it as the Last Year of Confusion. The first Julian Year commenced on the day of the first New Moon following the Winter Solstice of the Year.

The year 1752, when the Gregorian Calendar was adopted in Britain, was made several days short—the date jumping from the 3rd to the 14th of September. This brought the next Spring Equinox to the 21st of March. It is interesting to reflect that at so recent a date—only about 8 generations ago—there were folk who genuinely thought that by altering the calendar at the time, they were being deprived of 11 days of their lives.

The ancient Roman calendar was essentially a Lunar Calendar. The months commenced on the days of the New Moon. These days were known as the Calends, and the days of Full Moon, which occurred on the 14th or 15th day of each month, were known as the Ides.

The Moon is still used in the Ecclesiastical Calendar. The phase of the Moon is used in connection with the calculation of the date of Easter, which is the most important date in the Christian Calendar. It is from the date of Easter that the other important Christian feast days are calculated. Easter Day, in general, falls on the first Sunday after the Full Moon which follows the Spring Equinox. Easter therefore, must fall between March 21st and April the 27th. The date of the appropriate Full Moon is calculated from the Metonic Cycle, and it is sometimes different from the date of the astronomical Full Moon.

The Metonic Cycle is named after the Athenian astronomer Meton who first discovered it. It is a period of about 19 years after which the Moon's phases recur on the same day of the Solar Year. The number of the year in the Metonic Cycle is known as the Golden Number.

The Golden Number may be found by adding 1 to the year number and dividing the result by 19: the remainder is the Golden Number. This is used in conjunction with the Sunday or Dominical Letter, for determining the date of Easter. If the days of the week are lettered from A to G starting with A on the first of January, the letter for Sunday will change each year. The letter is called the Sunday or Dominical Letter. Leap Years have two Sunday Letters: one for the period up to the Leap Year Day, and the other for the remainder of the year.

Exercises on Chapter 29

1. Describe how the common units of time, namely the Day and the Year, are established.
2. What angles on the celestial sphere correspond to Solar Time and Sidereal Time at a given instant?
3. When do (*a*) the Solar Day, and (*b*) the Sidereal Day, commence.
4. Why is the length of the Apparent Solar Day not constant?
5. Explain why a Solar Day is longer than a Sidereal Day. Prove that the number of Sidereal Days in a year is exactly one more than the number of Solar Days in the year.
6. Explain clearly the derivation of a Mean Solar Day.
7. Explain the effect on daily life were Sidereal instead of Solar Time used as the basis of everyday time-keeping.
8. What is the Equation of Time? Prove that the Equation of time (e) is given by:

$$e = \text{H.A.M.S.} - \text{H.A.T.S.}$$

9. Explain why it is necessary to know the G.M.T. and the L.M.T. for a given instant in order to find Longitude.
10. Explain each of the two components which combine to form the Equation of Time.
11. Compare the lengths of the Mean Solar Day and the Sidereal Day.
12. Explain how time is kept on board ship.
13. Explain Zone Time. If a ship is in Longitude 132° E. and her Zone Time is 1300 hr. show that the G.M.T. at the same instant is 0400 hr.
14. What is meant by Standard Time?
15. Explain the Calendar in current use.
16. Which of the following are not Leap Years: 1600, 1800, 1904, 1944, 1972? (*Answer*—1800).
17. Prove that: G.H.A.$\star$ − L.H.A.$\star$ = Longitude of observer, where $\star$ denotes any heavenly body.
18. Compare the lengths of the Sidereal Year, the Tropical Year and the Anomalistic Year, in units of Mean Solar Time.
19. Show that when a Sidereal Clock and a Mean Solar Clock, both set and keeping perfect time, register 00 h. 00 m. 00 s., that the date is September the 23rd.
20. Show that when a Mean Solar Clock registers 12 h. 00 m. 00 s. at the same instant as a Sidereal Clock registers 13 h. 00 m. 00 s., the date is October 23rd, approximately.
21. Describe the motions of the Dynamical Mean Sun and the Astronomical Mean Sun.
22. Show that the Equation of Time is zero on four days each year.
23. What effect has the equation of time on the lengths of the forenoon and afternoon?
24. Write a brief account of U.T.C.

CHAPTER 30

THE MOON

1. The Moon and its Motions

The Earth's satellite, the Moon, is the closest celestial body to the Earth. Its diameter is about 2000 miles, this being about a quarter of the Earth's diameter. The Moon has the distinction among the satellites of the Solar System of being one of the nearest in size to its parent planet.

The Moon is often said to revolve around the Earth. More strictly the Earth and the Moon revolve around their common centre of gravity. This point, known as the Barycentre, lies on the straight line joining the centres of the Earth and Moon, and is located at a point about 1000 miles within the Earth.

The Moon's orbit is elliptical: its nearest approach to the Earth is about 222,000 miles, and its most remote point from the Earth lies at about 253,000 miles from the Earth. The names given to the points of nearest approach and greatest distance are Perigee and Apogee respectively.

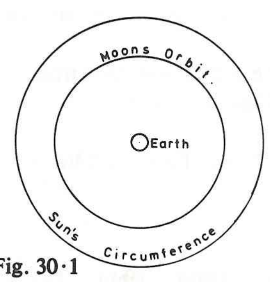


Fig. 30·1

Fig. 30·1 illustrates the relative dimensions of the Earth, Moon's Orbit, and the Sun. It is interesting to note that the diameter of the Sun is considerably greater than the diameter of the Moon's orbit.

As the Moon moves in its orbit it describes a great circle on the celestial sphere against the background of the fixed stars. It moves about 13° per day to the eastwards. This comparatively rapid motion may easily be observed in a relatively short space of time. In an hour, for exmple, the Moon moves through an arc of the sky, relative to the stars, approximately equal to its angular diameter.

To complete a circuit around the celestial sphere with respect to the stars, the Moon takes about 360° / 13 days. The actual time is $27^1/_3$ days, and this period is known as a Sidereal Period.

The easterly motion of the Moon relative to the stars causes the Moon to rise, culminate and set, later each day to the extent of an average interval of 50 minutes of time. This interval is known as the Retardation of the Moon's Rising and Setting.

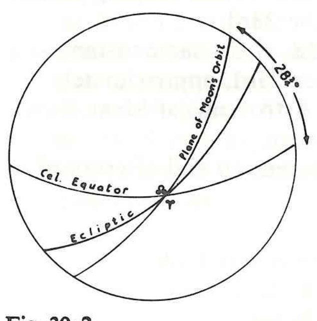


Fig. 30·2

The plane of the Moon's orbit is inclined at an angle of about 5¼° to the plane of the eliptic. The points on the celestial sphere where the two planes intersect are referred to as the Nodes of the Moon's Orbit. The node at which the Moon is located when it crosses from south to north of the ecliptic, in so doing changing her Celestial Latitude from South to North, is known as the Ascending Node. The other node is called the Descending Node.

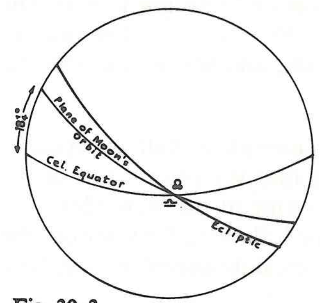


Fig. 30·3

The straight line, known as the Nodal Line, which joins the nodes of the Moon's orbit, has a retrograde motion along the ecliptic similar to the precession of the equinoxes. The nodes perform a complete revolution around the celestial sphere in 18·6 years. This is an important eclipse period known as the Saros. The effect of the nodal motion is that the limits of the Moon's declination in any given sidereal period change periodically with a period of 18·6 years. The limits of the Moon's declination during a given sidereal period are dependent upon the relative positions of the Nodes and the Equinoxes.

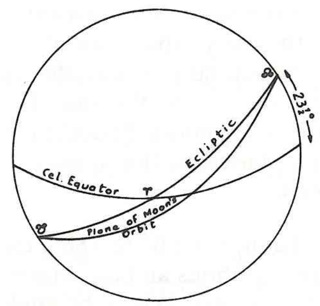


Fig. 30·4

Figs. 30·2, 30·3 and 30·4, illustrate the varying maximum values of the declination of the Moon during particular sidereal periods.

When the Ascending Node is at the Spring Equinox, as illustrated in fig. 30·2, the Moon's declination varies between (23½ + 5¼)°, that is 28¾° N. and S. during the sidereal period.

When the Ascending Node coincides with the Autumnal Equinox, as illustrated in fig. 30·3, the Moon's declination varies between (23½ − 5¼)°, that is, 18¼° N. and S.

When the nodes coincide with the Solstitial Points, as illustrated in fig. 30·4, the Moon's declinational limits are the same as those of the Sun, that is to say they are 23½° N. and S.

It is to be noted that, whatever may be the relative positions of the Moon's Nodes and the Equinoxes, the average rate of change of the Moon's declination is very rapid.

2. The Phases of the Moon

The Moon is rendered visible by reflected sunlight. The proportion of the Moon's illuminated hemisphere, visible at the Earth at any time, depends upon the relative positions of the Earth, Moon and Sun. The changing shapes of the part of the Moon's surface visible at the Earth are known as the Phases of the Moon.

When the Moon is in conjunction with the Sun, its illuminated hemisphere is directed away from the Earth. In this circumstance no part of the Moon's illuminated hemisphere is visible at the Earth. At this time the Moon and the Sun cross an observer's celestial meridian at the same instant of time. It follows that the time at which the Moon is in conjunction with the Sun must be 12 o'clock L.A.T. When this occurs the Moon is said to be New or at the Change, and its Age is said to be 00 d. 00 h. 00 m. 00 s.

When the Moon is in opposition with the Sun, its illuminated surface is facing the Earth. The whole of the Moon's illuminated hemisphere is, therefore, visible at the Earth. At this time the Moon appears as a disc of light and its phase is said to be Full. The Full Moon occurs when the Sun is on an observer's lower celestial meridian, the Moon being on the observer's upper meridian at the time. Full Moon, therefore, occurs at Midnight L.A.T. The angle between the Moon and the Sun, measured in the plane of the ecliptic, at the time of Full Moon, is 180°, so that the Moon rises at the time the Sun sets, and sets at the time the Sun rises.

From the time of New Moon to that of Full Moon the Moon completes half the cycle of her phases. During this time the Moon is said to Wax, meaning that the proportion of the Moon's illuminated surface visible at the Earth increases from nothing at the New Moon, to a full disc at Full Moon. From the time of Full Moon to that of the following New Moon, the Moon is said to Wane, for the reason that the proportion of its illuminated hemisphere visible on the Earth diminishes from maximum to zero.

It will be remembered that because of the Earth's orbital motion the Sun moves eastwards across the celestial sphere at the rate of about 1° per day. Thus, the daily separation of the Sun and Moon amounts to about 12° per day. It follows that the Moon takes about 360/12 days, approximately, to complete a circuit of the celestial sphere relative to the sun. The actual time is about 29½ days—an interval known as Lunation or Synodic Period. The Lunation is the time taken for the Moon to complete a cycle of its phases, so that it may be defined as the interval of time between two successive New Moons.

As the angle between the Moon and Sun increases after New Moon, from 0° to 180°, the Moon's visible shape appears successively as a crescentic, half Moon, gibbous and full Moon. During this half lunation the Moon rises after Sunrise and sets after Sunset. When the angle between the Moon and Sun is 90°, during the first half of a lunation, the Moon is said to be at the First Quarter. At this time the Moon rises at about 6 hours after the time of sunrise. It, therefore, crosses the upper celestial meridian of an observer at about 6 o'clock in the evening.

During the second half of a lunation the Moon rises before sunrise and sets after sunset. When the Moon is midway between Full and New, that is to say when it is at quadrature during the second half of the lunation, it is said to be at the Third Quarter. At this time the Moon rises at about six hours before sunrise, so that it crosses an observer's upper celestial meridian at about 6 o'clock in the morning.

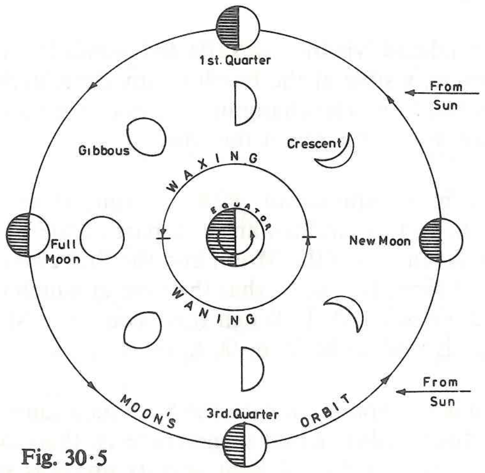


Fig. 30·5

The phases of the Moon are illustrated in fig. 30·5.

3. The Age of the Moon

The Age of the Moon is the interval of time that has elapsed since the last New Moon.

Age at New Moon = 0 days exactly
Age at 1st Qr. = 7 days approximately
Age at Full Moon = 15 days approximately
Age at 3rd Qr. = 22 days approximately

The Age of the Moon on the 1st of January is known as the Epact for the year. The Epact is used in the computation of the date of Easter. Because 12 lunations amount to 354 days, which is about 11 days short of the year, the Epact increases by 11 on successive years.

4. Winter and Summer Full Moons

Because the Sun moves in ecliptic, successive Full Moons take place in different parts of the celestial sphere. This has an effect on the duration of Moonlight.

In Summer, when the Sun has northerly declination, the Full Moon has southerly declination. In the northern hemisphere celestial bodies which have south declination are above the horizon for less than half a day. Therefore the Summer Full Moons are below the horizon for more than half the day.

In Winter, when the Sun has southerly declination, the Full Moons have northerly declination. They are, therefore, above the horizon for more than half the day. During the long Winter nights of the northern hemisphere the Full Moon is said to "ride high in the sky".

5. Spring and Autumn Full Moons

In Spring in the Northern Hemisphere, when the Sun is near the First Point of Aries, the Full Moons are near the First Point of Libra. In Autumn, when the Sun is near Libra, the Full Moons are near Aries. In September, the Moon's declination changes most rapidly during the lunation because it crosses the ecliptic.

In Spring, when the Sun's declination changes from South to North, the Full Moon's declination changes from North to South. This has the effect of speeding up the time of Moonset. It will be remembered that celestial bodies which have south declination are above the horizon of an observer in the northern hemisphere for less than half the day, and that the greater is the South declination the shorter is the period the body is above the horizon.

In Autumn the Full Moon's declination changes from South to North as the Sun's declination changes from North to South. The time of the setting of the Full Moon is, therefore, advanced. The retardation of the Moon's rising and setting, therefore, is offset by the northerly change in its declination. Therefore, in the Autumn, the Moon when near Full rises only slightly later on successive days. The interval between the times of sunset and moonrise is relatively small for several days near the time of Full Moon. Hence, before darkness sets in, the large Moon rises, and the reflected sunlight assists the northern farmers in the labour of the harvest. For this reason the Full Moon occurring nearest to the time of the Autumnal Equinox is called the Harvest Moon.

A similar phenomenon occurs during the few days on each side of the day of Full Moon following the Harvest Moon. At this time of the year, the harvest having been gathered, the sporting activities of the farmers are assisted by moonlight, and the Full Moon following the Harvest Moon is called the Hunter's Moon.

6. Earth-Shine

The Earth as observed from the Moon passes through phases which recur every 29½ days exactly as the Moon's phases. When the Moon is New at the Earth, the Earth is "New" at the Moon. At the time of Full Moon an observer on the Moon would observe "Full Earth". The Earth being a better reflector, area for area, than the Moon, and because of the Earth's atmosphere, the Earth is a magnificent spectacle to the intrepid space travellers who orbit the Earth or who land on the Moon.

When the Moon is crescent-shaped the portion of its surface which is not illuminated is facing the Earth. This dark hemisphere of the Moon, on those occasions when the sky is cloud-free and clear, may be observed to be slightly illuminated by sunlight which has been reflected from the Earth, and re-reflected from the Moon's surface. This illumination is called Earth-Shine, and the spectacle gives rise to the phenomenon, due to irradation, known as "The Old Moon lying in the New Moon's arms".

7. Moon's Librations

The Moon rotates on its axis once in a sidereal period, and the speed of this rotation is virtually uniform. Because of this, the same face of the Moon is always presented to the Earth.

Now although the rotation of the Moon takes place at a uniform rate, that of the Moon's orbital motion is not uniform. In obedience to Kepler's Second Law the Moon travels fastest when it is at perigee and slowest when it is at apogee. The average orbital speed is exactly equal to the uniform rotation speed. Thus, at perigee when the orbital speed is greater than the axial speed, a narrow strip of the Moon's surface beyond its normal western edge heaves into view. At apogee the reverse in the case and, in consequence, a part of the Moon's surface beyond its normal eastern edge becomes visible to terrestrial observers.

The Moon's axis of rotation is inclined to the plane of its orbit at an angle of about 84°. As the Moon revolves in its orbit, therefore, a continually changing aspect is presented to terrestrial observers. The Moon seems to "nod" to and fro once in a sidereal period. This allows observers on the Earth to observe parts of the lunar surface extending to as much as 6°—the complement of the angle of inclination of the Moon's spin axis to the plane of the Moon's orbit—over the Moon's poles.

These apparent irregularities in the Moon's motions are known as Librations. They result in about 59% of the Moon's surface being observable from the Earth.

8. Eclipses

An eclipse of the Sun occurs when the Moon lies on the straight line joining the Earth and

Sun. Because the Moon and the Earth are opaque bodies they cast shadows away from the direction of the Sun. At a Solar Eclipse the shadow of the Moon falls on the Earth.

An eclipse of the Moon occurs when the Moon passes through the shadow of the Earth.

Were the Earth's and Moon's orbits co-planar there would be an eclipse of the Sun at every New Moon and an eclipse of the Moon at every Full Moon. But the plane of the Moon's orbit is inclined at an angle of about 5¼° to the plane of the Earth's orbit. The declinations of the New and Full Moons are not, therefore, usually the same in magnitude as that of the Sun, so that eclipses of the Sun do not take place at every New Moon, and eclipses of the Moon do not take place at every Full Moon.

For an eclipse to occur the Moon must lie on the ecliptic: hence the name given to the great circle on the celestial sphere which is co-planar with the Earth's orbit. When the Moon is on the ecliptic it is at one of its nodes. So that for an eclipse to occur the Moon must be at or near a node and it must lie on the straight line joining the Earth to the Sun.

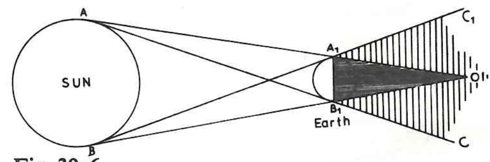


Fig. 30·6

In fig. 30·6 the two tangents AA_1 and BB_1 meet at the point O, and the cone of section A_1OB_1 is a region within which no sunlight enters. This dark shadow cone is known as the Umbra.

The tangents BA_1 and AB_1 mark out the plane section of the cone within which partial sunlight enters. This region is known as the Penumbra.

If the Moon enters the penumbra it is so slightly obscured that it is hardly noticeable. But if the Moon enters the umbra it is eclipsed. Sometimes the Moon does not pass entirely into the umbra, in which case a Partial Eclipse occurs. Should the Moon pass completely into the umbra a total Eclipse occurs.

During a total lunar eclipse the Moon is sometimes visible as a dull red disc—like a blood-drop hanging in the night sky. This is due to sunlight being refracted on passing through the Earth's atmosphere, and the red constituents of which, being refracted to a smaller degree than the higher frequency constituents, impinge upon and illuminate the eclipsed Moon.

A lunar eclipse is visible at all places which have the Moon above the horizon during the time it is in the Earth's shadow.

Because the Moon and the Sun have approximately the same angular diameters, the length of the Moon's shadow cone is roughly equal to the radius of the Moon's orbit. Thus, during a solar eclipse, the apex of the Moon's shadow cone sweeps out a narrow belt on the Earth's surface within which the eclipse is visible. Because of the eccentricities of the Earth's and Moon's orbits, it sometimes happens that the apparent diameter of the Moon is greater than that of the Sun. This is a case of the Earth being near aphelion at the same time as the Moon is near its perigee. A solar eclipse occurring in these circumstances is known as a Total Solar Eclipse, in which the whole of the Sun's disc is obscured during the period of totality. An eclipse of the Sun which occurs when the Moon's apparent diameter is smaller than that of the Sun's, is known as Annular Eclipse. During an annular eclipse a narrow ring, or annulus, of the Sun's surface is visible during the eclipse. Such an eclipse occurs when the Moon is near apogee at the same time the Earth is near perihelion.

An eclipse of the Sun is visible only within a narrow strip of the Earth's surface, the width of the strip depending upon the relative distances of the Moon and Sun. The strip is never more than about 170 miles in width. Within the penumbra, which may extend over a circular area of the Earth of radius 2000 miles, a Partial Eclipse of the Sun occurs.

Fig 30·7 illustrates a Total Solar Eclipse.

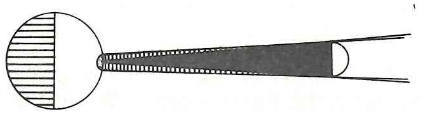


Fig. 30·7

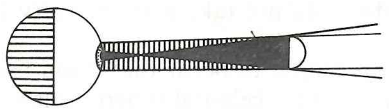


Fig. 30·8

Fig. 30·8 illustrates an Annular Eclipse.

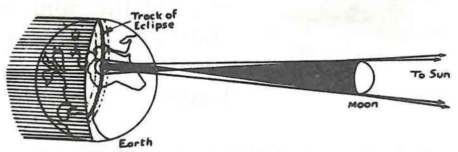


Fig. 30·9

Fig. 30·9 illustrates the track on the Earth of a Solar Eclipse.

9. Occultations

During the Moon's monthly circuit of the heavens it frequently passes over stars, and sometimes over planets. When this happens an Occultation takes place. For an occultation to occur the Moon and the occulted body must have the same celestial position.

It is interesting to observe an occultation of a star using the long glass or a good pair of binoculars. It will be seen that the star vanishes abruptly behind the edge of the Moon and reappears on the western edge, just as abruptly as it disappeared at the eastern edge some minutes before. The fact that a star disappears abruptly when being occulted by the Moon serves to show that the Moon is devoid of an atmosphere.

An occultation of a planet by the Moon is not nearly so dramatic as a star occultation: because of the planet's appreciable angular diameter it fades out gradually on being occulted.

Very occasionally a planet and a star occupy the same position in the celestial sophere. When this happens the star is said to be occulted by the planet.

Exercises on Chapter 30

1. Distinguish between a Sidereal Period and a Synodic period of the Moon.

2. Explain a Lunation.
3 Describe the phases of the Moon and explain their cause.
4. Explain why the maximum declination of the Moon varies from one lunation to another.
5. Define Ascending Node and Descending node.
6. In what circumstances would the Moon have a maximum declination during the lunation of 28¾° N. and S.?
7. At what latitude would an observer be able to see the Moon on the horizon at lower meridian passage at a time when the nodes coincide wth the Solstices?
8. Explain clearly why it is that the time of Moonrise occurs later each day by an amount known as the Retardation of the Moon.
9. What is meant by the Age of the Moon?
10. What is meant by the term Epact? If the epact is 4 on a certain year what is it on the following year?
11. Explain the expression: "The Full Moon rides high in Winter".
12. Explain the expression: "The Old Moon lies in the New Moon's arms".
13. Explain the Harvest Moon.
14. Explain Moon's Librations and indicate their effects.
15. Explain the causes of eclipses.
16. What conditions are necessary for a Solar Eclipse to occur?
17. What conditions are necessary for a Total Solar Eclipse, and for an Annular Eclipse of the Sun to occur?
18. What conditions are necessary for the period of totality of a Solar eclipse to be maximum?
19. What type of eclipse occurs if the Sun's S.H.A. and declination are equal to those of the Moon, and angular diameters of the Moon and Sun are 15·9′ and 16·2′ respectively.
20. Explain the appearance of the Moon during some total lunar eclipses.
21. Define Occultation. Describe the occultation of a planet by the Moon.

PART 5

NAUTICAL ASTRONOMY

Nautical Astronomy deals with the problems of finding a vessel's position when out of sight of land using astronomical principles.

The astronomical data required for solving the problems of Nautical Astronomy are contained in the *Nautical Almanac*, a very important instrument of nautical astronomy and one with which the navigator should be thoroughly familiar.

The basis of Nautical Astronomy is the relationship between a celestial body's position using co-ordinates of the horizon system with the body's position at the same instant of time using the co-ordinates of the celestial equatorial system.

The altitude of a celestial body, which is a co-ordinate of the horizon system, is measured by means of a sextant. The declination of an observed body, which is a celestial equatorial co-ordinate, may be lifted from the *Nautical Almanac*. Altitude, declination and latitude, are functions, respectively, of each of the three sides of a celestial triangle known as the Astronomical- or *PZX*-Triangle. In the general nautical astronomical problem a *PZX*-triangle has to be solved. The chapters in Part 5 of this book deal with the several aspects of this important branch of the practical work of a navigator.

The fundamental feature of modern Nautical Astronomy is the Astronomical Position Line. An astronomical position line is found from an astronomical observation or Sight, as such an observation is familiarly called. In by-gone days astronomical observations were made specifically for finding Latitude or Longitude. Nowadays, they are made essentially to determine astronomical position lines.

The astronomical methods of finding Latitude at sea have been used since the time when the Portuguese navigators of the Great Age of Discovery first applied astronomical methods for position-finding when out of sight of land.

Compared with the problem of finding Latitude at sea that of finding Longitude in early times was one of considerable complexity and difficulty. It was not until the mid-eighteenth century that a satisfactory solution of the longitude problem was discovered. Two methods for finding Longitude at sea became available at about the same time. The method most familiar to modern navigators requires the use of an accurate mechanical timekeeper. The first successful Chronometer, which is the name given to an accurate watch used for finding Longitude at sea, was the invention of an eighteenth-century horologist named John Harrison. Although Harrison's chronometer made its appearance during the early part of the eighteenth century, almost a century was to pass before these instruments were readily available on the score of expense. During this long period the second of the two general methods for finding Longitude at sea, known as the Lunar Method, was the more commonly used method.

The obsolete lunar method required the comparison of a measured Lunar Distance with predicted Lunar Distances given in the *Nautical Almanac*. The term Lunar Distance applies to the angle contained betwen the Moon and the Sun or some other celestial body lying in or near to the plane of the ecliptic. The early *Nautical Almanacs* were designed essentially for the lunar method of finding Longitude at sea. Since the early part of the present century, when predicted lunar distances ceased to be given, the *Nautical Almanac* has served essentially for the purpose of solving *PZX*-triangles.

G.M.T. is available to a navigator from his chronometer, the error of which may be obtained from Radio Time Signals transmitted at specified times throughout the day. The principle tables in the *Nautical Almanac* enable a navigator to find the declination and G.H.A. of any navigational celestial body for any given instant of G.M.T. We shall see in the following chapters how position lines are obtained from the remarkable and unfailing methods of Nautical Astronomy.

CHAPTER 31

FINDING THE TRUE ALTITUDE

1. The Altitude

The True Altitude of a celestial body is the angle at the centre of the Earth or an arc of a vertical circle contained between the body and the celestial horizon. A true altitude is found by applying certain Altitude Corrections to a measured altitude obtained by means of a sextant. The Sextant is the nautical astronomer's instrument for measuring arcs or angles. In Nautical Astronomy the principal arcs measured by means of a sextant are arcs of vertical circles.

The measured altitude is referred to as the Sextant Altitude. If the sextant possesses Index error (see Chapter 43), the sextant altitude must be corrected by applying a correction for the index error to give an angle known as the Observed Altitude. This is illustrated in fig. 31·1.

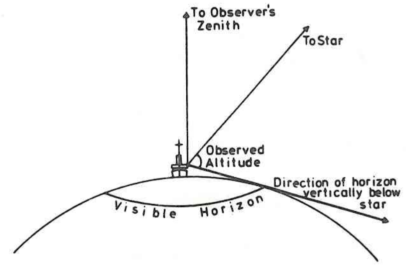


Fig. 31·1

The Observed Altitude of a celestial body is defined as the angle at the eye of the observer contained between the apparent directions, respectively, of the body and the visible horizon measured in the plane of the vertical circle on which the body lies. The distance of the visible horizon depends upon the height of the observer's eye above the level of the sea.

The Visible Horizon is a small circle on the Earth's surface which bounds an observer's view in the open sea. It may be defined as a circle every point on which the sea meets the sky. The radius of the visible horizon, as seen in fig. 31·2, increases as the height of the observer's eye increases.

In fig. 31·2, *B*'s visible horizon is a larger circle than *A*'s visible horizon. This follows because *B*'s eye is elevated to a greater extent than *A*'s.

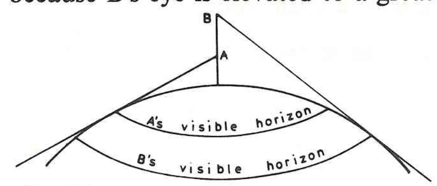


Fig. 31·2

An observer whose eye is at sea level has no visible horizon. The part of the celestial sphere which is visible to such an observer is bounded by a small circle on the celestial sphere which is parallel to the observer's celestial horizon, and on whose plane the observer's eye lies. This is called the observer's Sensible Horizon. It will be remembered that the celestial horizon is a great circle on the celestial sphere every point on which is 90° from the observer's zenith. The Earth's centre lies on the observer's celestial horizon.

2. Dip

The direction in any given vertical plane of the visible horizon is depressed below the direction of the sensible horizon in the same vertical plane by an angle which increases as the observer's height of eye increases. This angle of depression is known as Dip.

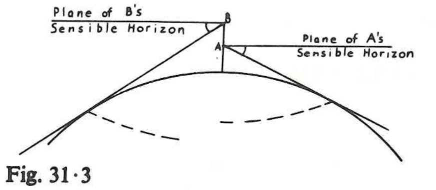


Fig. 31·3

In fig. 31·3 it will be seen that the dip of *B*'s visible horizon is greater than that of *A*'s. This follows because *B*'s height of eye is greater than *A*'s.

Ignoring the effect of atmospheric refraction the dip of the visible horizon in minutes of arc is equal to the distance of the visible horizon in nautical miles. This is proved with reference to fig. 31·4.

In fig. 31·4:

AB is the vertical height of the observer's eye above sea level.
AV is the distance of the observer's visible horizon in nautical miles.

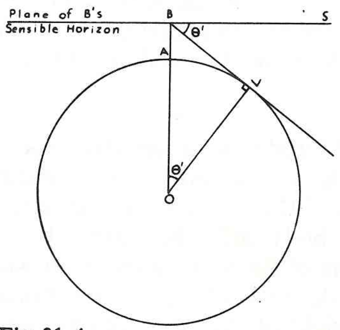


Fig. 31·4

arc $AV = \theta =$ Distance of visible horizon in miles
$SBV =$ Dip in minutes

In triangle *BOV*:
$BVO = 90°$
$BOV = \theta'$
$OBV = (90° - \theta')$

But:
$SBO = 90°$

Therefore:
$SBV = \theta'$

Thus:

Dip of visible horizon in mins. = Distance of visible horizon in mls.

In Part 3, Chapter 19, we proved that the distance of the theoretical horizon is equal to $1{\cdot}06\sqrt{h}$ miles, where *h* is the height of the observer's eye above sea level. Therefore:

$$\text{Dip of Theoretical Horizon} = 1{\cdot}06\sqrt{h}' \text{ of arc (in feet).}$$

$$\text{Dip of Theoretical Horizon} = 1{\cdot}93\sqrt{h}' \text{ (in metres).}$$

3. Refraction

The path of a ray of light, in passing from one medium to another of different optical density, changes its direction by an angle known as refraction. Refraction depends upon the relative optical densities of the two media and also upon the angle which the path of the ray of light makes with the common surface of the two media. When the ray strikes the surface

normally; that is to say, when the angle of incidence is 90°, refraction is zero. Refraction increases as the angle of incidence decreases.

A ray of light from a celestial body, in passing through the Earth's atmosphere, is refracted so that the path of the ray of light is a slight curve. The direction of this curved path at the surface of the Earth is such that the altitude of a celestial body is apparently greater than it would be if refraction did not exist.

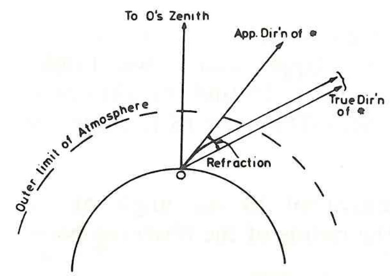


Fig. 31·5

Fig. 31·5 serves to illustrate that Atmosphere Refraction is equal to the angle between the true direction of a celestial body and its apparent direction.

The value of refraction depends upon the altitude of the celestial body. It is greatest for a body whose altitude is zero; that is to say, for a body on the horizon. Its maximum value is about 33′ of arc. Refraction of light from a body at the zenith; that is to say, a body whose altitude is 90°, is zero. This follows because the ray of light which enters the observer's eye from such a body, strikes the atmosphere normally.

Refraction is affected by changes in temperature and pressure of the atmosphere. In most collections of Nautical Tables, a table giving Mean Refraction is given for a so-called Standard Atmosphere in which the sea level temperature and pressure are assumed to have specified values. In addition to this table another is given in which are tabulated corrections to be applied to the Mean Refraction for temperatures and pressures different from those for which the Mean Refractions are computed.

Certain atmospheric and sea conditions may give rise to Abnormal Refraction. If this is suspected, results of observations of celestial bodies should be treated with caution.

4. Effect of Refraction on Dip

A ray of light from the sea surface at the visible horizon is refracted such that the distance of the visible horizon is slightly greater than that of the theoretical horizon. The distance of the visible horizon, when atmospheric refraction is normal, is about one-thirteenth of the theoretical distance greater than the theoretical distance. This gives $1{\cdot}15\sqrt{h}$ for the distance of the visible horizon. The actual dip of the visible horizon is less than the theoretical dip. When refraction is normal the dip of the visible horizon is $0{\cdot}98\sqrt{h}$'s of arc. The effect of refraction on dip and on distance of the horizon is illustrated in fig. 31·6.

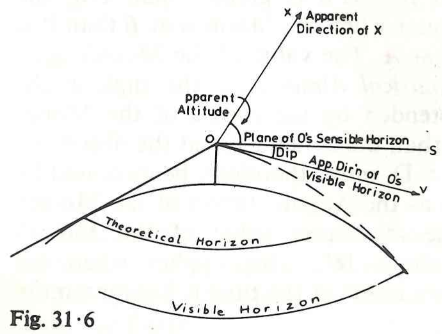


Fig. 31·6

Tables of Distance and Dip of the visible horizon are calculated from the formulae stated above and are inserted in most collections of Nautical Tables. A Dip table is also to be found in the *Nautical Almanac*.

The Observed Altitude of a celestial body, reduced by the dip of the visible horizon, gives the Apparent Altitude of the body. The Apparent Altitude of a celestial body is defined as the angle at the observer's eye contained between the apparent direction of the body and the plane of the observer's sensible horizon measured in the plane of the vertical circle on which the body lies.

In fig. 31·6 *XOV* is the observed altitude of the celestial body which lies in the direction indicated. *SOV* is the actual dip of the visible horizon, and *XOS* is the apparent altitude of the body.

5. Semi-Diameter

When the altitude of the Sun or the Moon is observed, the measured angle is the arc of a vertical circle between the top or bottom edge, known as the Upper and Lower Limbs, respectively, of the body and the vertical horizon vertically below it. To find the altitude of the centre of the Sun or the Moon, a correction known as Semi-Diameter (S.D.) must be applied to the altitude of the limb.

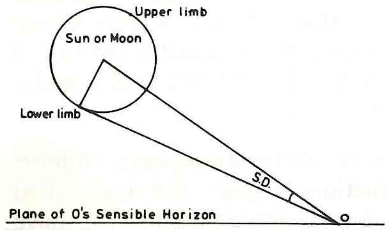


Fig. 31·7

Semi-diameter is equivalent to the angle at the observer subtended by the radius of the observed body, as illustrated in fig. 31·7.

It is to be noted that when the lower limb is observed, the S.D. correction is to be added to the altitude of the limb; and that when the upper limb is observed the S.D. correction is to be subtracted from the altitude of the limb.

The S.D. of both the Sun and Moon may be found from the daily pages of the *Nautical Almanac*. The Sun's S.D. varies between 15′·8 and 16′·3 during the course of the year: it is least when the Earth is at aphelion and greatest when the Earth is at perihelion, the dates of these events being, respectively, early July and early January. The Moon's S.D. varies between about 14′·7, when the Moon is at apogee, and about 16′·7 when the Moon is at perigee.

6. Augmentation of the Moon's Semi-Diameter

Because of the Moon's comparative nearness to the Earth, the distance of the Moon from an observer decreases appreciably as the Moon's altitude increases. This is illustrated in fig. 31·8.

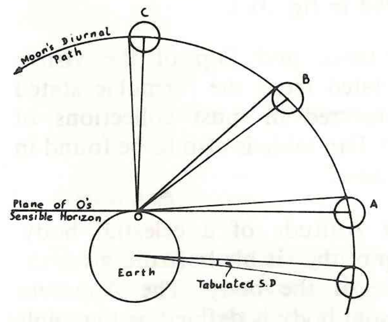


Fig. 31·8

Because *OA* in fig. 31·8 is greater than *OB*, the Moon's S.D. is greater when the Moon is at *B* than it is when the Moon is at *A*. The value of the Moon's S.D. tabulated in the *Nautical Almanac*, is the angle at the Earth's centre subtended by the radius of the Moon. This is always less than the actual S.D. at the observer's eye. The tabulated S.D. must, therefore, be increased by an amount known as the Augmentation of the Moon's semi-diameter. The maximum value of the Moon's augmentation is about 18″. This applies when the altitude of the Moon is 90° at the time it has maximum S.D.

The greatest S.D. for the day occurs when the Moon is at meridian passage; that is to say, at the instant when the Moon reaches its greatest altitude.

7. Parallax

Stars are so far distant from the earth that their directions measured from the earth's centre are sensibly the same as they are from any point on the Earth's surface.

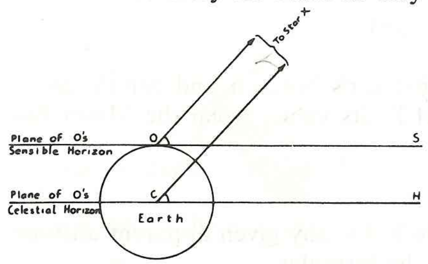


Fig. 31·9

In fig. 31·9:
Altitude of X above the sensible horizon $= XOS$
True Altitude of $X = XCH$
Because OX and CX are parallel: $XOS = XCH$

Therefore:
Altitude of X above sensible horizon = Altitude of X above celestial horizon.

Members of the Solar System, especially the Moon, are comparatively near to the Earth; and the respective directions of these bodies from the Earth's centre and the surface are not parallel as is the case with the fixed stars.

The true altitude of any body of the Solar System is always slightly greater than the altitude of the body above the sensible horizon by an angle which depends on:

(i) the distance of the body from the Earth
(ii) the apparent altitude of the body.

The angle by which the true altitude exceeds the altitude of the celestial body above the sensible horizon is known as Celestial Parallax. Parallax may be defined as the angle at the centre of the observed body contained between the respective directions of the observer and the centre of the Earth.

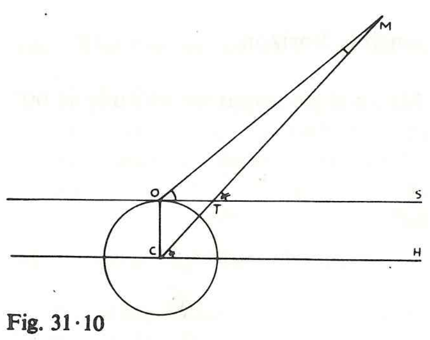


Fig. 31·10

Let M, in fig. 31·10, represent the Moon; O an observer; and C the centre of the Earth.

Altitude of M above sensible horizon $= MOS$
True Altitude $= MCH$
$= MTS$

Now: $MTS - MOS = OMT$

Therefore:

Alt. above Celestial Horizon − Alt. above Sensible Horizon = Parallax.

Parallax for a given celestial body is greatest when the body is on the sensible horizon of an observer. The value at this instant is known as the Horizontal Parallax (H.P.). The H.P. of any body may readily be deduced from the Earth's radius and the distance of the body from the Earth.

In fig. 31·11:

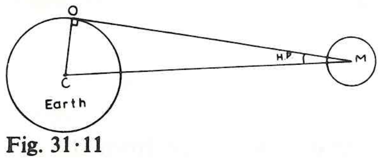


Fig. 31·11

Radius of the Earth = OC = 4000 ml. approx.
Distance of Moon = CM = 240,000 ml. approx.

$$\sin OMC = 4000/240{,}000 = 1/60 = OMC$$

Therefore:

$$\text{Moon's H.P.} = 1/60^c \text{ or } 57'{\cdot}3.$$

When the Moon is at any position between an observer's horizon and zenith, as at position B in fig. 31·12, the parallax is less than the H.P. Its value, when the Moon has altitude, is known as Parallax-in-Altitude.

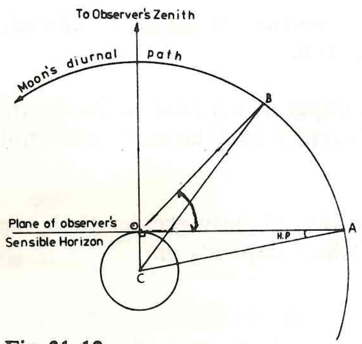


Fig. 31·12

Parallax-in-Altitude p, for any given apparent altitude A, can be found from the formula:

$$p = \text{H.P.} \cos A$$

This formula is proved with reference to fig. 31·12.

By the Plane Sine Formula:

$$\begin{aligned} \sin OBC/OC &= \sin COB/CB \\ \sin OBC &= OC/CB \,.\, \sin COB \\ &= OC/CB \,.\, \sin (90° + A) \\ &= OC/AC \,.\, \sin (90° + A) \end{aligned}$$

and: $\sin p = \sin \text{H.P.} \cos A$

Since p and H.P. are small angles, this formula reduces to:

$$p' = \text{H.P.}' \cos A$$

in which A denotes the altitude above the plane of the sensible horizon.

Example 31·1—Find the parallax-in-altitude of the Moon if its apparent altitude is 60° 00′, and its H.P. is 60′·2.

$$\begin{aligned} p &= \text{H.P.} \cos A \\ &= 60'{\cdot}2 \cos 60° \\ &= 60'{\cdot}2 \times ½ \\ &= 30'{\cdot}1 \end{aligned}$$

Answer—Moon's parallax-in-latitude = 30′·1.

8. Effect of Earth's Shape on Horizontal Parallax

The H.P. of the Moon, as tabulated in the *Nautical Almanac*, is the angle at the Moon's centre subtended by the equatorial radius of the Earth. Because of the oblate shape of the Earth, the Moon's H.P. for any other Latitude is slightly less than the tabulated value.

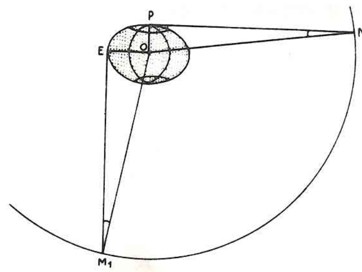


Fig. 31·13

In fig. 31·13, E denotes an observer on the equator. M_1 denotes the Moon on the observer's sensible horizon. The Moon's H.P. is angle OM_1E.

P denotes an observer at the Earth's north pole, and M_2 the Moon on his sensible horizon. The Moon's H.P., in this case, is angle OM_2P.

Because the Earth's equatorial radius is greater than her polar radius, the Moon's H.P. is greatest when the Latitude of the observer is 0°, and it decreases as the latitude increases.

The Reduction of the Moon's Horizontal Parallax for the Figure of the Earth, for all navigable latitudes, is tabulated in most collections of Nautical Tables. An examination of such a table will show that this reduction is never more than about 12″.

9. Irradiation

Irradiation is a physiological phenomenon in which a bright object viewed against a darker background appears larger than it is; and a dark object viewed against a lighter background appears smaller. The celestial bodies viewed against a relatively dark sky appear slightly larger than they really are because of irradiation. The visible horizon, on the other hand, because it is viewed against the relatively light sky, apppears to be depressed by this optical phenomenon.

When the Sun's lower limb is observed the two effects of irradiation of the Sun and the horizon, respectively, tend to neutralize each other, so that no resultant irradiation correction is neccessary. When, however, the Sun's upper limb is observed the two effects combine, and the irradation effect may be as much as a minute of arc or even more. In practice, therefore, the Sun's lower limb is to be preferred when observing the altitude of the Sun.

10. The Correction of Observed Altitudes

In the following examples altitudes are corrected using the Altitude Correction Tables given in Norie's or Burton's Nautical Tables. Many practical navigators use the tables given in the *Nautical Almanac*: readers are advised to familiarize themselves with these excellent tables.

11. Correcting Star Altitudes

Stars are so far distant from the Earth that they appear as mere pinpoints of light with no apparent size. Parallax is nil and so also is semi-diameter. The true altitude of a star is found by applying individual corrections for dip and refraction to the observed altitude.

All collections of Nautical Tables include a Star Total Correction Table which gives the combined values of dip and refraction against arguments Height of Eye and Observed Altitude. This table is invariably used in practice in preference to the individual correction tables.

Example 31·2—Find the true altitude of Aldebaran if its sextant altitude was 30° 21′·5, index error = − 1′30″, height of eye = 14 m. Use the individual correction tables.

Sextant Altitude =	30° 21′·5
Index Error =	− 1′·5
Observed Altitude =	30° 20′·0
Dip =	− 6′·6
Apparent Altitude =	30° 13′·4
Refraction =	− 1′·6
True Altitude =	30° 11′·8

Answer—True Altitude = 30° 11′·8.

Example 31·2—Find the true altitude of Sirius if the sextant altitude is 45° 38′·0, index error = nil, height of eye = 20·9 m. Use the total correction table.

Sextant Altitude =	45° 38′·0
Index Error =	− 0′·0
Observed Altitude =	45° 38′·0
Total Correction =	− 9′·0
True Altitude =	45° 29′·0

Answer—True Altitude = 45° 29′·0.

12. Correcting Planet Altitudes

The navigational planets are comparatively close to the Earth; but, because of their relatively small size, they are often assumed to be very small discs of light with no appreciable diameters. In practice, therefore, the true altitude of a planet is often found in exactly the same way as that of a star. If, however, great accuracy is required, parallax-in-altitude and a phase correction should be applied, in addition to corrections for dip and refraction. It will not, in general, be possible to apply these additional corrections without the aid of the *Nautical Almanac*.

The phase correction for a planet arises from the centre of the illuminated portion of the planet not coinciding with the centre of the planet's disc. The correction, which normally applies only to Mars and Venus, is a function of the relative positions of the Sun and the planet, and it varies as the cosine of the angle between the vertical circle through the planet and the great circle connecting the celestial positions of the Sun and the planet.

Example 31·4—Find the true altitude of Jupiter if its sextant altitude is 24° 50′·5, index error = + 1′·5, height of eye = 9·5 m.

Sextant Altitude =	24° 50′·5
Index Error =	+ 1′·5
Observed Altitude =	24° 52′·0
Dip =	− 5′·4
Apparent Altitude =	24° 46′·6
Refraction =	− 2′·1
True Altitude =	24° 44′·5

Answer—True Altitude = 24° 44′·5.

13. Correcting Sun Altitudes

The true altitude of the Sun is found by applying corrections for dip, refraction, semi-diameter; and, for upper limb observations, irradation (which should be assumed to be 1′·0) as well. If great accuracy is required a correction for parallax-in-altitude is also applied; but this correction, being negligibly small is usually ignored in practice.

Example 31·5—The sextant altitude of the Sun's lower limb was 30° 40′·5. Find the true altitude if the index error is nil, the height of eye is 12·3 metres and the date is 20th June.

Sextant Altitude =	30° 40′·5
Index Error =	− 0′·0
Observed Altitude =	30° 40′·5
Dip =	− 6′·2
Apparent Altitude of L.L. =	30° 34′·3
Refraction =	− 1′·6
True Altitude of L.L. =	30° 32′·7
Semi-Diameter =	+ 15′·8
True Altitude =	30° 48′·5

N.B.—Parallax-in-altitude for 20th June amounts to 0′·1.

If this is applied the true altitude is 30° 48′·6.

Answer—True Altitude = 30° 48′·6.

A Sun's Total Correction Table is provided in Norie's and Burton's Tables. This gives a combined correction for dip, refraction and a constant semi-diameter of 15·8′ and parallax in altitude. The table is entered with arguments Observed Altitude and Height of Eye. To allow for the variation in the Semi-diameter throughout the year an auxiliary table, known as the Monthly Correction Table, is provided.

For upper limb observations the Sun's diameter is first subtracted from the observed altitude, and this is used as an argument, with Height of Eye, in the Sun's Total Correction Table.

Example 31·6—The sextant altitude of the Sun's lower limb on 21st January was 35° 50′·5, index error = − 1′·5, height of eye = 16 metres. Find the Sun's true altitude using the total correction table from Norie's or Burton's Tables.

Sextant Altitude	=	30° 50′·5
Index Error	=	− 1′·5
Observed Altitude	=	35° 49′·0
Total Correction	=	+ 7′·6
Monthly Correction	=	+ 0′·4
True Altitude	=	35° 57′·0

Answer—True Altitude = 35° 57′·0.

Example 31·7—On 4th October the sextant altitude of the Sun's upper limb was 22° 03′·0, index error = 1′·0 off the arc, height of eye = 7·3 metres. Find the Sun's true altitude using the total correction table.

Sextant Altitude	=	22° 03′·0
Index Error	=	+ 1′·0
Observed Altitude	=	22° 04′·0
Dip	=	− 4′·8
Observed Altitude	=	21° 59′·2
Total Correction (UL)	=	− 18′·4
True Altitude	=	21° 40′·8

Answer—True Altitude = 21° 40′·8.

14. Correcting Moon Altitudes

The true altitude of the Moon is found by applying to the observed altitude, corrections for dip, refraction, semi-diameter and parallax-in-altitude. In practice it is not usual to apply correction for the reduction of the Moon's Horizontal Parallax and Augmentation of the Moon's Semi-diameter: these two minor corrections tend to neutralize each other—one being positive and the other negative.

Example 31·8—Find the true altitude of the Moon if the sextant altitude of the lower limb is 30° 40′·0, index error = 0′·5 on the arc, height of eye = 12·3 metres, Moon's H.P. and S.D. (from the *Nautical Almanac*) are, respectively, 56′·5 and 15′·4.

Sextant Altitude	=	30° 40′·0			
Index Error	=	− 0′·5			
Observed Altitude	=	30° 39′·5			
Dip	=	− 6′·2			
Apparent Altitude	=	30° 33′·3			
Refraction	=	− 1′·6			
		30° 31′·7			
Semi-diameter	=	+ 15′·4			
		30° 47′·1	Par-in-alt	=	H.P. cos A
Par-in-altitude	=	+ 48′·9		=	56′·5 × cos 30½°
				=	48′·9
True Altitude	=	31° 36′·0			

Answer—True Altitude = 31° 36′·0.

The above method of correcting the Moon's observed altitude is rarely used at sea. A Total Correction Table, such as that given in the *Nautical Almanac*, is normally used by practical navigators.

The Total Correction Table for Moon Altitudes given in Norie's and Burton's Tables is in two parts: one for Lower Limb, and the other for Upper Limb observations. It is constructed using a constant value 9′·8 of dip for the height of eye. Adjustments for heights different from that used in the construction of the table are given in an auxiliary table.

Although the Moon's H.P. and S.D. are constantly changing in magnitude, their values at all times are directly proportional to each other. The ratio between the Moon's S.D. and its H.P. is about 1 : 4 or, more accurately, 3 : 11. This is proved with reference to fig. 31·14.

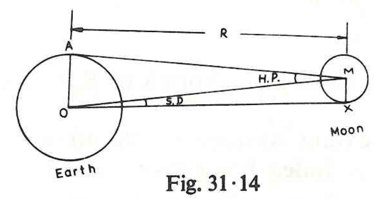


Fig. 31·14

In fig. 31·14:

Earth's radius = AO = 4000 ml. approx.
Moon's radius = MX = 1000 ml. approx.

Because angles AMO and MOX are small, we have:

AO/R = Moon's H.P. in radians
MX/R = Moon's S.D. in radians

Thus: $R = AO/\text{H.P.} = MX/\text{S.D.}$

Hence: $\dfrac{\text{Moon's S.D.}}{\text{Moon's H.P.}} = \dfrac{MX}{AO} = \dfrac{1000}{4000} = \dfrac{1}{4}$

This simple relationship facilitates the construction of the Moon's Total Correction Table.

Example 31·9—The sextant altitude of the Moon's upper limb was 30° 40′·0, index error = + 0′·5, height of eye = 9·3 metres, H.P. = 60′·0, S.D. = 15′·4. Find the Moon's true altitude.

(i) Using individual corrections.
(ii) Using Total Correction Table.

(i) Using Individual Corrections.

Sextant Altitude =	30° 40′·0		
Index Error =	+ 0′·5		
Observed Altitude =	30° 40′·5		
Dip =	− 5′·4		
Apparent Altitude =	30° 35′·1		
Refraction =	− 1′·6		
	30° 33′·5		
Semi-diameter =	− 15′·4	Par-in-alt =	H.P. × cos A
		=	60 × cos 30½°
	30° 18′·1	=	51′·7
Parallax-in-Alt. =	+ 51′·7		
True Altitude =	30° 09′·8		

Answer—True Altitude = 31° 09′·8.

(ii) Using Total Correction Table from Norie's or Burton's Tables:

Sextant Altitude =	30° 40′·0
Index Error =	+ 0′·5
Observed Altitude =	30° 40′·5
Total Correction =	+ 24′·8
Height Correction =	+ 4′·4
True Altitude =	31° 09′·7

Answer—True Altitude = 31° 09′·7.

Note—When using tables such as Altitude Correction Tables, it is very important to take care over the interpolation which is sometimes necessary if accurate results are required.

15. Back Angles

When a celestial body has an altitude of more than about 60° it is possible to measure with a sextant the obtuse angle in the vertical plane between the direction of the body and that of a point on the horizon which lies in a direction opposite to that of the body. This angle is called a Back Angle. Certain conditions may render it necessary or convenient to measure the back angle instead of the normally-measured "front" angle.

If the part of the horizon vertically below a body is indistinct, or if land lies in the same direction as that of the body, it may be necessary to observe the back angle instead of the altitude.

The following example illustrates how the true altitude of a celestial body is found from a back angle observation.

Example 31·10—The back angle of the Sun's upper limb was 118° 20′·0 by a sextant with no index error. Height of eye = 18·8 metres, Sun's S.D. = 15′·8. Find the Sun's true altitude.

Note—In a back angle observation of the Sun's upper limb the combined effect of Sun and Horizon-irradation may be taken as being nil.

In fig. 31·15:

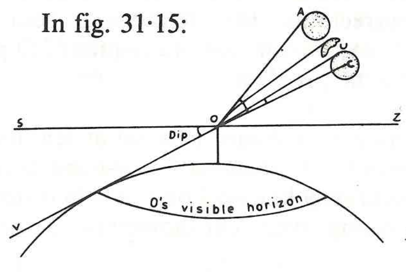


Fig. 31·15

Observed Altitude of the Sun's U.L. =	AOV
Dip =	SOV
Apparent Back Angle of the Sun's U.L. =	AOS
Apparent Altitude of the Sun's U.L. =	AOZ
Refraction =	AOU
Altitude of Sun's U.L. above Sensible Horizon =	UOZ
Semi-diameter =	UOC
Altitude of the Sun's centre above the S.H. =	COZ

Sextant Back Angle =	118° 20′·0
Index Error =	− 0′·0
Observed Back Angle =	118° 20′·0
Dip =	− 7′·6
Apparent Back Angle =	118° 12′·4
=	180° 0′
Apparent Alt. of L.L. =	61° 47′·6
Refraction =	− 0′·5
=	61° 47′·1
Semi-diameter =	− 15′·8
=	61° 31′·3
Parallax =	+ 0′·1
True Altitude =	61° 31′·4

Answer—True Altitude = 61° 31′·4.

16. The Artificial Horizon

If the visible horizon is not available because of fog or darkness, the true altitude of a celestial body may be found by employing an artificial horizon. The usual type of artificial horizon used ashore is a trough of opaque fluid, such as mercury or oil, which reflects the image of the observed body to the observer's eye. The observer measures with his sextant the angle between the true and reflected images of the body. The index error of the sextant, after being applied to the measured angle, gives an angle which is twice the apparent altitude of the observed body.

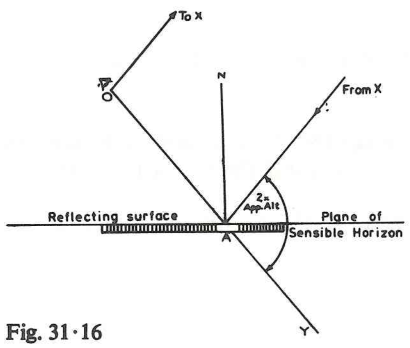


Fig. 31·16

In fig. 31·16 the ray of light from the celestial body X, which strikes the reflecting surface of the artificial horizon at A, is reflected to the observer's eye at O. The incident and reflected rays make the same angle with the normal AN, and the reflected image appears to lie in the direction OY. The angle XAY is the measured angle.

The artificial horizon lies in the plane of the observer's sensible horizon. Therefore, to find the true altitude, corrections for refraction, semi-diameter and parallax, are all that are required. Dip does not enter into the problem.

The mercury type of artificial horizon is practically useless on board a vessel at sea, on account of the inevitable vibration to which the mercury would be subjected. In by-gone days navigators often made observations ashore at places of accurately known Longitude in order to find the G.M.T., so as to provide a means of rating their chronometers. These observations were facilitated by the mercury horizon.

17. The Bubble Attachment

Some marine sextants are fitted with an Artificial Vertical in the form of a bubble. The Bubble Attachment enables a navigator to observe visible celestial bodies at night or in thick weather when the visible horizon is not available. Results obtained from using a bubble attachment are not as accurate as those obtained from good visual observations. Nevertheless, in some circumstances, such observations may provide the navigator with worthwhile fixes so alleviating possible anxiety. A Total Correction Table for use with Bubble Sextant Observations is included in Norie's or Burton's Nautical Tables. Instructions are also given in the *Nautical Almanac* for using the Altitude Correction Tables given therein for use with bubble sextant observations.

Exercises on Chapter 31

1. Define: Visible Horizon; Sensible Horizon; Celestial Horizon. Illustrate your definitions.
2. Define: True Altitude; Apparent Altitude; Observed Altitude.
3. What is Dip? Prove that dip of the horizon in minutes of arc is equal to the distance of the horizon in nautical miles, when atmospheric refraction is ignored.

4. Prove that: Theoretical Dip $= 1{\cdot}93\sqrt{h}$ where h is the observer's height of eye in metres.
5. What effect has atmospheric refraction on (i) dip of the visible horizon, (ii) distance of the visible horizon?
6. Compute the dip of the visible horizon for an observer whose height of eye is 127 metres.
7. Compute the distance of the visible horizon of an airman flying at 324 metres above the sea surface.
8. Define Mean Refraction. Using the "Correction to Mean Refraction Table" in Norie's or Burton's Tables, explain how atmospheric temperature and pressure affect refraction.
9. What is the maximum value of the atmospheric refraction for a standard atmosphere? What is refraction of light from a celestial body in the zenith?
10. What is Celestial Parallax? Why is Parallax-in-Altitude less than Horizontal Parallax?
11. Prove that: $p =$ H.P. . cos A, where p is parallax-in-altitude, and A is altitude.
12. Explain the effect of the Earth's shape on parallax. Define Equatorial Parallax.
13. Define Reduction to the Moon's Horizontal Parallax.
14. Define Augmentation of the Moon's Semi-diameter.
15. Why do the Sun's and Moon's S.D.s vary with time?
16. State the dates on which the Sun's S.D. is greatest and least during the year.
17. When correcting a sextant altitude, using individual corrections, what should be the order of applying the corrections in the case of (i) a star, (ii) the Sun, (iii) the Moon?
18. What is the relationship between the Moon's H.P. and S.D.? Explain how this facilitates the construction of the Total Correction Table for the Moon.
19. Explain why the Sun's diameter must be subtracted from an altitude of the Sun's Upper Limb before using the Total Correction Table for the Sun.
20. Explain irradation as it affects Sun observations of (i) the Lower Limb, (ii) the Upper Limb.
21. What is a "back angle"? State the circumstances when a back angle may be necessary.
22. Explain the artificial horizon.
23. No correction for dip is necessary when using an artificial horizon observation. Explain.
24. The sextant altitude of Antares was 35° 06′·0, index error = − 1′·0. Height of eye = 13·7 metres. Find the true altitude.
25. The sextant altitude of Fomalhaut was 36° 46′·5, index error = + 1′·5. Height of eye = 15·1 metres. Find the true altitude.
26. The sextant altitude of Sirius was 43° 43′·5, index error = 1′·5. off the arc. Height of eye = 15·4 metres. Find the true altitude.
27. The sextant altitude of Aldebaran was 50° 06′·5, index error = 2′·0 on the arc. Height of eye = 13·8 metres. Find the true altitude.
28. The sextant altitude of Polaris was 52° 41′·5, index error + 2′·0. Height of eye = 18·4 metres. Find the true altitude.
29. On 16th February, the sextant altitude of the Sun's lower limb was 32° 20′·5, index error nil, height of eye = 12·8 metres. Find the true altitude.
30. The sextant altitude of Mars was 25° 00′·5, index error 2′·0 off the arc. Find the true altitude given the height of the observer's eye = 12·2 metres.
31. Find the true altitude of the Sun on 21st July if the sextant altitude of the upper limb was 54° 54′·0, index error − 0′·5, height of eye = 18·4 metres.
32. The true altitude of a star was computed to be 36° 40′·0. Find the angle to set on a sextant if the index error is 1′·5 on the arc and the height of the observer's eye is 13·3 metres.
33. The true altitude of Canopus was computed to be 56° 40′·0. Find the angle to set on the sextant if the index error is 2′·0 off the arc and the observer's height of eye is 18·4 metres.

34. On 22nd March in cloudy weather, it is desired to set the sextant to the altitude of the Sun's lower limb for noon. The index error was 2′·0 off the arc; height of eye was 10·4 metres; and the computed true altitude was 34° 34′·0. Find the sextant altitude.
35. The sextant altitude of the Moon's lower limb was 42° 05′·5, index error nil, height of eye 17 metres and the Moon's H.P. was 56′·5. Find the true altitude.
36. The sextant altitude of the Moon's upper limb was 35° 53′·5, index error + 1′·5, height of eye 13·8 metres. Moon's H.P. = 60′·5, Moon's S.D. 15′·2. Find the true altitude.
37. The sextant altitude of the Moon's lower limb was 46° 54′·5, index error = − 2′·0, Moon's H.P. = 59′·8, Moon's S.D. = 15′·0. Height of eye = 21·4 metres. Find the true altitude.
38. On 5th October at 20 h. 00 m. G.M.T. the Moon's upper limb was 62° 08′·5 above the sea horizon as measured with a sextant the index error of which was 1′·5 off the arc. Find the Moon's true altitude if the height of the observer's eye was 6 metres. Moon's H.P. = 58′·4.
39. A back angle of Vega was 100° 42′·0, index error 0′·5 on the arc. Find the true altitude of the star if the observer's height of eye was 19·8 metres.
40. A back angle of the Sun's upper limb on 16th July was 115° 40′·0. Find the true altitude if the observer's height of eye was 9 metres.
41. Using an artificial horizon to observe Canopus the measured angle was 82° 45′·5, index error = − 0′·5, height of eye = 30·5 metres. Find the true altitude.
42. On 18th February the vertical angle between the Sun's lower limb and its reflection in an artificial horizon on shore was 96° 00′·5, index error 2′·0 on the arc, height of eye = 19·7 metres. Find the true altitude.

CHAPTER 32

THE ASTRONOMICAL POSITION LINE

1. The Geographical Position of a Heavenly Body

Imagine a straight line extending from a celestial body to the centre of the Earth. The point on the Earth's surface located on this imaginary line is known as the Geographical Position (G.P.) of the celestial body. If an observer is situated at the place at which the body is in his zenith, his terrestrial position coincides with the geographical position of the observed body.

Fig. 32·1 illustrates the celestial sphere with the Earth at its centre. *P* denotes the celestial pole and *p* the Earth's North Pole. Notice that *p* is at the G.P. of *P*.

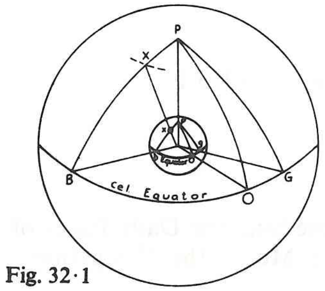


Fig. 32·1

pg represents the Greenwich Meridian
PG represents the Greenwich Celestial Meridian
po represents the Meridian of any Observer
PO represents the Observer's Celestial Meridian
X represents any celestial body
x represents the G.P. of X
px represents the Meridian of the G.P. of *X*.

Because the celestial meridian of *X* is in the same plane as the terrestrial meridian of the G.P. of *X*:

arc *BX* = arc *bx*

But: arc *BX* is the declination of *X*
and: arc *bx* is the Latitude of *x*
Therefore: Latitude of G.P. of *X* = Declination of *X*

From fig. 32·1 it will be seen that the angles *gpb* and GPB are equal.

Now: *gpb* is the West Longitude of the G.P. of *X*
and: *GPB* is the Greenwich Hour Angle of *X*
Therefore: West Longitude of G.P. of *X* = G.H.A. of *X*
= L.H.A. of *X* + W. Longitude of observer

The G.P. of any celestial body is, therefore, determined if the body's declination and G.H.A. are known. Provided that G.M.T. is known, the declination and G.H.A. of any navigational celestial body may be obtained from the *Nautical Almanac*.

Example 32·1—Find the G.P. of the Sun at 16 h. 00 m. G.M.T. on 30th December.

From *Nautical Almanac* Extracts:

Sun's declination = 23° 10′·3 S.
Sun's G.H.A. = 59° 22′·5
Therefore: Latitude of G.P. = 23° 10′·3 S.
Longitude of G.P. = 59° 22′·5 West of Greenwich

Answer—Lat. G.P. = 23° 10′·3 S.
Long. G.P. = 59° 22′·5 W.

Example 32·2—Find the G.P. of the Sun at 0430 L.M.T. on 22nd September. Observer's Longitude is 82° 30′·0 W.

L.M.T. = (22) 04 h. 30 m.
Long = 05 h. 30 m.
G.M.T. = (22) 10 h. 00 m.

From *Nautical Almanac* Extracts:

At 1000 hr. G.M.T. Sun's declination = 0° 26′·4 N.
Sun's G.H.A. = 331° 46′·7
Therefore: Lat. G.P. Sun = 0° 26′·4 N.
Long. G.P. Sun = 331° 46′·7 W, i.e. 28° 13′·3 E.

Answer—Lat. G.P. = 0° 26′·4 N.
Long. G.P. = 28° 13′·3 E.

In addition to tabulations of the declination and G.H.A. of the Sun, the Daily Pages of the *Nautical Amanac* give the declinations and G.H.A.s of the Moon; the Navigational Planets; and of the First Point of Aries (♈)

Example 32·3—Find the G.P. of the Moon at 1930 L.M.T. on 22nd September. Observer's Longitude is 52° 30′·0 E.

L.M.T. = (22) 19 h. 30 m.
Long = 03 h. 30 m. E.

G.M.T. = (22) 16 h. 00 m.

From the Nautical Almanac Extracts:

At 1600 hr. G.M.T. on 22nd September:

Moon's declination = 15° 04′·1 S.
Moon's G.H.A. = 299° 59′·6
Therefore: Lat. G.P. = 15° 04′·1 S
Long. G.P. = 299° 59′·6 W.
= 60° 00′·4 E.

Answer—Lat. G.P. = 15° 04′·1 S.
Long. G.P. = 60° 00′·4 E.

Example 32·4—Find the G.P. of Mars at 1800 hr. G.M.T. on 23rd September.

From the Nautical Almanac Extracts:

At 1800 hr. G.M.T. on 23rd September:

Declination of Mars = 18° 26′·6 N.
G.H.A. of Mars = 213° 13′·3
Therefore: Lat. G.P. Mars = 18° 26′·6 N
Long. G.P. Mars = 213° 13′·3 W.
= 146° 46′·7 E.

Answer—Lat. G.P. = 18° 26′·6 N.
Long. G.P. = 146° 46′·7 E.

To facilitate the problem of finding the G.P. of a navigational star, the G.H.A. of the First Point of Aries (G.H.A. ♈) is used. It is demonstrated in fig. 32·2 that the G.H.A. of a given star is equivalent to the sum of the G.H.A. ♈ and the Sidereal Hour Angle of Aries (S.H.A. ♈).

In fig. 32·2:

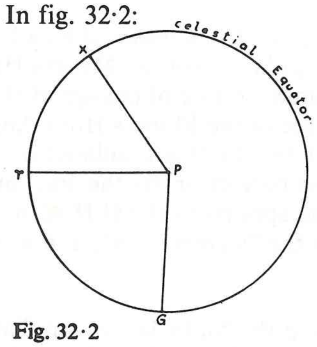


Fig. 32·2

P represents the Celestial Pole
PG represents the Celestial Meridian of Greenwich
P♈ represents the Celestial Meridian of ♈
PX represents the Celestial Meridian of a star ★

arc *G* ♈ = G.H.A. ♈
arc ♈*X* = S.H.A. ★
arc *GX* = G.H.A. ★
But: *GX* = *G*♈ + ♈*X*
Thus: G.H.A. ★ = G.H.A. ♈ + S.H.A. ★

Example 32·5—Find the G.P. of Aldebaran at 1600 hr. G.M.T. on 24th September.

From the *Nautical Almanac* Extracts:

At 1600 hr. G.M.T. on 24th September:

Declination of Aldebaran = 16° 25′·6 N.
G.H.A. ♈ = 242° 58′·7
S.H.A. Aldebaran = 291° 36′·6

Sum = 534° 35′·3
360° 00′·0

G.H.A. Aldebaran = 174° 35′·3

Therefore: Lat. G.P. Aldebaran = 16° 25′·6 N.
Long. G.P. Aldebaran = 174° 35′·3 W.

Answer—Lat. G.P. = 16° 25′·6 N.
Long. G.P. = 174° 35′·3 W.

In the above examples 32·1 to 32·5 inclusive, G.H.A.s and declinations were lifted direct from the appropriate Daily Page of the *Nautical Almanac* these quantities being tabulated for each integral hour of G.M.T.

When the G.M.T. is not an integral number of hours, recourse must be made to the Increment and Interpolation Table provided in the *Nautical Almanac*. Increments to the G.H.A.s of the Sun, Aries and the Moon, for each minute and second from 00 m. 00 s. to 60 m. 00 s. are tabulated in these tables.

The Hour Angle of the Mean Sun increases uniformly at the rate of 15° 00′·0 or 900′·0 per hour of Mean Solar Time. The Hour Angle of the True Sun increases at a variable rate, but the variation of this rate from its average rate of increase (which is equal to the rate of increase of the Mean Sun's Hour Angle), is very small. Allowance is made for this variation in the tabulated values of the Sun's G.H.A. The increment for the Sun's G.H.A. is, therefore, computed on the assumption that the True Sun's rate of change of Hour Angle is exactly 900′·0 per hour.

The Hour Angle of the First Point of Aries increases at the uniform rate of 15° 02′·5 per Mean Solar Hour, and the Increment Table for ♈ is computed for this rate.

The Hour Angle of the Moon, as well as that of any of the planets, increases at a variable rate, and it is necessary to allow for these variations in the interpolation for the Moon's Hour Angle. The Increment Table for the Moon is computed for a uniform rate of change of Hour Angle of 14° 19′·0 per hour. This is the minimum rate of increase of the Moon's Hour Angle. The excess of the Moon's hourly increase in hour angle over 14° 19′·0 is tabulated on the Daily Pages of the *Nautical Almanac* as "v". An additional correction to the increment obtained from the Moon's Increment Table is, therefore, to be applied to the G.H.A. of the Moon lifted from the Daily Page. This correction is known as the "v correction", and it is to be found from the Interpolation Tables.

The increment for the G.H.A. of a planet is found by using the Sun's Increment Table, and applying a "v correction" as in the case of the Moon. "v" for a planet, which is tabulated on the Daily Pages of the *Nautical Almanac*, is the excess, positive or negative, of the planet's hourly motion over 15° 00′·0.

When the G.M.T. is not an integral number of hours, and it is required to find the declination of the Sun, Moon, or a navigational Planet, it is necessary to apply a correction to the declination lifted from the Daily Page. To facilitate interpolating between tabulated values of the declination corresponding to G.M.T.s which embrace the given G.M.T., the differences between adjacent tabulated values of the declination are given. These differences are designated "d". The correction to apply to a tabulated value of declination found from the Daily Page is easily found from the Interpolation Tables, using as arguments "d" and the difference between the given G.M.T. and the lower tabulated G.M.T. The correction is known as the "d correction".

The quantity "d" is the mean hourly change in the declination of Moon, Sun or Planet. It is tabulated at one-hourly intervals for the Moon and at daily intervals for the Sun and Planets.

Example 32·6—Find the G.P. of the Moon at 21 h. 41 m. 45 s. on September 23rd.

From the Daily Page of the *Nautical Almanac* Extracts:

G.H.A. at 1400 G.M.T. =	0° 55′·6	v = 12′·7	Dec. =	11° 37′·8 S. d = 8.
Increment for 41 m 45 s. =	+9° 57′·7		Incr. =	− 5′·5
"v" correction =	+ 8′·8		Dec. =	11° 32′·3 S.
	001° 02′·1			
	360° 00′·0			
G.H.A. at 21 h. 41 m. 45 s. =	001° 02′·1			

Answer—Lat. G.P. = 11° 32′·3 S.
Long. G.P. = 01° 02′·1 W.

Example 32·7—Find the G.P. of Jupiter at 07 h. 42 m. 51 s. G.M.T. on 24th September.

From the Daily Page of the *Nautical Almanac* Extracts:

G.H.A. at 0700 G.M.T. =	256° 15′·2	v = 2′·0	Dec. =	11° 39′·5 S. d = 0′·2.
Increment for 42 m 51 s. =	10° 42′·8		Incr. =	+0′·1
"v" correction =	+ 1′·5		Dec. =	11° 39′·6 S.
	266° 59′·5			
	360° 00′·0			
	93° 00′·5			

Therefore: Lat. of G.P. of Jupiter = 11° 39′·6 S.
Long. of G.P. of Jupiter = 93° 00′·5 E.

Answer—Lat. G.P. = 11° 39′·6 S.
Long. G.P. = 93° 00′·5 E.

2. Circles of Equal Altitude

Imagine an observer to be at the G.P. of a star at a certain instant of time. The true Altitude of the star at the instant would be 90° 00′·0. Now suppose a second observer, to observe the same star at the same instant of time, and to find the true altitude to be 80° 00′·0. It follows that the second observer's zenith is 10° 00′·0 distant from that of the first observer, and that the second observer must be located on a small circle of radius 10° 00′·0 and centred at the first observer's position. Such a circle on the Earth's surface, at which at a given instant the altitude of a given celestial body is the same at all points on it, is called a Circle of Equal Altitude.

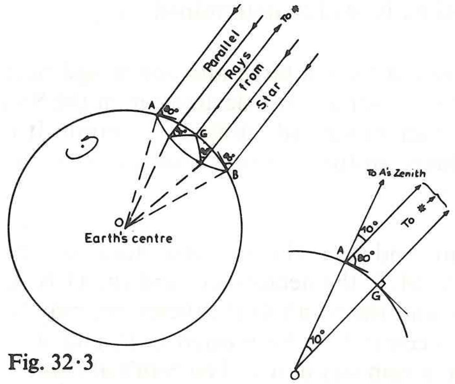


Fig. 32·3

The radius of a circle of equal altitude in nautical miles is equal to the number of minutes of arc in the angular distance of the observed celestial body from the observer's zenith. That is to say, it is equal to the complement of the altitude of the body, an angle known as the body's Zenith Distance.

Fig. 32·3 illustrates a circle of equal altitude of radius 10° or 600 miles.

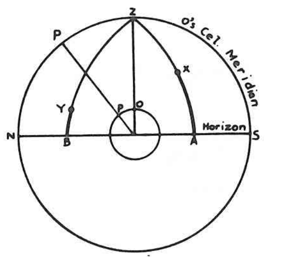


Fig. 32·4

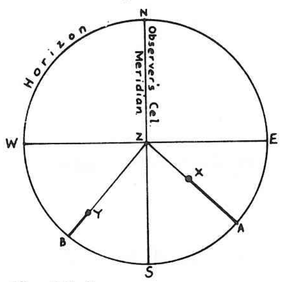


Fig. 32·5

Fig. 32·4 represents the celestial sphere projected onto the plane of the celestial meridian of an observer whose zenith is at *Z*. Fig. 32·5 represents the celestial sphere illustrating the same conditions as in fig. 32·4, but projected, in this case, onto the plane of the observer's celestial horizon.

In figs. 32·4 and 32·5:

AX = Altitude of X; ZX = Zenith distance of X
BY = Altitude of Y; ZY = Zenith Distance of Y;

If the zenith distance of a celestial body is small, and its declination also is small, the corresponding circle of equal altitude may be plotted on a Mercator Chart as a circle centred at the G.P. of the observed body. The observer's position may be fixed on such a circle, for which the reason it is known as a position Circle.

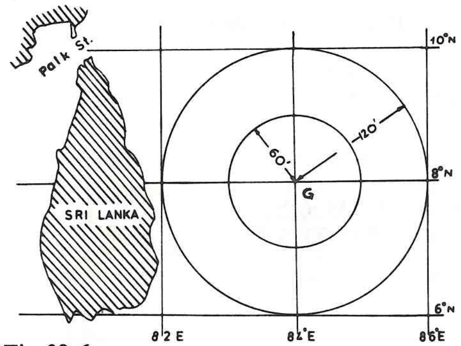


Fig. 32·6

Fig. 32·6 represents a portion of a Mercator Chart. Suppose the G.P. of the Sun at a given instant to be at *G* in Latitude 08° 00'·0 N., Longitude 84° 00'·0 E. If the True Altitude of the Sun at the instant is 89° 00'·0, the observer must lie on a circle of equal altitude centred at *G* and of spherical radius 1° or 60 miles.

The Latitude of the G.P. of a celestial body changes if the declination of the body changes. The G.P. of a star remains on the same parallel of Latitude because a star's declination does not change over short periods of time.

The Longitude of the G.P. of a Celestial body changes at the same rate as that of the body's Hour Angle. For the Mean Sun this is 360° per Mean Solar Day, or 15° per hour; 15' per minute or 15" per second of time. For a start it is slightly greater than this. The rate at which a body is changing its Hour Angle may readily be found from the *Nautical Almanac*, so that the rate of change of Longitude of a body's G.P. is readily determined.

It happens, when located in the tropics, that the Sun attains a large altitude at and near the time of its meridian passage. If the Latitude of the observer and the declination of the Sun are equal in name and magnitude, the Sun is at the observer's zenith at apparent noon. It is possible in a circumstance when the Sun has a very large altitude, to find a vessel's position very easily and very speedily in the following manner.

The Sun's altitude is measured with a sextant and the chronometer time of the observation noted, and the G.M.T. found. With the G.M.T. the declination and the G.H.A. of the Sun may be lifted from the *Nautical Almanac* and the Sun's G.P., therefore, may be plotted on the chart. A position circle is then drawn centred at the plotted G.P. and with radius in miles equal to the zenith distance of the sun in minutes of arc. The Sun's altitude is again observed two or three minutes after the first observation, and a second position circle is

drawn on the chart. The vessel's position is then fixed at the intersection of the two positions circles appropriate to the vessel's D.R. position, thus removing the ambiguity arising for the circles intersecting at two positions.

The timing of the observations must be such that the two position circles intersect at a good angle of cut. Three observations produce three position circles which intersect at a common point so revolving any ambiguity. If the interval between observations is more than a few minutes it may be necessary to transfer the first position circle for the run between the sights.

This method of finding a vessel's position has severe limitations. It fails at times when considerable error would result from the distortion of the plotted circles of equal altitude, which should be assumed to be circles on a Mercator Chart only in cases in which the zenith distance of the observed body is small and the Latitude of the G.P. of the body is also small. The method does not, therefore, solve the general problem in Nautical Astronomy, in which the zenith distance of the observed body is generally many tens of degrees, and the radius of the corresponding circle of equal altitude many hundreds, or even thousands of miles.

Any small arc of a circle of equal altitude when ploted on a Mercator Chart may be assumed to be a straight line lying at right angles to the direction of the observed body from the observer's position. Such a line is known as an Astronomical Position Line.

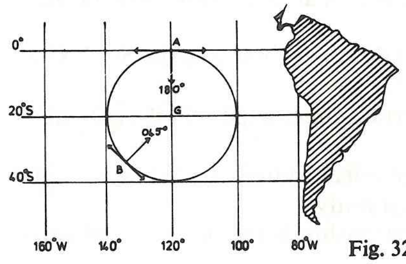


Fig. 32·7

In fig. 32·7 an observer located at *A* observes the body whose G.P. is at *G* to bear due South. The position line obtained from this observation lies 090°–270°. An observer at *B* lies on a position line the direction of which is 135°–315°. This is so because the G.P. of the observed body bears 045° from the observer.

Suppose that at a certain time an observer ascertains the zenith distances of two celestial bodies *A* and *B* whose G.P.s are located at *a* and *b* respectively. Fig. 32·8 illustrates the two circles of equal altitude corresponding to the two observations. The observer's position, which must lie on both circles, must be at *X* or *Y*. No difficulty is experienced in deciding which of these two possible positions is the position of the observer.

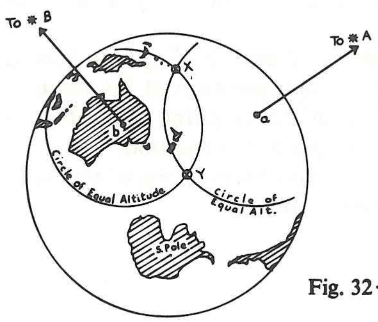


Fig. 32·8

The general problem in Nautical Astronomy involves:

(i) Finding the latitude and longitude of a point on a circle of equal altitude and:

(ii) Finding the Direction of the circle at this point.

With this information a position line is determined, and the vessel's position is located on it.

The various methods of ascertaining astronomical position lines will be investigated in the following chapters.

Exercises on Chapter 32

1. Define Geographical Position of a Heavenly Body.
2. Prove that the Latitude of the G.P. of a heavenly body is equal to the declination of the body; and that the Longitude of the G.P. of the body is equal to its G.H.A.
3. Using the *Nautical Almanac* Extracts, find the G.P. of the Sun:
   (i) at 10 h. 00 m. 00 s. G.M.T. on September 24th.
   (ii) at 09 h. 00 m. L.M.T. on September 22nd. Longitude 60° W.
   (iii) at 21 h. 00 m. L.M.T. on September 23rd. Longitude 45° W.
4. Using the *Nautical Almanac* Extracts, find the G.P. of the Sun:
   (i) at 23 h. 42 m. 25 s. G.M.T. on 12th June.
   (ii) at 02 h. 43 m. 56 s. L.M.T. on 14th June. Longitude 30° 15′ E.
5. At what position on the Earth will the Sun be at the zenith at 12 h. 00 m. 00 s. G.M.T. on 16th June.
6. At what position on the Earth will the Sun be at the zenith at 16 h. 13 m. 40 s. L.M.T. on 14th June, given that the Longitude of the observer is 37° 50′ E.?
7. Find the G.P. of Alioth at 16 h. 40 m. 36 s. on 15th June.
8. Find the G.P. of Hamal at 20 h. 41 m. 45 s. on 17th June.
9. Find the G.P. of Schedar at 07 h. 40 m. 45 s. L.M.T. on 16th June given that the observer's Longitude is 29° 58′ W.
10. Find the G.P. of the Moon at 06 h. 41 m. 37 s. G.M.T. on 22nd September.
11. At what position on the Earth will the Moon be at the zenith at 20 h. 36 m. 46 s. G.M.T. on 22nd September?
12. If the Sun's G.P. is 23° 16′·0 N., 165° 42′·0 E. at a certain time, what was the Sun's G.P. 01 h. 15 m. 20 s. before this time?
13. If the Sun's G.P. is 23° 20′·0 S., 19° 22′·0 W., at a certain time, what will it be 01 h. 43 m. 54 s. later.
14. Define: Circle of Equal Altitude. Explain why the spherical radius of a circle of equal altitude is equal to the zenith distance of the observed body.
15. Define: Astronomical Position Circle. What is the relationship between a circle of equal altitude and an astronomical position line?
16. Define: Astronomical Position Line. Explain why the direction of an astronomical position line is at right angles to the bearing of the observed body.
17. Explain a simple method of fixing from observations of a celestial body at a large altitude.
18. Two astronomical position circles intersect at two points X and Y. Explain how a navigator may determine which of these positions is the actual position of his vessel.
19. A fifteenth-century method of finding a vessel's position at sea required the use of a terrestrial globe and two altitude observations of stars. Explain this method.
20. Devise a method of fixing a vessel at sea using a globe and two observations of the Sun, given the date and the interval of time between the observations.

CHAPTER 33

MERIDIAN ALTITUDE OBSERVATIONS

1. Latitude by Meridian Altitude

A star, in performing its apparent diurnal motion, attains its greatest altitude at the instant it bears due North or due South on the upper celestial meridian of a stationary observer. This greatest daily altitude is called the Meridian Altitude. The meridian altitude is named North or South according as the star bears, respectively, North or South from the observer. By subtracting the meridian altitude from 90° 00′ the Meridian Zenith Distance (M.Z.D.) is found. The M.Z.D. is named according to the direction of the zenith from the observed body. Thus, if the meridian altitude is named North the meridian zenith distance is named South, and *vice versa.*

A position line obtained from an observation of a celestial body at meridian passage lies along a parallel of Latitude; its direction, therefore, is 090°–270°. A meridian altitude observation enables an observer to ascertain the exact Latitude of his vessel from a single observation.

Fig. 33·1

In fig. 33·1, which represents the Earth, P is the North Pole and QQ_1 is the equator. Imagine G to be the geographical position of a star which bears due South of an observer at O. The observer's position lies on a circle of equal altitude of radius GO. This radius, in angular units, is equal to the M.Z.D. of the star.

The latitude of G is equivalent to the declination of the star. If the declination is known, the Latitude of the observer may readily be found. The Latitude of the observer is equal to the arc RO, and this is the sum of the arcs OG and GR. Arc OG is equal to the M.Z.D. of the star, and the arc GR is equal to the declination of the star. Thus:

Latitude of Observer = M.Z.D. ★ + Declination ★

This relationship is illustrated in figs. 33·2 and 33·3.

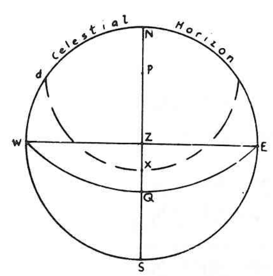


Fig. 33·2

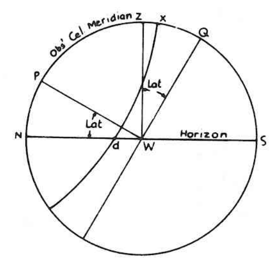


Fig. 33·3

Fig. 33·2 is a projection of the celestial sphere onto the plane of the celestial horizon of an observer whose zenith is projected at Z. Fig 33·3 is a projection of the celestial sphere on the plane of the observer's celestial meridian. From both figures it may readily be seen that:

$$\begin{aligned}\text{Latitude of Observer} &= \text{Altitude of Celestial Pole}\\ &= NP\\ &= ZQ\\ &= ZX + XQ\\ &= \text{M.Z.D.} \bigstar + \text{Declination} \bigstar\end{aligned}$$

In the above example the Latitude of the observer is the SUM of the M.Z.D. and the declination of the observed body. In some cases the latitude is found by taking the DIFFERENCE between the M.Z.D. and declination of the observed body. In all cases the Latitude is a combination of the M.Z.D. and the declination.

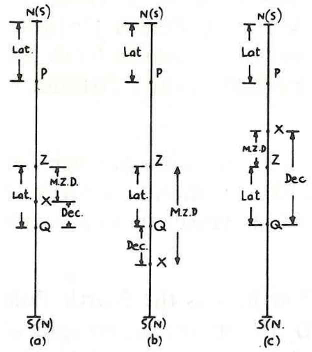


Fig. 33·4

Fig. 33·4 in which NZS represents the observer's celestial meridian on the plane of his celestial horizon, illustrates the relationship between the Latitude of Observer, and M.Z.D. and Declination of an observed body.

Referring to fig. 33·4:
In case (i): Latitude = M.Z.D. ★ + Declination ★
In case (ii): Latitude = M.Z.D. ★ − Declination ★
In case (iii): Latitude = Declination ★ − M.Z.D.

In every case:

$$\text{Latitude} = \text{M.Z.D.} \bigstar \pm \text{Declination} \bigstar$$

When solving meridian altitude problems it is advisable to draw diagrams, such as those in fig. 33·4, to assist in the solutions.

It should be noted that, essentially, a meridian altitude observation yields a position line which lies 090°–270° along the parallel of the computed, or the "Observed", Latitude, in the vicinity of the meridian of the observer. This important matter will be developed in the remaining pages of this chapter. The following examples serve to illustrate the relationship between Latitude of Observer, M.Z.D. and Declination of an observed celestial body.

Example 33·1—The True Meridian Altitude (T.M.A.) of Alpheratz, whose declination is 28° 43′·0 N., is 62° 07′·0 S. Find the Latitude of the observer.

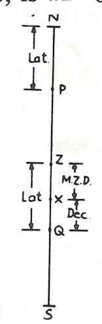


Fig. 33·5

In fig. 33·5:

(SX)	=	T.M.A.	= 62° 07′·0 S.
			90° 00′·0
(ZX)	=	M.Z.D.	= 27° 53′·0 N.
(QX)	=	Declination	= 28° 43′·0 N.
(ZQ)	=	Latitude	= 56° 36′·0 N.

Answer—Latitude = 56° 36′·0 N.

Example 33·2—The T.M.A. of Betelguese (declination 07° 24′·0 N.) was 42° 10′·0 N. Find the latitude of the observer.

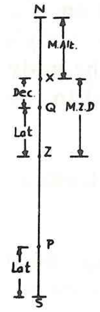


Fig. 33·6

In fig. 33·6:

(NX)	=	T.M.A.	= 42° 10′·0 N.
			90° 00′·0
(ZX)	=	M.Z.D.	= 47° 50′·0 S.
(QX)	=	Declination	= 7° 24′·0 N.
(ZQ)	=	Latitude	= 40° 26′·0 S.

Answer—Latitude = 40° 26′·0 S.

Example 33·3—Compute the T.M.A. of Antares whose declination is 26° 17′·0 S., if the observer's Latitude is 30° 20′·0 N.

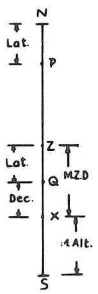


Fig. 33·7

In fig. 33·7:

(ZQ)	=	Latitude	= 30° 20′·0 N.
(QX)	=	Declination	= 26° 17′·0 S.
(ZX)	=	M.Z.D.	= 56° 37′·0 N.
			90° 00′·0
(SX)	=	T.M.A.	= 33° 23′·0 S.

Answer—True Meridian Altitude = 33° 23′·0 S.

Example 33·4—The T.M.A. of a star, observed by an observer in Latitude 40° 55′·0 N. was 54° 22′·0 S. Find the declination of the star.

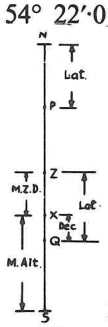


Fig. 33·8

In fig. 33·8:

(SX)	=	T.M.A.	= 54° 22′·0 S.
			90° 00′·0
(ZX)	=	M.Z.D.	= 35° 38′·0 N.
(ZQ)	=	Latitude	= 40° 55′·0 N.
(QX)	=	Declination	= 05° 17′·0 N.

Answer—Declination = 05° 17′·0 N.

2. Latitude by Meridian Altitude of a Body at Lower Meridian Passage

In the above examples, we considered cases in which the observed body is at Upper Meridian Passage. We shall now discuss the meridian altitude observation of a celestial body at Lower Meridian Passage.

A circumpolar body is above the horizon at its lower transit, at which time its altitude is least for the day and is less than the altitude of the celestial pole. A body at lower transit is said to be "On the Meridian Below the Pole". To find the Latitude from an observation of a body on the meridian below the pole, the body's true altitude is simply added to the body's polar distance, the polar distance being the arc of a great circle contained between the body and the celestial pole. In the case of a circumpolar body, the polar distance is equal to the complement of the declination of the body.

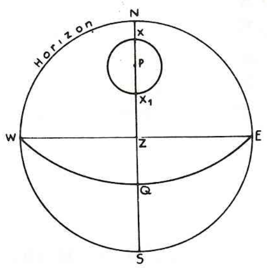


Fig. 33·9

Fig. 33·9 and fig. 33·10 illustrate how Latitude is found from an observation of a body at lower meridian passage.

Fig. 33·9 and 33·10 are projections of the celestial sphere onto the planes of the observer's celestial horizon and meridian, respectively. In fig. 33·9, the small circle is the projection of the diurnal circle of a circumpolar star. When this star is at lower meridian passage, at X, its true altitude is NX. This arc added to PX gives NP, the altitude of the celestial pole, which is equal to the Latitude of the observer.

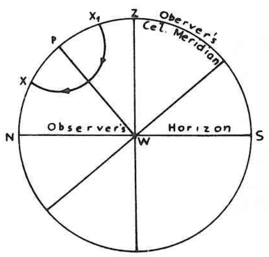


Fig. 33·10

$$PX = PX_1$$
$$= 90° - QX_1$$
$$= 90° - \text{Declination of } X$$
$$= \text{Polar Distance of } X$$

Therefore:

$$\text{Latitude} = NP$$
$$= NX + PX$$
$$= \text{True Altitude of } X + \text{Polar Distance of } X$$

Example 33·5—The true altitude of Kochab (declination 74° 26′·0 N.) on the meridian below the pole, was 20° 40′·0 N. Find the Latitude of the observer.

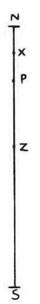


Fig. 33·11

In fig. 33·11:

NX = True Altitude	=	20° 00′·0 N.
PX = Polar Distance	=	15° 34′·0
NP = Latitude	=	36° 14′·0 N.

Answer—Latitude = 36° 14′·0 N.

3. Effect of Observer's Motion on Meridian Altitude Observations

When a celestial body is at upper meridian passage it attains its greatest altitude for the day. When a celestial body is at lower transit its altitude is least for the day. This applies strictly to a stationary observer observing a celestial body whose declination is constant. We shall now investigate the effect of the observer's motion over the Earth's surface.

It is not uncommon practice, when observing the meridian altitude of a body for Latitude, to measure the maximum altitude of the body and to take this as the meridian

altitude. This practice may introduce appreciable and unnecessary error. When a celestial body is near the meridian its rate of change of altitude is usually very small. An observer on a fast moving vessel heading northerly or southerly, is increasing or decreasing his distance from the G.P. of the observed body. Therefore, the body, when at meridian passage, is changing its altitude at a rate which is equal to the rate at which the observer is changing his Latitude.

If a vessel is travelling towards the G.P. of a celestial body the altitude of the body continues to increase after it has culminated until the rate of change of the observer's Latitude is equal to the rate of change of the body's altitude. At this instant the body is said to "Dip". If a vessel is travelling away from the G.P. of an observed body the body dips before the time of its meridian passage.

Not only does a vessel's northerly or southerly motion cause the maximum altitude to occur at a time different from that of its meridian altitude; any change in the declination of the observed body produces the same effect. To a stationary observer observing the meridian altitude of a body which transits North of the observer's zenith, northerly change in the body's declination results in the maximum altitude occurring AFTER meridian altitude. Southerly change in the body's declination results in the maximum altitude occurring BEFORE meridian altitude. For an observer observing the meridian altitude of a body which transits south of the observer's zenith the reverse applies.

4. G.M.T. of Sun's Meridian Passage

To ensure, when observing for Latitude, that the correct meridian altitude is measured, the G.M.T. of the meridian passage of the body should be computed and the observation made precisely at that time. We shall now examine the method of finding the time of meridian passage of a celestial body.

When the True Sun is on an observer's upper celestial meridian its L.H.A. is 00 h. 00 m. 00 s. The observer's Longitude applied to this gives the G.H.A. of the True Sun corresponding to its meridian passage at the observer. Now the G.H.A. of the True Sun is tabulated against G.M.T. in the *Nautical Almanac* so that if G.H.A. is known the G.M.T. is determined.

Example 33·6—Find the G.M.T. of the Sun's meridian passage across the meridian of 101° 24′ W. on June 16th.

L.H.A.T.S. at meridian passage	=	00° 00′·0
Longitude	=	101° 24′ W.
G.H.A.T.S. at meridian passage	=	101° 24′·0

G.M.T.	=	18 h. 00 m.	→	G.H.A.	=	89° 52′
Increment	=	46 m.	←	Increment	=	11° 32′
Required G.M.T.	=	18 h. 46 m.				

Answer—Required G.M.T. = 18 h. 46 m.

An alternative, and more practical, method is to apply the longitude direct to the time of the Sun's meridian passage given at the foot of the right-hand Daily Page of the *Nautical Almanac*. This time is strictly the G.M.T. of the True Sun's meridian passage at Greenwich, but it may be taken as the L.M.T. of the Sun's meridian passage over any meridian without introducing material error.

Example 33·7—Find the G.M.T. of the Sun's meridian passage over the meridian of 120° 00′·0 W. on 17th June.

From *Nautical Almanac* Extracts:

L.M.T. of Sun's meridian passage =	(17)	12 h. 00 m.
Longitude =		08 h. 00 m. W.
G.M.T. of Sun's meridian passage =	(17)	20 h. 00 m.

Answer—G.M.T. = 20 h. 00 m.

Example 33·8—Find the G.M.T. of the Sun's meridian passage over the meridian of 90° 00′·0 E. on 31st December.

From *Nautical Almanac* Extracts:

L.M.T. of Sun's meridian passage =	(31)	12 h. 03 m.
Longitude =		06 h. 00 m. E.
G.M.T. of Sun's meridian passage =	(31)	06 h. 03 m.

Answer—G.M.T. = 06 h. 03 m.

5. G.M.T. of Moon's Meridian Passage

The G.M.T. of the upper and lower meridian passages of the Moon over the meridian of Greenwich may be found from the right-hand Daily Pages of the *Nautical Almanac*. It will be remembered that the interval between successive transits of the Moon over the meridian of a stationary observer is always more than 24 hours of Mean Solar Time. For example, the Moon was on the upper meridian of Greenwich on 16th June at 11 h. 18 m. G.M.T. On the 17th June the Moon was at upper meridian passage at Greenwich at 12 h. 10 m. G.M.T. The interval between these times of transit is 24 h. 52 m. The excess of the Lunar Day over the Mean Solar Day is, therefore, 52 m.

Suppose that an observer is located in Longitude 90° W., the Moon will cross his meridian at a quarter of a lunar day after it has crossed the Greenwich meridian. It follows, therefore, that on the 16th of June the Moon will cross the meridian of 90° W. (90/360 of 24 h. 52 m.), which is 06 h. 13 m., after it has crossed the Greenwich meridian. In other words the Moon will cross the meridian of 90° W. on 16th June at 17 h. 31 m. G.M.T. This time is 06 h. 13 m. later than 11 h. 18 m., which is the G.M.T. of the Moon's upper meridian passage at Greenwich on 16th June.

On 17th June the Moon will cross the meridian of 90° E. (270/360 of 24 h. 52 m.), that is, 18h. 47 m. after it has crossed the Greenwich meridian on 16th June; or (90/360 of 24 h. 52 m.) before it will cross the Greenwich meridian on 17th June.

In practice, when it is necessary to find the G.M.T. of the Moon's meridian passage, it is usual first to find the L.M.T. of the Moon's meridian passage at the observer and then, by applying the Longitude in time, to find the G.M.T. of meridian passage.

The L.M.T. of the Moon's meridian passage is found by applying a Correction for Longitude to the tabulated G.M.T. of the Moon's meridian passage at Greenwich. The Correction for Longitude is a fraction of the excess of the Lunar Day over 24 hours, and it is proportional to the observer's Longitude.

In the interval between successive transits of the Moon, the Moon passes over 360° of Longitude. The L.M.T. of the Moon's meridian passage to an observer in any West Longitude is later than the G.M.T. of its meridian passage at Greenwich by an amount which is equal to:

W. Long./360 × Excess of Lunar Day over 24 hours.

Similarly for any East Longitude the L.M.T. of the Moon's meridian passage occurs earlier than its passage across the Greenwich meridian by an amount which is equal to:

E. Long./360 × Excess of Lunar Day over 24 hours.

Example 33·9—Find the G.M.T. of the Moon's upper meridian passage over the meridian of 60° W. on 31st December.

From *Nautical Almanac* Extracts:

G.M.T. of mer. pass. at Greenwich =	(31)	04 h. 04 m.
Corr. for Long. (60/360 × 48 m.) =		+ 08 m.
L.M.T. of mer. pass at Observer =	(31)	04 h. 12 m.
Longitude in time =		04 h. 00 m. W.
G.M.T. of mer. pass at Observer =	(31)	08 h. 12 m.

Answer—G.M.T. = 08 h. 12 m.

Example 33·10—Find the G.M.T. of the Moon's upper meridian passage over the meridian of 36° 00′ E. on 23rd September.

From *Nautical Almanac* Extracts:

G.M.T. of mer. pass. at Greenwich =	(23)	20 h. 56 m.
Corr. for Long. (36/360 × 46 m.) =		− 5 m.
L.M.T. of mer. pass at Observer =	(23)	20 h. 51 m.
Longitude in time =		02 h. 24 m. E.
G.M.T. of mer. pass at Observer =	(23)	18 h. 27 m.

Answer—G.M.T. = 18 h. 27 m.

Should the local time of the Moon's meridian passage be near midnight on any day, the next meridian passage will occur at a very early time during the morning of the next but one day. For example, the Moon is on the meridian of Greenwich at 23 h. 08 m. on 31st May. The following meridian passage at Greenwich takes place at 00 h. 07 m. on 2nd June. In other words there was no meridian passage at Greenwich on the 1st June. This does not mean that there has not been a meridian passage over other meridians on that date. For instance, the Moon will cross certain meridians East of the Greenwich meridian provided that the Correction for Longitude is greater than 07 minutes. All meridians to the East of that of Longitude (360/59× 7 m.), that is 42° E. approximately, will experience a meridian passage of the Moon on 1st June.

Example 33·11—Find the G.M.T. of the Moon's upper meridian passage over the meridian of 120° 00′ W. on 1st July.

From *Nautical Almanac* Extracts:

G.M.T. of Moon's mer. pass. at Greenwich on 30th June = 23 h. 48 m.
G.M.T. of Moon's mer. pass. at Greenwich on 1st July = — — —
G.M.T. of Moon's mer. pass. at Greenwich on 2nd July = 00 h. 43 m.

G.M.T. of mer. pass. at Greenwich	=	(30)	23 h. 48 m.
Correction for Longitude (120/360× 55 m.)	=		+ 18 m.
L.M.T. of mer. pas. at Observer	=	(1)	00 h. 06 m.
Longitude in time	=		08 h. 00 m. W.
G.M.T. of mer. pas. at Observer	=	(1)	08 h. 06 m.

Answer—G.M.T. = (1) 08 h. 06 m.

Example 33·12—Find the G.M.T. of the Moon's upper meridian passage over the meridian of 60° 00′ E. on 1st June.

From *Nautical Almanac* Extracts:

G.M.T. of Moon's mer. pass. at Greenwich on 31st May = 23 h. 08 m.
G.M.T. of Moon's mer. pass. at Greenwich on 1st June = — — —
G.M.T. of Moon's mer. pass. at Greenwich on 2nd June = 00 h. 07 m.

G.M.T. of mer. pass. at Greenwich	=	(2)	00 h. 07 m.
Long. Corr. (60/360× 60 m.)	=		− 10 m.
L.M.T. of mer. pas. at Observer	=	(1)	23 h. 57 m.
Longitude in time	=		04 h. 00 m. E.
G.M.T. of mer. pas. at Observer	=	(1)	19 h. 57 m.

Answer—G.M.T. = (1) 19 h. 57 m.

6. G.M.T. of Planet's Meridian Passage

The G.M.T. of a planet's meridian passage at Greenwich is given to the nearest minute of time, for every third day throughout the year, on the left-hand Daily Pages of the *Nautical Almanac*. The tabulated time is the approximate L.M.T. of the planet's meridian passage over every other meridian. If it is required to find the exact L.M.T. (to the nearest minute of time) of meridian passage of a planet, it is necessary to apply a Correction for Longitude to the G.M.T. of meridian passage at Greenwich, as in the case for the Moon.

Example 33·13—Find the G.M.T. of the upper meridian passage of Jupiter over the meridian of 90° 00′ W. on 14th June.

From *Nautical Almanac* Extracts:

G.M.T. of mer. pass. at Greenwich	=	(14)	19 h. 52 m.
Corr. for Long. (90/360× 4 m.)	=		− 01 m.
L.M.T. of mer. pas. at Observer	=	(14)	19 h. 51 m.
Longitude in time	=		06 h. 00 m. W.
G.M.T. of mer. pas. at Observer	=	(15)	01 h. 51 m.

Answer—G.M.T. = (15) 01 h. 51 m.

Example 33·14—Find the G.M.T. of the upper meridian passage of Saturn over the meridian of 90° 00′ E. on 15th June.

From *Nautical Almanac* Extracts:

G.M.T. of mer. pass. at Greenwich	=	(15)	23 h. 53 m.
Long. Corr. (90/360× 6 m.)	=		+ 01 m.
L.M.T. of mer. pas. at Observer	=	(15)	23 h. 54 m.
Longitude in time	=		06 h. 00 m. E.
G.M.T. of mer. pas. at Observer	=	(15)	17 h. 54 m.

Answer—G.M.T. = (15) 17 h. 54 m.

Observation Notes relating to the planets, appropriate for the whole year, together with a diagram illustrating the relative positions of the planets and the Sun, are given on pages 8 and 9 of the *Nautical Almanac*.

7. G.M.T. of Star's Meridian Passage

When a celestial body is on the upper meridian of an observer, its L.H.A. is 00 h. 00 m. 00 s. Therefore, the L.S.T.—which is equivalent to the L.H.A. of γ —at the time of a star's meridian passage is equal to (360° − S.H.A. of the star).

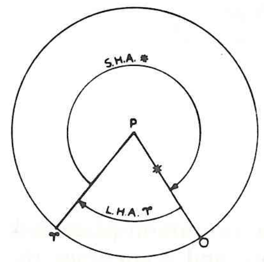


Fig. 33·12

Fig. 33·12, which represents the celestial sphere projected onto the plane of the celestial equator serves to show the relationship between S.H.A. ★ and L.H.A. of γ when the star is on the observer's upper celestial meridian.

The L.H.A. of γ at the time of a star's meridian passage may be found by subtracting the star's S.H.A. from 360° 00′. By applying the Longitude of the observer to the L.H.A. of γ, the G.H.A. of γ is found. The G.M.T. corresponding to any given G.H.A. of γ may be found by interpolation using the *Nautical Almanac*.

Example 33·15—Find the G.M.T. of the upper meridian passage of Achernar over the meridian of 11° 00′ W. on 15th June.

S.H.A. of Achernar =	35° 58′	
	360° 00′	
360° − S.H.A. =	24° 02′ =	L.H.A. mer. pass.
Longitude =	11° 00′ W.	
G.H.A. mer. pass. =	35° 02′	

From *Nautical Almanac* Extracts:

G.M.T. when G.H.A. is 23° 06′ =	(15)	08 h. 00 m. 00 s.
Increment for (35° 02′ − 23° 06′) =		47 m. 44 s.
G.M.T. of star's mer. pass. =	(15)	08 h. 47 m. 44 s.

Answer—G.M.T. = (15) 08 h. 47 m. 44 s.

In practice, the G.M.T. of a star's meridian passage may be found by applying (360° − S.H.A. star) to the G.M.T. of meridian passage of the First Point of Aries. The G.M.T. of the meridian passage of Aries is tabulated every third day on the left-hand Daily Pages of the *Nautical Almanac.*

The G.M.T. of the passage of Aries over the meridian of Greenwich is nearly the same as the L.M.T. of its passage over every other meridian. (It will be remembered that the sidereal day is 4 minutes shorter than a solar day). Therefore, the approximate L.M.T. of a star's meridian passage may be found by applying (360° − S.H.A. ★) to the G.M.T. of meridian passage of Aries found from the *Nautical Almanac.*

Example 33·15, using this method, is solved as follows:

From *Nautical Almanac* Extracts:

G.M.T. of mer. pass. of Aries at Greenwich =	(15)	06 h. 28 m.
(360° − 335° 58′) = 24° 02′ =		01 h. 36 m.
Approx. L.M.T. of mer. pas. at Observer =	(15)	08 h. 04 m.
Longitude in time =		00 h. 44 m. W.
Approx. G.M.T. of mer. pas. at Observer =	(15)	08 h. 48 m.

Answer—G.M.T. = (15) 08 h. 48 m.

8. Position Line from Meridian Altitude Observation

Finding Latitude from a meridian altitude observation of the Sun is a common-place task of practical navigation. Occasionally a navigator observes the Moon, and sometimes the

planet Venus or Jupiter (these planets, when suitably placed, being visible during the daytime), when at meridian passage, in order to find his latitude. It is seldom, however, that the meridian altitude of a star is observed at sea. Problems involving star meridian altitudes are largely of academic interest only.

The following examples illustrate how the latitude and position line are deduced from meridian altitude observations.

Example 33·16—16th June in E.P. Latitude 36° 05′ N., Longitude 16° 00′ E. Find the G.M.T. at which the star Formalhaut is at meridian passage. If the meridian altitude observed was 24° 18′·0 S., and the height of the observer's eye was 15·5 metres, find the Latitude and position line.

From *Nautical Almanac* Extracts:

G.M.T. of mer. pass. of Aries =	(15)	06 h. 28 m.
360° − S.H.A. Formalhaut =		22 h. 55 m.
L.M.T. mer. pas. Formalhaut =	(16)	05 h. 23 m.
Longitude in time =		01 h. 04 m. E.
G.M.T. mer. pas. Formalhaut =	(16)	04 h. 19 m.

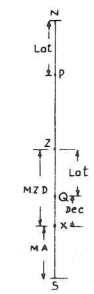


Fig. 33·13

Observed Meridian Altitude =	24° 18′·0 S.	
Total Correction =	− 9′·0	
T.M.A. =	24° 09′·0 S.	(*SX*)
	90° 00′·0	
M.Z.D. =	65° 51′·0 N.	(*ZX*)
Declination =	29° 50′·3 S.	(*QX*)
Latitude =	36° 00′·7 N.	(*ZQ*)
Azimuth =	180°	

Answer— { Position Line runs 090° − 270° through { Lat. 36° 00′·7 N. Long. 16° 00′·0 E. ; G.M.T. of local meridian passage = 04 h. 19 m. }

Example 33·17—16th June in E.P. Latitude 41° 30′·0 S., Longitude 03° 00′·0 W. Find the G.M.T. of the meridian passage of Mars. If the sextant altitude of Mars at meridian passage was 48° 02′·0 bearing north, find the Latitude and position line. Index error 1′·0 off the arc. Height of eye 9·1 metres.

From *Nautical Almanac* Extracts:

Approx L.M.T. mer. pass. of Mars =	(16)	06 h. 51 m.	
Longitude in time =		12 m. W.	
Approx G.M.T. mer. pass. of Mars =	(16)	07 h. 03 m.	
Sextant Meridian Altitude =		48° 02′·0 N.	
Index Error =		+ 1′·0	
Observed Meridian Altitude =		48° 03′·0 N.	
Total Correction =		− 6′·2	
T.M.A. =		47° 56′·8 N.	(*NX*)
		90° 00′·0	
M.Z.D. =		42° 03′·2 S.	(*ZX*)
Declination =		00° 32′·3 N.	(*QX*)
Latitude =		41° 30′·9 S.	(*ZQ*)
Azimuth =		000°	

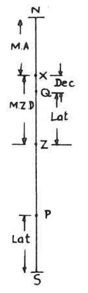


Fig. 33·14

Answer— Position Line runs 090° − 270° through { Lat. 41° 30′·9 S. Long. 03° 00′·0 W. }
G.M.T. of meridian passage = 07 h. 03 m.

Example 33·18—30th December in E.P. Latitude 39° 10′·0 N., Longitude 50° 15′·0 W. Find the G.M.T. of the Moon's upper meridian passage. The observed meridian altitude of the Moon's upper limb was 59° 23′·0. Height of eye 12·1 metres. Find the observer's Latitude and position line.

From *Nautical Almanac* Extracts:

G.M.T. of Moon's upper mer. pass. at Greenwich =	(30)	03 h. 15 m.
Long Correction (50/360 × 49 m.) =		+ 7m.
L.M.T. of Moon's mer. pass. at Observer =	(30)	03 h. 22 m.
Longitude in time =		03 h. 21 m. W.
G.M.T. mer. pass. at Observer =	(30)	06 h. 43 m.

Moon's H.P. =	57′·7	
Moon's declination at 06 h. 00 m. =	08° 42′·2 N.	d = 9′·7
'd' correction for 43 m. =	− 7′·0	
Moon's declination at 06 h. 43 m. =	08° 49′·2 N.	

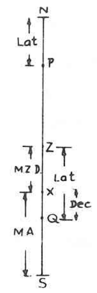


Fig. 33·15

	Observed Meridian Altitude Moon's U.L. =	59° 23′·0 S.
	Total Correction (+ 3·3 + 3·6) =	+ 6′·9
(*SX*)	T.M.A. =	59° 29′·9 S.
		90° 00′·0
(*ZX*)	M.Z.D. =	30° 30′·1 N.
(*QX*)	Declination =	08° 35′·2 N.
(*ZQ*)	Latitude =	39° 05′·3 N.

Azimuth = 180°

Answer— Position Line runs 090° − 270° through { Lat. 39° 19′·3 N. Long. 50° 15′·0 W. }
G.M.T. of Moon's transit = 06 h. 43 m.

Example 33·19—30th December in E.P. Latitude 26° 35′·0 N., Longitude 55° 15′·0 W. Find the G.M.T. of the Sun's upper meridian passage. If the observed meridian altitude of the Sun's Lower limb was 40° 02′·5 find the Latitude and position line.. Height of eye 13·8 metres.

From *Nautical Almanac* Extracts:

L.M.T. of Sun's transit =	(30)	12 h. 02 m.	
Longitude in time =		03 h. 41 m. W.	
G.M.T. of Sun's transit =	(30)	15 h. 43 m.	
Declination at 15 h. 00 m. G.M.T. =		23° 10′·5 S.	d = 0′·2
'd' correction for 43 m. =		— 0′·1	
Declination for 15 h. 43 m. G.M.T. =		23° 10′·4 S.	

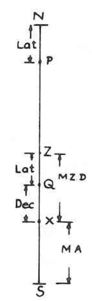


Fig. 33·16

	Observed Meridian Altitude of Sun's L.L. =	40° 02′·5 S.
	Total Correction (+ 8′·3 + 0′·5) =	+ 8′·8
(*SX*)	T.M.A. =	40° 11′·3 S.
		90° 00′·0
(*ZX*)	M.Z.D. =	49° 48′·7 N.
(*QX*)	Declination =	23° 10′·4 S.
(*ZQ*)	Latitude =	26° 38′·3 N.

Azimuth = 180°

Answer— Position Line runs 090° − 270° through { Lat. 26° 38′·3 N. Long. 55° 15′·0 W. }
G.M.T. of transit = 15 h. 43 m.

Exercises on Chapter 33

1. What is the relationship between Observer's Latitude, Meridian Zenith Distance of a celestial body, and Declination of the body?
2. Explain clearly how an observer may find his Latitude from a sextant observation of a celestial body on his upper celestial meridian.
3. Explain how Latitude is found from an observation of a celestial body at lower meridian passage.
4. Explain how an observer may find his Latitude by making sextant observations of a circumpolar body of unknown declination.
5. Distinguish between Meridian Altitude and Maximum Altitude. Why is it important to observe the meridian altitude and not the maximum altitude when observing for Latitude?
6. Find the G.M.T. of the Sun's meridian passage:
   (i) in Long. 30° E. on 16th June
   (ii) in Long. 150° W. on 17th June
   (iii) in Long. 170° E. on 22nd September.
7. Find the G.M.T. of the Moon's meridian passage:
   (i) in Long. 30° W. on 16th June
   (ii) in Long. 155° E. on 23rd September
   (iii) in Long. 90° E. on 31st December.
8. Find the G.M.T. of the meridian passage of:
   (i) Venus in Long. 70° W. on 23rd September
   (ii) Mars in Long. 90° E. on 16th June.
9. Find the G.M.T. of the upper meridian passage of:
   (i) Achernar in Long. 25° W. on 15th June
   (ii) Fomalhaut in Long. 20° E. on 17th June
   (iii) Alpheratz in Long. 90° W. on 30th December.
10. Find the observer's Latitude if the true meridian altitude of Hamal was 50° 10′ S.
11. Find the latitude of the observer if the true meridian altitude of Bellatrix was 35° 08′ N.
12. 16th June in estimated Long. 78° 45′ E. the meridian altitude of the Sun's lower limb was 32° 48′·0 N. Index error 1′·0 on the arc. Height of eye 13·8 metres. Find the Latitude and position line.
13. 23rd September in estimated Long. 29° 45′ W. the meridian altitude of the Sun's lower limb was 45° 00′·5 S. Index error − 1′·0. Height of eye 15·5 metres. Find the Latitude and position line.
14. 16th June in estimated Longitude 160° 45′ W. the meridian altitude of the Sun's lower limb was 63° 11′·0 S. Index error nil. Height of eye 9·2 metres. Find the Latitude and position line.
15. 1st January in Long. 156° 00′ E. the meridian altitude of the Sun's lower limb was 80° 20′·0 N. Index error 0′·5 off the arc. Height of eye 12·2 metres. Find the Latitude and position line.
16. 16th June in estimated Long. 53° 15′ W. the meridian altitude of the Moon's lower limb was 51° 22′·0 S. Index error nil. Height of eye 8·9 metres. Find the Latitude and position line.
17. 17th June in estimated Long. 70° 00′ W. Altair bearing due South had an observed altitude of 48° 20′·0. Index error was − 1′·0. Height of eye 8·9 metres. Find the Latitude and position line.
18. 31st December during evening twilight the meridian altitude of Achernar was 62° 20′·0 S. Index error nil. Height of eye 15·5 metres. Find the Latitude and position line given that the estimated Longitude was 75° 00′ E.

19. 15th June in estimated Longitude 152° 00′ E. the meridian altitude of Avior was 26° 03′·5 S. Index error 1′·0 on the arc. Height of eye 9·2 metres. Find the Latitude and position line.
20. 12th June in estimated Long. 120° 00′ W. the meridian altitude of Arcturus was 41° 05′·0 S. Index error nil. Height of eye 7·9 metres. Find the Latitude and position line.
21. 30th December during morning twilight in estimated Longitude 23° 00′ W. Pollux bore 000° and had an observed altitude of 61° 45′·0. Index error 1′·5 off the arc. Height of eye 10·6 metres. Find the Latitude and position line.

CHAPTER 34

THE ASTRONOMICAL TRIANGLE AND SIGHT REDUCTION

1. Introduction

An observation, or Sight, of a celestial body enables a navigator to plot an astronomical position line on his chart. The direction of an astronomical position line is at right angles to that of the observed body at the time of the sight.

A position line obtained from a sight of a celestial body on an observer's meridian lies 090° — 270° along a parallel of Latitude. The meridian altitude problem for obtaining an East-West position line is relatively simple, as we have seen from Chapter 33. The problem of finding a position line from a sight of a celestial body which lies out of the observer's celestial meridian involves solving a spherical triangle by a process known as Sight Reduction. The spherical triangle solved in reducing a sight is a celestial triangle known as the Astronomical- or *PZX*-triangle.

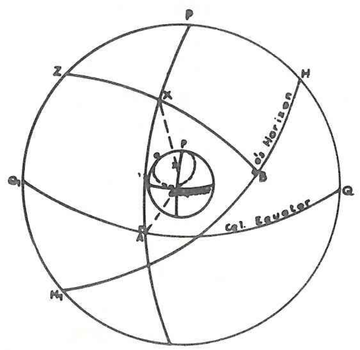


Fig. 34·1

Fig. 34·1 illustrates a typical *PZX*-triangle. In fig. 34·1, *p* represents the Earth's North Pole and *P* the celestial pole. *o* represents the observer and *Z* his zenith. *x* represents the geographical position of a star or other celestial body denoted by *X*. The great circles QQ_1 and HH_1 represent, respectively, the celestial equator and the celestial horizon of the observer.

The spherical triangle, the three angles of which are *P*, *Z* and *X*, respectively, is the Astronomical Triangle. Notice that the spherical triangle *pxo* on the Earth is geometrically similar to the astronomical triangle *PZX*. Referring to fig. 34·1:

	PH =	Altitude of Celestial Pole
	=	Observer's Latitude
But since:	ZH =	90°
Therefore:	PZ =	Observer's co-Latitude
	AX =	Declination of *X*
And since:	AP =	90°
Therefore:	PX =	Polar Distance of *X*
	BX =	Altitude of *X*
But since:	BZ =	90°
Therefore:	ZX =	Zenith Distance of *X*.

The three sides of the *PZX*-triangle are:

PZ = Co-Latitude of an observer whose zenith is at *Z*
ZX = Zenith Distance of the Observed Body
PX = Polar Distance of the Observed Body.

The three angles of the *PZX*-triangle are:

P = Hour Angle of Observed Body at observer whose zenith is at *Z*
Z = Azimuth of Observed Body to an observer whose zenith is at *Z*
X = Parallactic Angle (of relatively minor account in practical Nautical Astronomy).

The zenith distance of the observed body *X*, in minutes of arc, is equal to the great circle distance, in nautical miles, between the observer and the geographical position of the observed body. The distance, *xo* in fig. 34·1, is the radius of a circle of equal altitude of the body *X*. A small arc of this circle, when projected onto a Mercator Chart or plotting sheet, is a Position Line somewhere on which the navigator may fix his vessel's position.

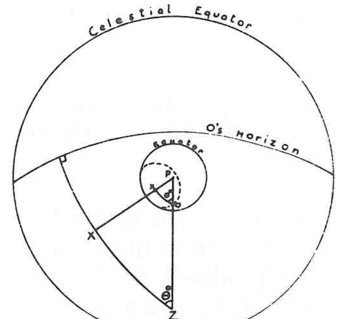


Fig. 34·2

Fig. 34·2 illustrates a typical *PZX*-triangle projected onto the plane of the celestial equator. The direction of *P* from *o*, which denotes the observer on the Earth, is N. θ° W., where θ is the azimuth of the body indicated by the angle *PZX* in the Astronomical Triangle. The direction of the circle of equal altitude at *o*, denoted by the pecked line, is at right angles to the direction of *x*, the G.P. of *X*, at *o*. This direction is determined if the azimuth of *X* is known.

In our discussion of the *PZX*-triangle above, we have made no mention of the important fact that an observer at sea is generally unaware of his precise position. It is not possible, therefore, for him to form a *PZX*-triangle such as that illustrated in figs. 34·1 and 34·2. This gives rise to a problem in computing a position line when the observer knows neither his Latitude nor Longitude. We shall see later how this problem is overcome.

There are two general methods of computing a position line from an astronomical observation. These are known, respectively, as the Longitude Method and the Intercept Method.

2. The Longitude Method

The essence of the Longitude Method of sight reduction is the computation of the Hour Angle of an observed body at a position where a circle of equal altitude is intersected by a parallel of chosen Latitude. By comparing the computed Hour Angle with the Greenwich Hour Angle of the body at the time of the observation, the Longitude of the position—the so-called Calculated Longitude—is ascertained. The azimuth of the observed body is then found, and the required position line is plotted on the chart at right angles to the azimuth and through a position having the Chosen Latitude and the Calculated Longitude. The principle of the method is illustrated in fig. 34·3.

In fig. 34·3, the point c lies on the chosen parallel of Latitude. The zenith at c is denoted by Z_c.

$cp = PZ_c =$ Co-Latitude of c
$px = PX =$ Polar Distance of X
$xc = XZ_c =$ Zenith Distance of X at c, or at any other point on the circle of equal altitude through c.

Hence, a PZX-triangle is formed from the sides PZ_c, PX, and XZ_c.

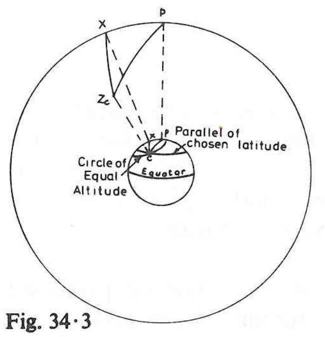


Fig. 34·3

The angle XPZ_c in the PZX-triangle formed is the Hour Angle of the observed body X at any point on c's meridian. This angle is readily found by solving the PZX-triangle given the three sides. The Longitude of the meridian of c is then easily found by comparing the Calculated Hour Angle with the Greenwich Hour Angle at the time of the observation. The G.M.T. of the observation is, of course, obtained from the chronometer the error of which is known. The G.M.T. also allows the observer to lift the declination of the observed body from the *Nautical Almanac*.

The angle XZ_cP is the azimuth of the observed body at c, and this angle may readily be found by computing the PZX-triangle given the initial three sides; or, more conveniently, by the spherical sine formula using the computed Hour Angle.

If the observer's position is not at c—and this will generally be the case—the azimuth of the observed body (which is necessary for the determination of the direction of the position line) will generally be different from what is at c. The observer's position (which he is endeavouring to discover) does, however, lie on a circle of equal altitude having a radius in miles equal to the zenith distance of the observed body in minutes of arc. Provided that c is near to the actual but unknown position of the observer, the azimuth computed from the PZX-triangle may be taken to be the same at the observer's position as it is at c. This is considered in relation to fig. 34·4.

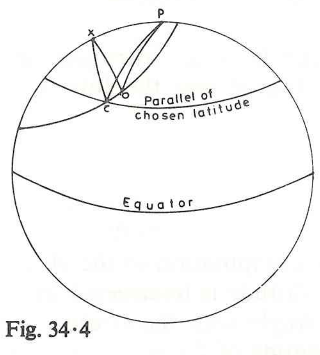


Fig. 34·4

Suppose that o in fig. 34·4 represents an observer's actual position, and that c is the position computed in the way we have described above. The radius of the circle of equal altitude through o and c is usually in the order of many hundreds or even thousands of miles. The direction of x from o is different from that at c; but, provided that c and o are close to each other, the directions of ox and cx are so slightly different from each other that, bearing in mind that an azimuth to the nearest half a degree or so is sufficiently accurate for nautical astronomical purposes, they may be considered to be the same without introducing error.

The chosen Latitude must, of necessity, be near to the actual, although unknown, Latitude of the observer at the time of the sight. In this circumstance the resulting position

line may be projected onto the chart as a straight line at right angles to the calculated azimuth and through a position the Latitude of which is equal to the chosen Latitude, and the Longitude of which is equal to the calculated Longitude.

The Longitude Method is summarized as follows:

1. The altitude of a celestial body is measured with a sextant and the G.M.T. of the sight noted.
2. The measured altitude is corrected to give a True Altitude, which, subtracted from 90° gives the zenith distance of the observed body, or the side ZX of the PZX-triangle.
3. The declination of the observed body is lifted from the *Nautical Almanac*. By combining the declination with 90° (subtracting it from 90° when the Latitude and declination have the same name, and adding it to 90° when the Latitude and declination have different names) the polar distance of the observed body is found, and this gives side PX of the PZX-triangle.
4. A latitude near to the actual but unknown Latitude of the vessel is chosen. This is subtracted from 90° to give the co-Latitude and hence the side PZ of the PZX-triangle.
5. Angle P of the PZX-triangle is computed using a direct trigonometrical method, or, more practically, by using a Short-Method or inspection table (see Chapter 38).
6. The computed angle P—Hour Angle of observed body—is compared with the Greenwich Hour Angle of the body at the time of sight. The difference between the G.H.A. (obtained from the *Nautical Almanac*) and the computed H.A. is the Longitude of a point on the chosen parallel through which to project the required position line.
7. Angle Z of the PZX-triangle is found—almost invariably by inspection from Azimuth Tables. The direction of the required position line is at right angles to the azimuth, and the position line is projected in this direction through a point having a Latitude equal to the chosen Latitude and a Longitude equal to the calculated Longitude.

3. General Remarks on the Longitude Method

When the chosen Latitude and the observed body's declination are both North or both South, that is to say, when they have the same name, the polar distance of the body is found by subtracting the declination from 90° 00′.

When the observed body is West of the observer, the Hour Angle of the body is less than 180° 00′.

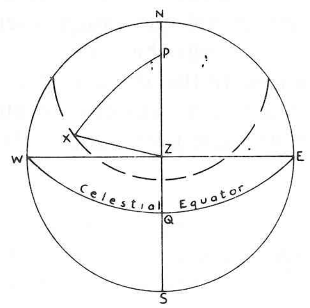


Fig. 34·5

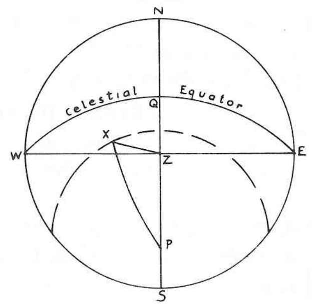


Fig. 34·6

Figs. 34·5 and 34·6 illustrate typical *PZX*-triangles projected onto the plane of an observer's celestial horizon. In fig. 34·5 the observer's Latitude and the body's declination are both North: in fig. 34·6 they are both South. In both figs. the body is West of the meridian in which case the angle *P* is less than 180° 00′.

When a celestial body is East of the meridian of an observer its Hour Angle is greater than 180°. In these cases the angle *P* of the *PZX*-triangle is found by subtracting the Hour Angle from 360° 00′.

When the chosen Latitude and the declination of an observed body have different names, the polar distance of the body is found by adding the declination to 90° 00′.

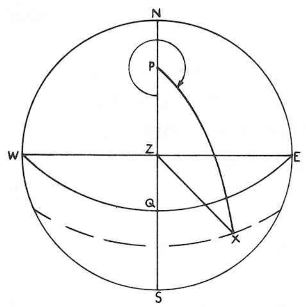


Fig. 34·7

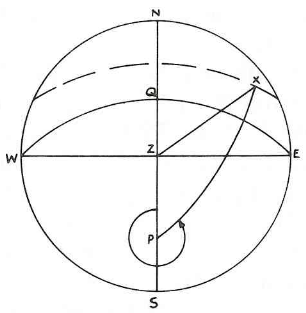


Fig. 34·8

Figs. 34·7 and 34·8 illustrate typical *PZX*-triangles in which the declination of the observed body *X* and the Latitude of an observer have different names. In both figs. the Hour Angle is greater than 180°—the observed body lying East of the meridian—and the angle *P*, therefore, is obtained by subtracting the Hour Angle from 360° 00′.

In practice, when computing a *PZX*-triangle using a direct method of spherical trigonometry, it is usual to choose a Latitude corresponding to the Latitude by estimation in order to determine the side *PZ* of the *PZX*-triangle. When using Short-Method or Inspection Tables, it is usually necessary to choose a Latitude having an integral value.

When a sight has been reduced and a calculated Longitude found, it is important to realize that the calculated Longitude is not necessarily the Longitude of the observer at the time of the observation. It is not uncommon to hear inexperienced navigators refer to the Calculated Longitude as the Vessel's Longitude: it cannot too strongly be emphasized that a single astronomical observation yields only a single position line. In the absence of additional information a navigator cannot possibly tell where on such a line his vessel is located. The calculated Longitude is the actual Longitude only if the chosen Latitude happens to be the actual Latitude at the time of the observation.

If a sight is solved several times using a different Latitude for each solution it will be found that, in general, a different Longitude will be computed in each case. The exception to the general rule applies to the case when the azimuth of the observed body is due East or due West. It is very instructive to carry out this exercise and to verify that every position, having a computed Longitude and its corresponding chosen Latitude, will lie on a smooth curve, the

direction at every point on which is equal to the azimuth at the position of the point. Provided that the zenith distance of the observed body is large and that the azimuth of the body is not too near due North or due South, the "smooth curve" will be projected on the Mercator Chart as a straight line.

4. The Longitude Method in Practice

The following examples are solved using a direct method popularized by P. L. H. Davis of the *Nautical Almanac* Office. Davis is credited with having introduced the arrangement of the Haversine Table in which Natural and Logarithmic Haversines are tabulated abreast of one another, thus facilitating computation using the so-called Haversine Method.

Example 34·1—17th June in D.R. position Lat. 40° 02′·0 N., Long. 54° 00′·0 W., at about 0800 hr. on board, the observed altitude of the Sun's lower limb was 38° 20′·0. Height of eye 7·9 metres. Chronometer time 11 h. 32 m. 26 s. Chronometer error 08 m. 40 s. slow on G.M.T. Using the Longitude method of sight reduction ascertain the position line on which the observer is located at the time of the observation.

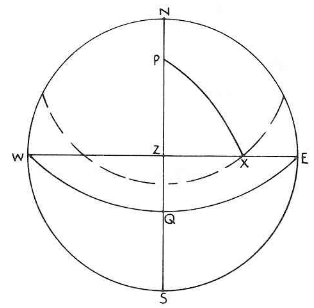


Fig. 34·9

Fig. 34·9 illustrates the *PZX*-triangle for this problem.

The first step in this problem is to ascertain whether the G.M.T. of the observation is 11 hr. or 23 h. remembering that the dial of a chronometer extends only to 12 hr. This is the reason why the time on board, namely 0800 hr. approx., is given.

Approx. L.M.T. =	08 h. 00 m.	Chron. Time =	11 h. 32 m. 26 s.
Approx. Long. W. =	03 h. 36 m.	Chron. Error =	+ 8 m. 40 s.
Approx. G.M.T. =	11 h. 36 m.	G.M.T. (17th) =	11 h. 41 m. 06 s.
		Obs. Altitude =	38° 20′·0
		Dip. =	— 4′·9
Dec. at 11 h. G.M.T. =	23° 22′·4 N.	Apparent Altitude =	38° 15′·1
'd' correction for 41 m. =	+ 0′·1	Total correction =	+ 14′·8
Dec. at 11 h. 41 m. 06 s. =	23° 22′·5 N.	True Altitude =	38° 29′·9
	90° 00′·0		90° 00′·0
Polar Distance (*PX*) =	66° 37′·5	Zen. Distance (*ZX*) =	51° 30′·1

Latitude =	40° 02′·0 N.	G.H.A. at 11 h. =	344° 49′·7
	90° 00′·0	Incr. for 41 m. 06 s. =	10° 16′·5
co-Lat. (*PZ*) =	49° 58′·0	G.H.A. =	355° 06′·2
(*PX*) =	66° 37′·5	Calc. H.A. =	301° 29′·7
(*PX* ~ *PZ*) =	16° 39′·5	Calc. Long. =	53° 36′·5 W.

hav *P* = { hav *ZX* − hav (*PZ* ~ *PX*) } cosec *PZ* cosec *PX*

nat hav *ZX* =	0·18876		
nat hav (*PZ* ~ *PX*) =	0·02098		
nat hav θ =	0·16778		
log hav θ =	$\bar{1}$·22473		
log cosec *PZ* =	0·11596	Azimuth (from Azimuth Tables) =	090°
log cosec *PX* =	0·03719	(Sun is on P.V.)	
log hav *P* =	$\bar{1}$·37788		
Calc. H.A. =	301° 29′·7		

Position Line runs 000° – 180° through Lat. 40° 02′·0 N., Long. 53° 36′·5 W.

Example 34·2—24th September in D.R. position Lat. 50° 20′·0 N., Long. 10° 30′·0 W., at about 3 p.m. on board, the sextant altitude of the Sun's lower limb was 25° 26′·5. Index error 0′·5 off the arc. Height of eye 12·1 metres. Chronometer time 3 h. 40 m. 20 s. Chronometer error 00 m. 22 s. slow on G.M.T. Ascertain, using the Longitude method, the position line on which the vessel was located at the time of sight.

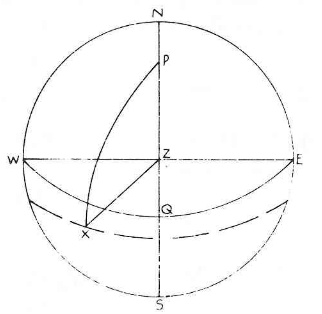


Fig. 34·10

Fig. 34·10 illustrates this problem.

Approx. L.M.T. =	15 h. 00 m.	Chron. Time =	15 h. 40 m. 20 s.
Approx. Long. =	00 h. 42 m.	Chron. Error =	+ 00 m. 22 s.
Approx. G.M.T. =	15 h. 42 m.	G.M.T. (24th) =	15 h. 40 m. 42 s.

N.B.—Sun is West of Meridian

Dec. at 15 h. =	00° 25′·1 S.	d = 1′·0	
Increment =	+ 0′·7	Sext. Alt. =	25° 26′·5
Dec. =	00° 25′·8 S.	Index Er. =	+ 0′·5
	90° 00′·0	Obsd. Alt. =	25° 27′·0
		Dip. =	− 6′·1
		Apparent Altitude =	25° 20′·9
PX =	90° 25′·8	Total correction =	+ 14′·0
Lat. =	50° 20′·0 N.	True Altitude =	25° 34′·9
	90° 00′·0		90° 00′·0
PZ =	39° 40′·0	*ZX* =	64° 25′·1
PX =	90° 25′·8		
(*PX* − *PZ*) =	50° 45′·8		

hav *P* = { hav *ZX* − hav (*PX* − *PZ*) } cosec *PZ* cosec *P*

nat hav *ZX* =	0·28410	G.H.A. at 15 h. =	46° 58′·3
nat hav (*PX* − *PZ*) =	0·18374	Increment =	10° 10′·5
nat hav θ =	0·10036	G.H.A. =	57° 08′·8
log hav θ =	$\bar{1}$·00157	Calc. H.A. =	46° 43′·3
log cosec *PZ* =	0·19496	Calc. Long. =	10° 25′·5 W.
log cosec *PX* =	0·00001	Azimuth (From Azimuth Tables) =	233½°
log hav *P* =	$\bar{1}$·19654		
Calc. H.A. =	46° 43′·3		

Position Line runs 143° – 323½° through Lat. 50° 20′·0 N., Long. 10° 25′·5 W.

Example 34·3—23rd September in D.R. position Lat. 45° 10′·0 S., Long. 118° 05′·0 W. the sextant altitude of the star Alphard was 32° 30′·0. Index error + 1′·0. Height of eye 9·5 metres. Chronometer time 13 h. 45 m. 10 s. Chronometer error 02 m. 05 s. fast on G.M.T. Ascertain the position line on which the observer is located.

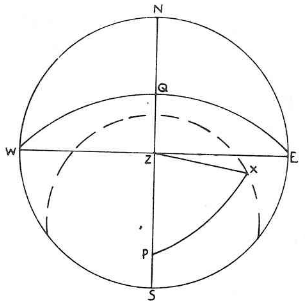


Fig. 34·11

Fig. 34·11 illustrates this problem.

Chron. Time =	13 h. 45 m. 10 s.	S.H.A. Alphard =	218° 36′·7
Chron. Error =	− 02 m. 05 s.		360° 00′·0
G.M.T. (23rd) =	13 h. 43 m. 05 s.	φ =	141° 23′·3
		=	9 h. 25 m.

G.M.T. of Transit of γ =	(22)	23 h. 46 m.
φ =		09 h. 25 m.
G.M.T. of Transit of ★	(23)	09 h. 11 m.

Therefore the ★ is East of the meridian and its H.A. is > 180°.

Lat. =	45° 10′·0 S.	Sext. Altitude =	32° 30′·0
PZ =	44° 50′·0.	Index Error =	+ 1′·0
Dec. =	8° 28′·7 S.	Obs. Altitude =	32° 31′·0
PX =	81° 31′·3	Dip =	− 5′·4
(*PX* − *PZ*) =	36° 41′·3	Apparent Altitude =	32° 25′·6
		Total correction =	− 1′·5
		True Altitude =	32° 24′·1
		ZX =	57° 35′·9

$$\text{hav } P = \{ \text{hav } ZX - \text{hav } (PX - PZ) \} \text{ cosec } PX \text{ cosec } PZ$$

nat hav *ZX* =	0·23208	G.H.A. γ at 13 h. =	196° 52′·2
nat hav (*PX* − *PZ*) =	0·09920	Increment =	10° 48′·0
nat hav θ =	0·13288	G.H.A. γ =	207° 40′·2
log hav θ =	$\bar{1}$·12347	S.H.A. ★ =	218° 36′·7
log cosec *PZ* =	0·15178	G.H.A. ★ =	426° 16′·9
log cosec *PX* =	0·00477	Calc. H.A. =	308° 14′·1
log hav *P* =	$\bar{1}$·28002	Calc. Long. =	118° 02′·8 W.
Calc. H.A. =	308° 14′·1		

Azimuth (From Azimuth Tables) = 113°

Position Line runs 023°–203° through lat. 45° 10′·0 S., Long. 118° 02′·8 W.

Example 34·4—24th September in D.R. position Lat. 35° 10′·0 N., Long. 165° 15′·0 E., during morning twilight at about 0550 hr. on board, the observed altitude of Hamal was 39° 55′·0. Height of eye 7·5 metres. Chronometer time 6 h. 44 m. 10 s. Chronometer error 01 m. 53 s. fast on G.M.T. Ascertain the position line on which the observer is located.

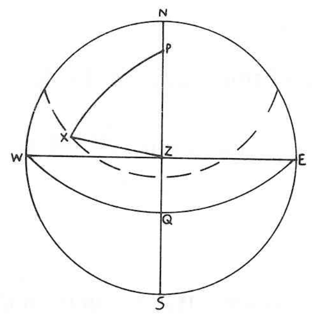


Fig. 34·12

Fig. 34·12 illustrates this problem.

Approx. Local Time =	(24)	05 h. 50 m.	S.H.A. ★ =	328° 46′·9
Approx. Longitude =	E.	11 h. 00 m.		360° 00′·0
Approx. G.M.T. =	(23)	18 h. 50 m.	φ =	31° 13′·1
			=	02 hr. 05 m.
Chron. Time =		18 h. 44 m. 10 s.	G.M.T. of Transit of γ =	23 h. 50·7 m.
Chron. Error =		− 1 m. 53 s.	φ =	02 h. 05 m.
G.M.T. (23rd) =		18 h. 42 m. 17 s.	G.M.T. of Transit of ★ (24th) =	01 hr. 55·7 m.

Therefore ★ is West of the meridian and H.A. is < 180°.

Lat. = 35° 10′·0 N.	Observed Altitude = 39° 55′·0
PZ = 54° 50′·0.	Dip = − 4′·8
Dec. = 23° 16′·1 N.	Apparent Altitude = 39° 50′·2
PX = 66° 43′·9	Total correction = − 1′·2
$(PX - PZ)$ = 11° 53′·9	True Altitude = 39° 49′·0
	ZX = 50° 11′·0

$$\text{hav } P = \{ \text{hav } ZX - \text{hav } (PX - PZ) \} \text{ cosec } PZ \text{ cosec } PX$$

nat hav ZX = 0·17983	G.H.A. γ at 18 = 272° 04′·5
nat hav $(PX - PZ)$ = 0·01075	Increment = 10° 36′·0
nat hav θ = 0·16908	G.H.A. γ h. = 282° 40′·5
log hav θ = 1·22811	S.H.A. ★ = 328° 46′·9
log cosec PZ = 0·08752	G.H.A. ★ = 251° 27′·4
log cosec PX = 0·03684	Calc. H.A. = 56° 39′·2
log hav P = 1·35247	Calc. Longitude = 165° 11′·8 E.
Calc. H.A. = 56° 39′·2	

Azimuth (From Azimuth Tables) = 273°

Position Line runs 003°–183° through Lat. 35° 10′·0 N., Long. 165° 11′·8 E.

5. The Intercept Method

The radius of a circle of equal altitude is equivalent to the great circle distance between an observer and the G.P. of an observed body. This spherical distance in minutes of arc (or nautical miles) on the Earth's surface is equal to the zenith distance of the observed body in minutes of arc, and this may readily be found from an astronomical observation or "sight" of the body.

In the Intercept Method, which was first given by the French Naval Officer, Captain (later Admiral) Marcq St. Hilaire in 1875, a position near to the actual but unknown position of the vessel is chosen. In the Longitude Method, it will be remembered, only a Latitude is chosen. The radius of the circle of equal altitude on which the chosen position lies is computed using the angle P opposite to the side ZX, and the other two sides, viz., PZ and PX., of the PZX-triangle. This computed Zenith Distance is compared with the True Zenith Distance obtained from the altitude observation of the body. The difference between the computed and observed zenith distances is known as the "intercept". The chosen position is plotted on the chart, and the intercept is projected in a direction from the plotted chosen position corresponding to the azimuth of the observed body. The position line is then

projected through the end of the intercept in a direction at right angles to the azimuth of the body. Fig. 34·13 serves to illustrate the principle of the Intercept or "Marcq St. Hilaire Method" of sight reduction.

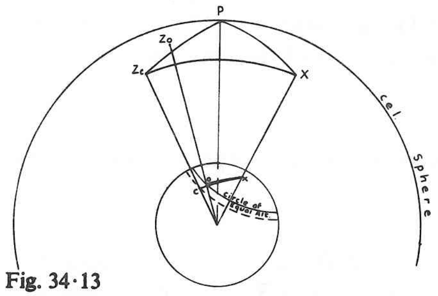


Fig. 34·13

Referring to fig. 34·13: o denotes an observer and Z_0 his zenith. c is a chosen position which is near to o; and x is the G.P. of an observed celestial body. Because the distance oc is small (never more than 40 or so miles in practice), and the distance from o (or c) to x is great (usually in the order of many hundreds, or even thousands of miles), the direction of x from o is almost the same as that of x from c. No error is introduced in practice by assuming these directions to be the same. It is to be noted that the direction of x from o (or c) is equivalent to the azimuth Z in the PZX-triangle.

The Intercept method is summarized as follows:

1. The altitude of a celestial body is observed and the G.M.T. of the observation noted.

2. The zenith distance of the observed body at the observer's actual, but unknown, position is found by subtracting the true altitude of the body from 90° 00′·0.

3. A position (Latitude and Longitude) near to the observer's position is chosen.

4. The Latitude of the chosen position subtracted from 90° 00′·0 gives the side PZ_c of the astronomical triangle which has to be solved.

5. The Longitude of the chosen position allows the observer to find the angle XPZ_c of the astronomical triangle. This angle is found simply by combining the Longitude of the chosen position with the G.H.A. of the observed body determined by means of the *Nautical Almanac*.

6. The declination of the body at the time of the obvservation is found from the *Nautical Almanac*, and the polar distance gives the side PX of the astronomical triangle.

7. The side XZ_c of the astronomical triangle is computed using the angle P and the adjacent sides PZ_c and PX.

8. The azimuth of the body is found, usually from Azimuth Tables.

9. The position line is plotted on the chart through the end of the intercept which is found by taking the difference between the Computed Zenith Distance and the Observed Zenith Distance or True Zenith Distance (T.Z.D.). The direction of the position line is at right angles to the azimuth.

Great care must be taken to plot the intercept in the correct direction. If the Computed Zenith Distance (C.Z.D.) is greater than the Observed Zenith Distance (T.Z.D.) the observer must be nearer to the G.P. of the observed body than is the Chosen Position. In this case the intercept is named TOWARDS. If, on the other hand, the C.Z.D. is less than the (T.Z.D.)

the observer must be farther from the G.P., of the observed body than is the Chosen Position. In this case the intercept is named AWAY. The following examples illustrate this.

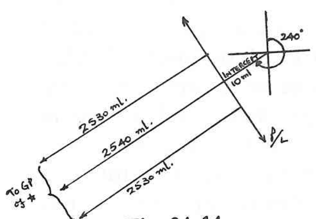


Fig. 34·14

Example 34·5— The T.Z.D. of a star was 42° 20′·0 and the C.Z.D. was 42° 30′·0. The azimuth of the star was 240°. Plot the position line.

Fig. 34·14 illustrates that the intercept in this case is named TOWARDS.

Example 34·6—The Sun's T.Z.D. was 20° 00′·0 and the C.Z.D. was 19° 50·0. The azimuth was 330°. Plot the position line.

Fig. 34·15 illustrates that, in this case, the intercept is named AWAY.

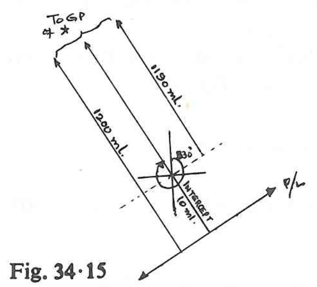


Fig. 34·15

It is clear from examples 34·5 and 34·6, that the intercept is named AWAY when the T.Z.D. is greater than the C.Z.D., and that it is named TOWARDS when the C.Z.D. is greater than the T.Z.D. Most practical navigators use a mnemonic for finding, without effort, whether an intercept is AWAY or TOWARDS. A common mnemonic is 'TTT' meaning True Tiny Towards or 'TAG' meaning True Greater Away. These mnemonics often serve useful purposes, but it is always a source of satisfaction to understand the basis of the mnemonic.

6. The Intercept Method in Practice

Example 34·7—13th June during evening twilight at about 1950 hr., the sextant altitude of the star Denebola was 61° 02′·0. Index error 0′·5 on the arc. Height of eye 6·5 m. Chronometer time 22 hr. 43 m. 50 s. Chronometer error 00 m. 05 s. fast on G.M.T. Using Latitude 36° 10′·0 N., Longitude 44° 00′·0 W., ascertain the position line on which the observer was located using the Intercept Method.

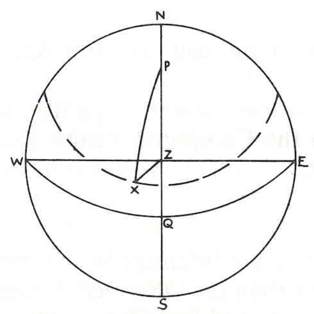


Fig. 34·16

Chron. Time =	22 h. 43 m. 50 s.	G.H.A. γ at 22 h. =	231° 42′·2
Chron. Error =	− 00 m. 05 s.	Increment =	10° 58′·0
G.M.T. (13th) =	22 h. 43 m. 45 s.	G.H.A. γ =	242° 40′·2
		S.H.A. ★ =	183° 15′·8
Latitude =	36° 10′·0 N.	G.H.A. ★ =	65° 56′·0
PZ =	53° 50′·0	Longitude =	44° 00′·0 W.
Declination ★ =	14° 48′·3 N.	H.A. ★ =	21° 56′·0
PX =	75° 11′·7		
(*PX* − *PZ*) =	21° 21′·7		

$$\text{hav } ZX = \text{hav } P \sin P \sin PZ \sin PX + \text{hav } (PZ \sim PX)$$

log hav *P* =	$\bar{2}$·55859	Sext. Altitude =	61° 02′·0
log sin *PZ* =	$\bar{1}$·90704	Index Error =	— 0′·5
log sin *PX* =	$\bar{1}$·98634	Observed Alt. =	61° 01′·5
log hav θ =	$\bar{2}$·45097	Dip =	— 4′·5
nat hav θ =	0·02824	Apparent Altitude =	60° 57′·0
nat hav (*PX* − *PZ*) =	0·03435	Total corr. =	— 0′·5
nat hav *ZX* =	0·06259	True Altitude =	60° 56′·5
ZX =	28° 58′·5	T.Z.D. =	29° 03′·5
		C.Z.D. =	28° 58′·5
		Intercept =	5′·0 AWAY

Azimuth (from Azimuth Tables) = 228½°

Position Line runs 138½° – 318½° through a point 5·0 miles 048½° from Latitude 36° 10′·0 N., Longitude 44° 00′·0 W.

Example 34·8—31st December at about 0800 hr. on board, the sextant altitude of the Sun's lower limb was 36° 07′·0. Index error — 1′·0. Height of eye 22·8 m. Chronometer time 3 hr. 47 m. 10 s. Chronometer error 05 m. 06 s. fast on G.M.T. Ascertain the position line on which the observer was located using the Intercept Method and a chosen position in Latitude 40° 05′·0 S., Longitude 63° 30′·0 E.

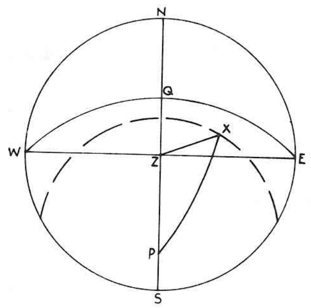


Fig. 34·17

Fig. 34·17 illustrates Example 34·8.

Approx. Local Time =	(31st) 08 h. 00 m.
Longitude E. =	04 h. 14 m.
Approx. G.M.T. =	(31st) 03 h. 46 m.
Chron. Time =	03 h. 47 m. 10 s.
Chron. Error =	— 05 m. 06 s.
G.M.T. (31st) =	03 h. 42 m. 04 s.
Dec. (03 h.) =	23° 08′·5 S. d = 0′·2
Increment =	— 0′·1
Dec. =	23° 08′·4 S.
PX =	66° 51′·6

G.H.A.T.S. at 03 h. =	224° 19′·2
Increment =	10° 31′·0
G.H.A.T.S. =	234° 50′·2
Longitude =	63° 30′·0 E.
H.A. =	298° 20′·2
P =	61° 39′·8
Latitude =	40° 05′·0 S.
PZ =	49° 55′·0
(*PX* − *PZ*) =	16° 56′·6

$$\text{hav } ZX = \text{hav } P \sin PZ \sin PX + \text{hav } (PX \sim PZ)$$

log hav *P* =	$\bar{1}$·41941
log sin *PZ* =	$\bar{1}$·88372
log sin *PX* =	$\bar{1}$·96357
log hav θ =	$\bar{1}$·26670
nat hav θ =	0·18480
nat hav (*PX* − *PZ*) =	0·02170
nat hav *ZX* =	0·20650
ZX =	54° 03′·4

Sext. Altitude =	36° 07′·0
Index Error =	— 1′·0
Obs. Altitude =	36° 06′·0
Dip =	— 8′·4
Apparent Altitude =	35° 57′·6
Total corr. =	+ 14′·9
True Altitude =	36° 12′·5
T.Z.D. =	53° 47′·5
C.Z.D. =	54° 03′·4
Intercept =	15′·9 TOWARDS

Azimuth (from Azimuth Tables) = 091½°

Position Line runs 001½° – 181½° through a point 15·9 miles 091½° from Lat. 40° 05′·0 S., Long. 63° 30′·0 E.

Example 34·9—30th December at about 3 p.m. on board, the sextant altitude of the Sun's lower limb was 18° 24′·5. Index error 1′·0 off the arc. Height of eye 21·4 m. Chronometer time 01 hr. 34 m. 50 s. Chronometer error 08 m. 05 s. slow on G.M.T. Find, using the Intercept Method, the position line on which the observer was located, the chosen position being Lat. 35° 10′·0 N., Long. 161° 15′·0 W.

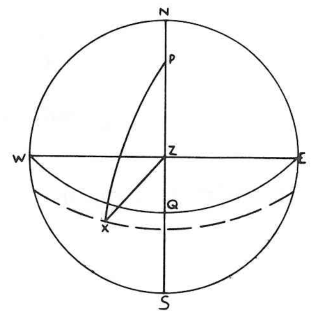


Fig. 34·18

Fig. 34·18 illustrates Example 34·8.

Approx. Local Time =	(30th) 15 h. 00 m.
Longitude =	10 h. 45 m.
Approx. G.M.T. =	(31st) 01 h. 45 m.

Chron. Time =	01 h. 34 m. 50 s.
Chron. Error =	+ 08 m. 05 s.
G.M.T. (31st) =	01 h. 42 m. 55 s.
Dec. (01 h.) =	23° 08′8 S. d = 0′·2
Increment =	— 0′·1
Dec. =	23° 08′·7 S.
PX =	113° 08′·7

G.H.A.T.S. at 01 h. =	194° 19′·8 E.
Increment =	10° 43′·8
G.H.A.T.S. =	205° 03′·6
Longitude =	161° 15′·0
H.A. =	43° 48′·6
Lat. =	35° 10′·0 N.
PZ =	54° 50′·0
(*PX* − *PZ*) =	58° 18′·7

$$\text{hav } ZX = \text{hav } P \sin PZ \sin PX + \text{hav } (PX - PZ)$$

log hav P =	$\bar{1}$·14358	Sextant Alt. =	18° 24′·5
log sin PZ =	$\bar{1}$·91248	Index Error =	+ 1′·0
log sin PX =	$\bar{1}$·96358	Obs. Altitude =	18° 25′·5
log hav θ =	$\bar{1}$·01964	Dip =	− 8′·1
nat hav θ =	0·10463	Apparent Altitude =	18° 17′·4
nat hav $(PX \sim PZ)$ =	0·23736	Total correction =	+ 13′·4
nat hav =	0·34199		
ZX =	71° 34′·7	True Altitude =	18° 30′·8
		T.Z.D. =	71° 29′·2
		C.Z.D. =	71° 34′·7
		Intercept =	5′·5 TOWARDS

Azimuth (from Azimuth Tables) = 214°

Position Line runs 124° - 304° through a point 5·5 miles 214° from Lat. 35° 10′·0 N., Long. 161° 15′·0 W.

7. The Modified Formula

In the Longitude Method of sight reduction the angle P is found from the formula:

$$\text{hav } P = \frac{\text{hav } ZX - \text{hav } (PX \sim PZ)}{\sin PX \sin PZ}$$

This formula is simplified by a modification illustrated in fig. 34·19.

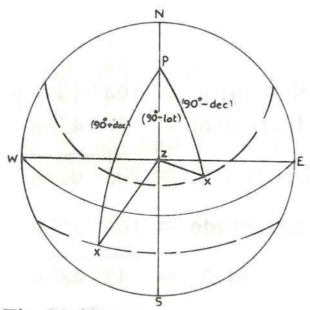


Fig. 34·19

Referring to fig. 34·19:

$$PZ = (90° - NP) = 90° - \text{Lat.})$$

Therefore:

$$\sin PZ = \cos \text{lat.}$$
$$PX = (90° \pm QY) = (90° \pm dec.)$$

Therefore:

$$\sin PX = \cos \text{dec.}$$

$$\text{hav } (PX \sim PZ) = \text{hav } \{ (90° \pm \text{dec.}) \sim (90° - \text{Lat.}) \}$$
$$= \text{hav (Lat.} \sim \text{dec.) when Lat. and dec. have different names}$$
$$= \text{hav (Lat.} + \text{dec.) when Lat. and dec. have same name}$$

In general, therefore:

$$\text{hav } (PX \sim PZ) = \text{hav (Lat.} \pm \text{dec.)}$$

The Haversine Formula given above may, therefore, be modified to:

$$\text{hav } P = \frac{\text{hav } ZX - \text{hav (Lat.} \pm \text{dec.)}}{\text{cos. Lat. cos. dec.}}$$

or: $$\text{have } P = \{ \text{hav } ZX - \text{hav (Lat.} \pm \text{dec.)} \} \text{ sec. Lat. sec. dec.}$$

By transposition, we have:

$$\text{hav } ZX = \text{hav}P \text{ cos. Lat. cos dec.} + \text{hav (Lat.} \pm \text{dec.)}$$

this being the corresponding formula for use with the Intercept Method of sight reduction.

The following examples illustrate the use of the Modified Haversine Formula.

Example 34·10—23rd September during evening twilight the observed altitude of Jupiter West of the meridian was 16° 42′·0. Height of eye 8·6 m. Chronometer time 20 hr. 44 m. 10 s. Chronometer error 07 m. 04 s. fast on G.M.T. Using the modified Haversine Formula and the Intercept Method, ascertain the position line on which the observer was located given the chosen position in Lat. 35° 20′·0 N., Long. 40° 30′·0 W.

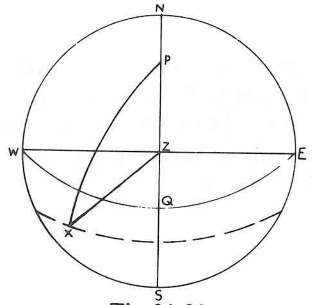


Fig. 34·20

Fig. 34·20 illustrates Example 34·10.

Approx. Local Time =	(23rd) 18 h. 45 m.		Chron. Time =	20 h. 44 m. 10 s.	
Approx. Longitude =	W. 2 h. 42 m.		Chron. Error =	− 07 m. 04 s.	
Approx. G.M.T. =	(23rd) 20 h. 42 m.		G.M.T. (23) =	20 h. 37 m. 06 s.	
Dec. at 20 h. =	11° 37′·5 S.	d = 0′·2	G.H.A. at 20 h. =	90° 53′·4	v = 2′·0
Increment =	+ 0′·1		Increment =	9° 16′·5	
Dec. =	11° 37′·6 S.		v corr. =	+ 1′·3	
			G.H.A. Jupiter =	100° 11′·2	
Latitude =	35° 20′·0 N.		Longitude =	40° 30′·0 W.	
(Lat. + dec.) =	46° 57′·6		H.A. =	59° 41′·2	

hav ZX = hav P cos Lat. cos dec. + hav (Lat. + dec.)

log hav P =	$\bar{1}\cdot39381$	Obs. Altitude =	16° 42′·0
log cos Lat. =	$\bar{1}\cdot91158$	Dip =	—5′·2
log cos dec. =	$\bar{1}\cdot99100$	Apparent Altitude =	16° 36′·8
log hav θ =	$\bar{1}\cdot29639$	Total correction =	—3′·2
nat hav θ =	0·19788	True Altitude =	16° 33′·6
nat hav (1+d) =	0·15875	T.Z.D. =	73° 26′·4
nat hav ZX =	0·35663	C.Z.D. =	73° 20′·2
ZX =	73° 20′·2	Intercept =	6′·2 AWAY

Azimuth (from Azimuth Tables) = 243°

Position Line runs 133° - 313° through a point 6·2 miles 043° from Latitude 35° 20′·0 N., Longitude 40° 30′·0 W.

Example 34·11—13th June at about 0545 hr. on board, the sextant altitude of the Moon's upper limb was 38° 47′·4. Index error 0′·5 on the arc. Height of eye 9·2 m. Chronometer time 15 hr. 14 m. 20 s. Chronometer error 29 m. 22 s. slow on G.M.T. Using the modified formula and the Intercept Method, ascertain the position line on which the observer was located given the chosen position in Lat. 40° 20′·0 N., Long. 150° 00′·0 W.

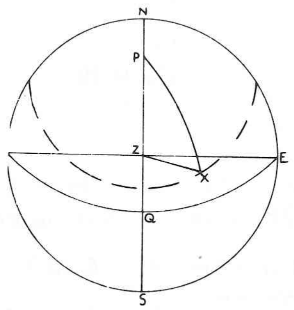


Fig. 34·21

Fig. 34·21 illustrates Example 34·11.

Approx. Local Time =	(13) 05 h. 45 m.	Chron. Time =	15 h. 14 m. 20 s.
Longitude W. =	10 h. 00 m.	Chron. Error =	+ 29 m. 22 s.
Approx. G.M.T. =	(13) 15 h. 45 m.	G.M.T. (13th) =	15 h. 43 m. 42 s.

Dec. at 15 h. =	13° 50′·4 N. d = 7′·0	G.H.A. =	89° 10′·0	v = 13′·2
Incr. =	+ 5′·1	Incr. =	10° 25′·6	
Dec. =	13° 55′·5 N.	v corr. =	+ 9′·6	
		G.H.A. =	99° 45′·2	
Lat. =	40° 30′·0 N.	Long. W. =	150° 00′·0	
(Lat. - dec.) =	26° 24′·5	H.A. =	309° 45′·2	

hav ZX = hav P cos Lat. cos dec. + hav (Lat. - dec.)

log hav P =	$\bar{1}\cdot25589$	Sext. Altitude =	38° 47′·4
log cos Lat. =	$\bar{1}\cdot88105$	Index Error =	—0′·5
log cos Dec. =	$\bar{1}\cdot98704$	Obs. Altitude =	38° 46′·9
log hav θ =	$\bar{1}\cdot12398$	Total corr. =	+21′·4
		(From Nories Table H.P. = 54′·7)	
nat hav θ =	0·13304	True Altitude =	39° 08′·3
nat hav (1 - d) =	0·05217	T.Z.D. =	50° 51′·7
nat hav ZX =	0·18521	C.Z.D. =	50° 58′·9
ZX =	50° 58′·9	Intercept =	4′·6 TOWARDS

Azimuth (from Azimuth Tables) = 106°

Position Line runs 016°–196° through a position 7·2 miles 106° from Latitude 40° 20′·0 N., Longitude 150° 00′·0 W.

8. The Azimuth

In the early days of position line navigation the customary method of finding the azimuth was to apply the relatively simple Spherical Sine Formula to the PZX-triangle. The following example illustrates the method.

Example 34·12—The altitude of Alphard (dec. = 8° 28′·7 S.) was 32° 24′. The observer was in Latitude 45° 10′·0 S. Find the azimuth of the star at the time when its H.A. was 308° 14′.

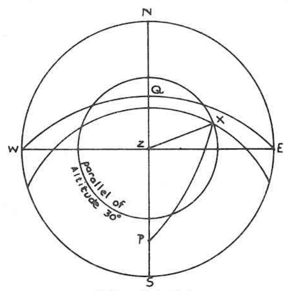


Fig. 34·22

Referring to fig. 34·22:

$$\sin Z = \sin PX \, \sin P / \sin ZX$$

or:

$$\sin Z = \cos \text{dec.} \sin P \sec \text{alt.}$$

$$\log \cos \text{dec.} = \bar{1}{\cdot}99525$$
$$\log \sin P = \bar{1}{\cdot}89515$$
$$\log \sec \text{alt.} = 0{\cdot}07349$$

$$\log \sin Z = \bar{1}{\cdot}96389$$

Therefore: $Z =$ S. 113° E.

N.B.—Care was necessary in using this formula because the sine of an angle is equal to the sine of its complement; and, in the example given, it is not an uncommon mistake to call the azimuth 67° instead of its complement, viz. 113°.

This example should be compared with Example 34·3.

Answer—Azimuth = S. 113° E.

At the time when position line navigation was discovered iron, and later steel, vessels were becoming increasingly common. The difficulties associated with the magnetic compass on board iron and steel vessels paved the way for the introduction of pre-computed Azimuth Tables of the Davis and Burdwood type which are triple-entry tables giving azimuths against Latitude, hour angle and declination. The Davis and Burdwood Tables, although designed specifically for checking magnetic compasses, were readily adapted for position line navigation. They are still commonly used today for this purpose as well as for checking compasses. Although straightforward in use care is important in the necessary interpolation.

Another type of azimuth table is the *ABC* table which we shall discuss in detail in Chapter 38.

9. Compass Error by Azimuth

Compass Error is found by comparing an observed Compass Bearing of a body whose True Bearing is known. In the days before pre-computed Azimuth Tables, the true bearing of a celestial body was computed by the observer. This required the solution of the *PZX*-triangle for the angle *Z*. Celestial bodies are more conveniently observed for azimuth

when their altitudes are not great. Azimuth Mirrors and other devices used for observing azimuths or bearings are subject to error when observing bodies at altitudes in excess of about 30°.

True Azimuths at the present time are almost always obtained from Azimuth Tables or *ABC* Tables.

10. Compass Error by Amplitude

The angle at the observer's position between the East or West point of his celestial horizon and the direction of a heavenly body at rising or setting is known as the body's Rising- or Setting-Amplitude. To facilitate finding compass error from an observation of a body at rising or setting. Amplitude Tables are included in collections of tables such as those of Norie's or Burton's. Amplitude Tables are double-entry tables giving amplitudes against arguments Latitude and declination. Amplitudes are readily computed by applying Napier's Rules to the *PZX*-triangle, which is a quadrantal triangle in cases in which the True Altitude of the body is 0°. Fig. 34·23 illustrates the so-called Amplitude Formula.

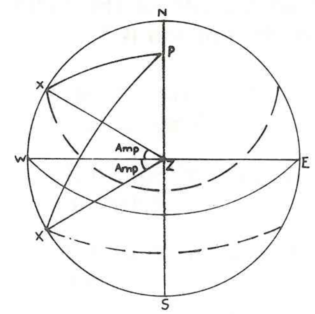


Fig. 34·23

In fig. 34·23:

$$ZX = 90°$$
$$PZ = \text{co-latitude of observer}$$
$$PX = 90° \pm \text{declination of observed body}$$
$$PZX = 90° \pm \text{amplitude of observed body} = (90° \pm A°)$$

By Napier's Rules:

$$\cos PX = \cos(90° \pm A) \cos(90° - PZ)$$

That is:

$$\sin \text{dec.} = \sin A \cos \text{Lat.}$$

or:

$$\sin A = \sin \text{dec.} \sec \text{Lat.}$$

Amplitudes are always named from East on rising and from West on setting towards North or South, the same as the name of Declination.
e.g. E. 10° N., or W. 5° S. etc.

Example 34·13—Find the Sun's rising amplitude when its declination is 20° S., and the observer's Latitude is 40° N.

$$\begin{aligned} \sin \text{Amp.} &= \sin \text{dec.} \sec \text{Lat.} \\ \log \sin \text{dec.}\ (20°) &= \bar{1}{\cdot}53405 \\ \log \sec \text{Lat.}\ (40°) &= 0{\cdot}11574 \\ \log \sin \text{Amp.} &= \bar{1}{\cdot}64979 \\ \text{Rising Amplitude} &= \text{E. } 26\tfrac{1}{2}° \text{ S.} \end{aligned}$$

Answer—Amplitude = E. 26½° S.

It is important to bear in mind that when observing the Sun or Moon in an amplitude observation that the centre of the observed body should be on the celestial horizon of the observer, and to remember that the visible horizon does not coincide with the celestial horizon. The effects of refraction, parallax and dip, results in the Sun's and Moon's lower limb having an altitude of several minutes of arc at the time its centre is on the celestial horizon. The following examples demonstrate this.

Example 34·14—Find the observed altitude of the Sun's lower limb at a time when its semi-diameter is 16′ and the dip of the horizon is 5′.

True Altitude of Sun's Centre =	00° 00′
Semi-diameter =	− 16′
True Altitude of Sun's L.L. =	− 16′
Refraction =	+ 33′
Apparent Altitude of Sun's L.L. =	+ 17′
Dip =	+ 5′
Observed Altitude of Sun's L.L. =	+ 22′

Answer—Observed Altitude of Sun's L.L. = 00°·22′.

Example 34·15—Find the observed altitude of the Moon's lower limb at a time when the Moon's H.P. and S.D. are, respectively 64′ and 16′, and the dip of the horizon is 5′.

True Altitude of Moon's centre =	00° 00′
Semi-diameter =	− 16′
True Altitude of Moon's L.L. =	− 00° 16′
Parallax =	− 01° 04′
Refraction =	+ 33′
Apparent Altitude of Moon's L.L. =	− 47′
Dip =	+ 5′
Observed Altitude of Moon's L.L. =	− 42′

Answer—Observed Altitude of Moon's L.L. = − 42′.

It is particularly important when using amplitude tables to make the observation at the precise time at which the body's centre is on the celestial horizon, especially when the latitude is high. The reason for this stems from the fact that the higher the Latitude the sharper the angle a body's diurnal circle makes with the horizon. In these circumstances the rate at which a body is changing its azimuth is relatively large, and a small change in altitude results in a large change in azimuth. In practice it is better to treat all observations made to find compass error as straightforward azimuth problems and to ignore the amplitude table.

Exercises on Chapter 34

1. Explain the term "sight" used in nautical astronomy.
2. Explain carefully the sides and angles of a typical *PZX*-triangle.
3. Explain the principles of astronomical position line navigation.
4. Describe Captain Sumner's discovery.
5. Analyse the Longitude Method of obtaining a position line, and explain the circumstances in which the method falls down.
6. Explain carefully the intercept method of sight reduction.
7. Explain the basis of naming the intercept TOWARDS or AWAY.
8. Demonstrate that the intercept method is superior to the longitude method of sight reduction.
9. Explain why a single sight yields only a position line and not a position.
10. Show how the haversine formula applied to the *PZX*-triangle for finding (i) angle *P*, (ii) side *ZX*, is modified by using Latitude and declination instead of sides *PZ* and *PX* respectively.
11. Explain the Amplitude Table and its construction.
12. What precautions are necessary when making observations of (i) Azimuths, and (ii) Amplitudes?
13. Explain why great care is necessary when making amplitude observations in high Latitudes.
14. 16th June, at about 0730 hr. on board, the observed altitude of the Sun's lower limb was 38° 05′·0. Height of eye 10·8 m. Chronometer time 11 h. 30 m. 05 s. Chronometer error 8 m. 12 s. slow on G.M.T. Using Latitude 40° 00′·0 N., Longitude 53° 30′·0 W. reduce the sight using the intercept method, and ascertain the position line on which the vessel was located.
15. 22nd September, during the afternoon, the observed altitude of the Sun's lower limb was 26° 30′·0. Height of eye 12 m. Chronometer time 15 h. 35 m. 20 s. Chronometer error 4 m. 02 s. slow on G.M.T. Using Lat. 50° 00′·0 N., Long. 10° 00′ W. reduce the sight using the intercept method, and ascertain the position line on which the vessel was located.
16. 15th June, at about 1430 hr. on board, the observed altitude of the Sun's lower limb was 12° 30′·0. Height of eye 10·5 m. Chronometer time 0 h. 59 m. 20 s. Chronometer error 19 m. 10 s. fast on G.M.T. The sight was reduced using Lat. 45° 10′·0 S., Long. 29° 30′·0 E., find the intercept and position line.
17. 30th December, a.m. on board, the observed altitude of the Sun's lower limb was 44° 46′·0. Height of eye 19 m. Chronometer time 4 h. 22 m. 10 s. Chronometer error 20 m. 05 s. slow on G.M.T. The sight was reduced using Lat. 35° 10′·0 S., Long. 59° 30′·0 E. Ascertain the position line.
18. 22nd September, a.m. on board, the observed altitude of Alphard, east of the meridian, was 30° 12′·0. Height of eye 13·5 m. Chronometer time 1 h. 35 m. 20 s. Chronometer

error 5 m. 02 s. slow on G.M.T. The sight was reduced using Lat. 44° 50′·0 S., Long. 120° 00′·0 W. Find the intercept and position line.

19. 14th June, during morning twilight, the observed altitude of Mars was 64° 02′·0. Index error 0′·5 off the arc. Height of eye 7·7 m. Chronometer time 10 h. 55 m. 23 s. Chronometer error 13 m. 04 s. fast on G.M.T. Using Lat. 25° 10′·0 S., Long. 50° 00′·0 W., find the intercept and position line. Verify that the Longitude method of sight reduction breaks down for this problem.
20. 22nd September, during evening twilight, the observed altitude of Jupiter was 19° 32′·0. Height of eye 8·8 m. Chronometer time 7 h. 01 m. 23 s. Chronometer error 18 m. 14 s. fast on G.M.T. Reduce the sight using Lat. 45° 00′·0 N., Long. 25° 00′·0 W., and ascertain the interceept and position line.
21. 14th June, during morning twilight, the observed altitude of Altair was 31° 44′·0. Height of eye 13·4 m. Chronometer time 08 h. 00 m. 00 s. Chronometer error 16 m. 04 s. fast on G.M.T. Using Lat. 25° 00′·0 S., Long. 32° 00′·0 W., find the intercept and position line.
22. 15th June, during morning twilight, the observed altitude of Achernar was 51° 47′·0. Height of eye 15 m. Chronometer time 07 h. 40 m. 03 s. Chronometer error 2 m. 20 s. slow on G.M.T. Using Lat. 27° 00′·0 S., Long. 28° 00′·0 W., find the intercept and position line.
23. 23rd September, during morning twilight, the observed altitude of Regulus was 40° 40′·0. Height of eye 11 m. Chronometer time 5 h. 40 m. 20 s. Chronometer error 4 m. 15 s. fast on G.M.T. Using Lat. 26° 00′·0 N., Long. 165° 00′·0 W., ascertain the position line using the intercept method.
24. 23rd September, p.m. on board, the observed altitude of the Moon's upper limb was 12° 11′·3. Height of eye 12 m. Chronometer time 18 h. 40 m. 20 s. Chronometer error 01 m. 02 s. slow on G.M.T. Reduce the sight using the intercept method using Lat. 54° 00′·0 N., Long. 15° 00′·0 W., and ascertain the intercept and position line.
25. 31st December, a.m. on board, the observed altitude of the Moon's upper limb was 35° 48′·7. Height of eye 12 m. Chronometer time 8 h. 36 m. 02 s. Chronometer error 00 m. 04 s. slow on G.M.T. Using Lat. 48° 00′·0 N., Long. 30° 00′·0 W. reduce the sight using the intercept method and ascertain the intercept and position line.

Note—It would be a useful and illuminating exercise to solve questions 14 to 25 inclusive using the Longitude method of sight reduction, and to verify that the same position line is found in every case.

CHAPTER 35

POSITION LINES AND PLOTTING CHART

1. Introduction

Not uncommonly, a navigator observes the Sun in the morning and solves his sight using the Longitude Method. It is important to realize, as we have stressed in Chapter 34, that the calculated Longitude is not generally the observer's actual Longitude at the time at which the observation is made. The calculated Longitude is the observer's actual Longitude only if the Latitude used in the computation is the actual Latitude of the observer at the time of the sight. Because, in general, the Latitude of a vessel is not known, the Longitude of the vessel cannot be determined from a single sight. It is important to appreciate that the result of a morning Sun-sight is NOT a longitude but a POSITION LINE. The Longitude calculated is that of a point located on the position line which also lies on the parallel of the Latitude used in reducing the sight; and the direction of the position line is at right angles to the bearing of the observed body at the time of the sight.

Referring to fig. 35·1, suppose an observation of the Sun bearing 070° gives a calculated Longitude $A°$ when Latitude $a°$ is used in the computation; then the calculated Longitude would be $B°$ had Latitude $b°$ been used; Longitude $C°$ had latitude $c°$ been used; and so on. Notice that, in every case, although a different Longitude is computed, the same position line is determined.

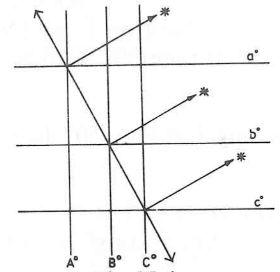


Fig. 35·1

Referring to fig. 35·2, suppose that a navigator aboard a stationary vessel in Latitude 50° 00′·0 N., Longitude 30° 00′·0 W., observes the Sun, having North declination, when it is successively at positions *d, e* and *f.* Suppose that these observations are reduced by the Longitude Method using, in turn, Latitude 50° 00′·0 N.—the true Latitude of the vessel—and Latitude 50° 10′·0 N.

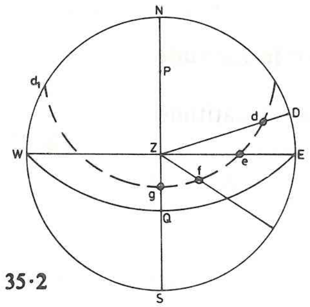


Fig. 35·2