Vessel A is a closed shelter deck ship.
Vessel B is a raised quarter deck ship.

It is essential for a vessel with small freeboard, such as an oil tanker, to have a large metacentric height and thus extend the range of stability.

If a vessel is initially unstable it will not remain upright but will either heel to the *Angle of Loll* or will capsize depending upon the degree of instability and the shape of the stability curve (Fig. 62).

Vessel A will heel to an angle of loll of about 8° but still remains a fairly stable ship, and while this heel would be very inconvenient, the vessel would not be in a dangerous condition.

If vessel B is unstable it will capsize since at all angles the righting lever is negative.

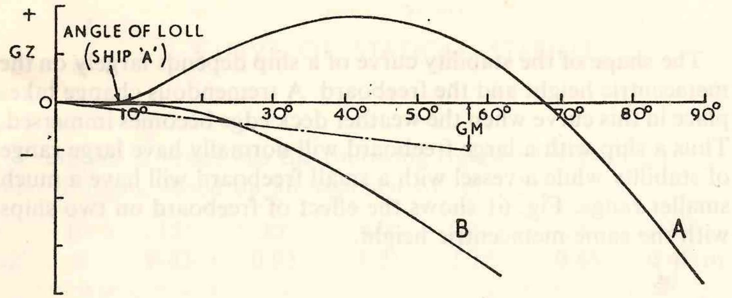


Fig. 62

Example. The righting levers of a ship of 15 000 tonne displacement at angles of heel of 15°, 30°, 45° and 60° are 0·29, 0·70, 0·93 and 0·90 m respectively. Calculate the dynamical stability of the ship at 60° heel.

Angle	*GZ*	SM	Product for area
0	0	1	0
15°	0·29	4	1·16
30°	0·70	2	1·40
45°	0·93	4	3·72
60°	0·90	1	0·90
			7·18

Note: The common interval must be expressed in *radians*.

$$h = \frac{15}{57{\cdot}3}$$

$$\text{Area under curve} = \tfrac{1}{3} \cdot \frac{15}{57{\cdot}3} \cdot 7{\cdot}13$$
$$= 0{\cdot}6012$$
$$\text{Dynamical stability} = 15\,000 \times 9{\cdot}81 \times 0{\cdot}6012$$
$$= 88{\cdot}47 \times 10^3 \text{ kJ}$$
$$= 88{\cdot}47 \text{ MJ}$$

STABILITY OF A WALL SIDED-SHIP

If a vessel is assumed to be wall-sided in the vicinity of the waterplane, the righting lever may be estimated from the expression.

$$GZ = \sin\theta\,(GM + \tfrac{1}{2}\,BM\,tan^2\,\theta)$$

This formula may be regarded as reasonably accurate for vessels which are deeply loaded, up to the point at which the deck edge enters the water.

If a vessel is initially unstable it will either capsize or heel to the angle of loll. At this angle of loll θ the vessel does not tend to return to the upright or incline to a greater angle. The righting lever is therefore zero. Hence if the vessel is assumed to be wall-sided:

$$0 = \sin\theta\,(GM + \tfrac{1}{2}\,BM\,tan^2\theta)$$

and since sin θ cannot be zero unless θ is zero:

$$0 = GM + \tfrac{1}{2}BM\,tan^2\theta$$
$$\tfrac{1}{2}BM tan^2\theta = -GM$$
$$\tan^2\theta = -2\,\frac{GM}{BM}$$

From which
$$\tan\theta = \pm\sqrt{-2\,\frac{GM}{BM}}$$

Since GM must be negative for this condition, $-2\frac{GM}{BM}$ must be *positive*.

Thus the angle of loll may be determined for any given unstable condition.

Example. A ship of 12 000 tonne displacement has a second moment of area about the centreline of 72×10^3 m^4. If the metacentric height is $-0{\cdot}05$ m, calculate the angle of loll.

$$\nabla = \frac{12\,000}{1{\cdot}025} \text{ m}^3$$

$$BM = \frac{I}{\nabla}$$

$$= \frac{72 \times 10^3 \times 1{\cdot}025}{12\,000} \text{ m}$$

$$= 6{\cdot}150 \text{ m}$$

$$\tan\theta = \pm\sqrt{-2\,\frac{(-0{\cdot}05)}{6{\cdot}15}}$$

$$= \pm\, 0{\cdot}1275$$

From which $\theta = \pm\, 7^\circ\, 16'$

i.e. the vessel may heel 7° 16′ either to port or to starboard.

A more practical application of this expression may be found when the vessel is listing at sea. It is necessary first to bring the ship upright and then provide sufficient stability for the remainder of the voyage. Thus it is essential to estimate the negative metacentric height causing the angle of loll in order to ensure that these conditions are realised.

Example. At one point during a voyage the above vessel is found to have an angle of loll of 13°. Calculate the initial metacentric height.

From above $BM = 6{\cdot}150$ m

$$\tan\theta = \pm\sqrt{\frac{-2GM}{BM}}$$

$$0{\cdot}2309 = \pm\sqrt{\frac{-2GM}{6{\cdot}15}}$$

$$GM = -\frac{0{\cdot}2309^2 \times 6{\cdot}15}{2}$$

Initial metacentric height $= -\,0{\cdot}164$ m.

TEST EXAMPLES 5

1. A ship displaces 12 000 tonne, its centre of gravity is 6·50 m above the keel and its centre of buoyancy is 3·60 m above the keel If the second moment of area of the waterplane about the centreline is $42{\cdot}5 \times 10^3$ m^4 find the metacentric height.

2. A vessel of 10 000 tonne displacement has a second moment of area of waterplane about the centreline of 60×10^3 m^4. The centre of buoyancy is 2·75 m above the keel. The following are the disposition of the masses on board the ship.

4000 tonne 6·30 m above the keel
2000 tonne 7·50 m above the keel
4000 tonne 9·15 m above the keel

Calculate the metacentric height.

3. A vessel of constant rectangular cross-section has a breadth of 12 m and metacentric height of one quarter of the draught. The vertical centre of gravity lies on the waterline. Calculate the draught.

4. A raft is made from two cylinders each 1·5 m diameter and 6 m long. The distance between the centres of the cylinders is 3 m. If the draught is 0·75 m, calculate the transverse BM.

5. A vessel of constant rectangular cross-section is 7·2 m wide.

(a) Draw the metacentric diagram using 0·5 m intervals of draught up to the 4·0 m waterline.

(b) If the centre of gravity is 3 m above the keel, determine from the metacentric diagram the limits of draught between which the vessel will be unstable.

6. A vessel of constant triangular cross-section is 9 m wide at the deck and has a depth to deck of 7·5 m. Draw the metacentric diagram using 0·5 m intervals of draught up to the 3·0 m waterline.

7. An inclining experiment was carried out on a ship of 8000 tonne displacement. A mass of 10 tonne was moved 14 m across the deck causing a pendulum 8·5 m long to deflect 110 mm. The transverse metacentre was 7·15 m above the keel. Calculate the metacentric height and the height of the centre of gravity above the keel.

8. An inclining experiment was carried out on a ship of 4000 tonne displacement, when masses of 6 tonne were moved transversely through 13·5 m. The deflections of a 7·5 m pendulum were 81, 78, 85, 83, 79, 82, 84, and 80 mm respectively.
Calculate the metacentric height.

*f*9. A ship of 5000 tonne displacement has a rectangular double bottom tank 9 m wide and 12 m long, half full of sea water. Calculate the virtual reduction in metacentric height due to free surface.

*f*10. A ship of 8000 tonne displacement has its centre of gravity 4·5 m above the keel and transverse metacentre 5·0 m above the keel when a rectangular tank 7·5 m long and 15 m wide contains sea water. A mass of 10 tonne is moved 12 m across the deck. Calculate the angle of heel.:
(a) if there is no free surface of water,
(b) if the water does not completely fill the tank.

*f*11. A ship of 6000 tonne displacement has its centre of gravity 5·9 m above the keel and transverse metacentre 6·8 m above the keel. A rectangular double bottom tank 10·5 m long, 12 m wide and 1·2 m deep is now half-filled with sea water. Calculate the metacentric height.

*f*12. An oil tanker 24 m wide displaces 25 000 tonne when loaded in nine equal tanks, each 10 m long, with oil rd 0·8. Calculate the total free surface effect with:
(a) no longitudinal divisions,
(b) a longitudinal centreline bulkhead,
(c) twin longitudinal bulkheads forming three equal tanks.
(d) twin longitudinal bulkheads, the centre compartment having a width of 12 m.

*f*13. A ship of 12 500 tonne displacement and 15 m beam has a metacentric height of 1·10 m. A mass of 80 tonne is lifted from its position in the centre of the lower hold by one of the ship's derricks, and placed on the quay 2 m from the ship's side. The ship heels to a maximum angle of 3·5° when the mass is being moved.
(a) Does the *GM* alter during the operation?
(b) Calculate the height of the derrick head above the original centre of gravity of the mass.

*f*14. The righting levers of a ship, for an assumed *KG* of 3·5 m, are 0, 0·25, 0·46, 0·51, 0·39, 0·10 and −0·38 m at angles of heel of 0, 15°, 30°, 45°, 60°, 75° and 90° respectively.
When the ship is loaded to the same displacement the centre of gravity is 3·0 m above the keel and the metacentric height 1·25 m. Draw the amended curve of statical stability.

*f*15. The righting moments of a ship at angles of heel of 0, 15°, 30°, 45° and 60° are 0, 1690, 5430, 9360 and 9140 kN m respectively. Calculate the dynamical stability at 60°.

*f*16. A ship of 18 000 tonne displacement has *KB* 5·25 m, *KG* 9·24 m and second moment of area about the centreline of 82×10^3 m^4.
Using the wall-sided formula calculate the righting levers at intervals of 5° heel up to 20° and sketch the stability curve up to this angle.

*f*17. A ship of 7200 tonne displacement has *KG* 5.20 m, *KB* 3.12 m and *KM* 5.35 m. 300 tonne of fuel at *Kg* 0.6 m are now used. Ignoring free surface effect and assuming the *KM* remains constant, calculate the angle to which the vessel will heel.

CHAPTER 6

TRIM

CHANGE IN DRAUGHTS DUE TO ADDED MASSES

TRIM is the difference between the draughts forward and aft. Thus if a ship floats at draughts of 6 m forward and 7 m aft, it is said to trim 1 m by the *stern*. If the draught forward is greater than the draught aft the vessel is said to trim by the *head*.

CENTRE OF FLOTATION (LCF) is the centroid of the waterplane and is the axis about which a ship changes trim when a mass is added, removed or moved longitudinally.

If a small mass m is added to a ship at the centre of flotation, there is an increase in mean draught but no change in trim, since the centre of gravity of the added mass is at the same position as the centre of the added layer of buoyancy. A large mass (e.g. one exceeding, say, one twentieth of the displacement) will cause a considerable increase in draught and hence a change in waterplane area and centre of flotation.

MEAN DRAUGHT

The mean draught of a vessel is the draught at which the vessel would lie in level keel conditions. Since the vessel changes trim about the LCF, the draught at this point remains constant for any given displacement whether the vessel is at level keel or trimmed. Hence the mean draught may be taken as the draught at the LCF.

The mean of the end draughts may be compared with the actual draught amidships to determine whether the vessel is hogging or sagging, but is of little relevance in hydrostatic calculations.

EFFECT OF ADDING SMALL MASSES

It is useful to assume that when a small mass is added to the ship it is first placed at the centre of flotation and then moved forward or aft to its final position. Thus the effect of an added mass on the draughts may be divided into:

(a) a bodily increase in draught

(b) a change in trim due to the movement of the mass from the centre of flotation to its final position.

The bodily increase in draught may be found by dividing the mass by the TPC.

The change in trim due to any longitudinal movement of mass may be found by considering its effect on the centre of gravity of the ship.

Consider a ship of displacement $\triangle$ and length L, lying at waterline WL and having a mass m on the deck (Fig. 63). The centre of gravity G and the centre of buoyancy B lie in the same vertical line.

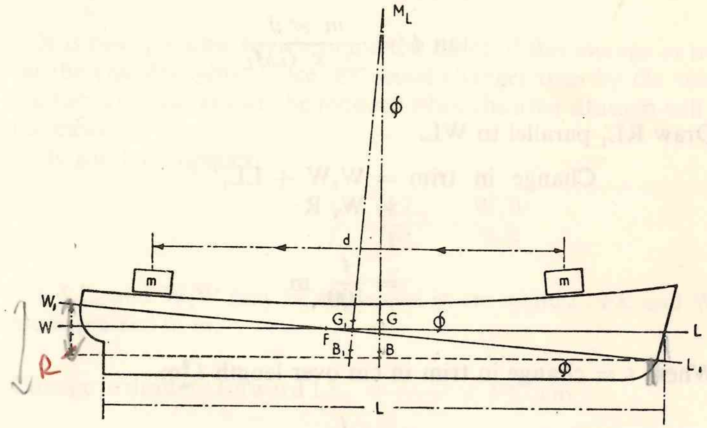


Fig. 63

If the mass is moved a distance d aft, the centre of gravity moves aft from G to G_1, and

$$GG_1 = \frac{m \times d}{\triangle}$$

The ship then changes trim through the centre of flotation F until it lies at waterline W_1L_1. This change in trim causes the centre of buoyancy to move aft from B to B_1, in the same vertical line as G_1. The vertical through B_1 intersects the original vertical through B at M_L, the *longitudinal metacentre*. GM_L is known as the *longitudinal metacentric height*,

$$GM_L = KB + BM_L - KG$$

$$BM_L = \frac{I_F}{\nabla}$$

Where I_F = second moment of area of the waterplane about a transverse axis through the centre of flotation F.

If the vessel trims through an angle ϕ, then

$$GG_1 = GM_L \tan\phi$$

and $$GM_L \tan\phi = \frac{m \times d}{\triangle}$$

$$\tan\phi = \frac{m \times d}{\triangle \times GM_L}$$

Draw RL_1 parallel to WL.

$$\text{Change in trim} = W_1W + LL_1 = W_1R = \frac{t}{100}\text{ m}$$

Where t = change in trim in cm over length L m.

But $$\tan\phi = \frac{t}{100L}$$

$$\therefore \quad \frac{t}{100L} = \frac{m \times d}{\triangle \times GM_L}$$

$$t = \frac{m \times d \times 100L}{\triangle \times GM_L}\text{ cm}$$

The change in trim may therefore be calculated from this expression. $m \times d$ is known as the trimming moment.

It is useful to know the moment which will cause a change in trim of one cm.

$$m \times d = \frac{t \times \triangle \times GM_L}{100\,L}\text{ tonne m}$$

Let $$t = 1\text{ cm}$$

Then moment to change trim one cm

$$\text{MCTI cm} = \frac{\triangle \times GM_L}{100L}\text{ tonne m}$$

Change in trim $$t = \frac{\text{trimming moment}}{\text{MCTIcm}}\text{ cm} = \frac{m \times d}{\text{MCTIcm}}\text{ cm by the stern}$$

It is now possible to determine the effect of this change in trim on the end draughts. Since the vessel changes trim by the stern, the forward draught will be reduced while the after draught will be increased.

By similar triangles.

$$\frac{t}{L} = \frac{LL_1}{FL} = \frac{W_1W}{WF}$$

t, LL_1 and W_1W may be expressed in cm while L, FL and WF are expressed in m.

$$\text{Change in draught forward } LL_1 = -\frac{t}{L} \times \text{FL cm}$$

$$\text{Change in draught aft } W_1W = +\frac{t}{L} \times \text{WF cm}$$

Example. A ship of 5000 tonne displacement, 96 m long, floats at draughts of 5·60 m forward and 6·30 m aft. The TPC is 11·5, GM_L 105 m and centre of flotation 2·4 m aft of midships.

Calculate:

(a) the MCTIcm

(b) the new end draughts when 88 tonne are added 31 m forward of midships.

(a) $$\text{MCTI cm} = \frac{\triangle \times GM_L}{100L} = \frac{5000 \times 105}{100 \times 96} = 54{\cdot}69\text{ tonne m}$$

(b)

$$\text{Bodily sinkage} = \frac{88}{11\cdot5}$$
$$= 7\cdot65 \text{ cm}$$
$$d = 31 + 2\cdot4$$
$$= 33\cdot4 \text{ m from } F$$
$$\text{Trimming moment} = 88 \times 33\cdot4 \text{ tonne m}$$
$$\text{Change in trim} = \frac{88 \times 33\cdot4}{54\cdot69}$$
$$= 53\cdot74 \text{ cm by the head}$$

$$\text{Distance from } F \text{ to fore end} = \frac{96}{2} + 2\cdot4$$
$$= 50\cdot4 \text{ m}$$

$$\text{Distance from } F \text{ to after end} = \frac{96}{2} - 2\cdot4$$
$$= 45\cdot6 \text{ m}$$

$$\text{Change in trim forward} = +\frac{53\cdot74}{96} \times 50\cdot4$$
$$= +28\cdot22 \text{ cm}$$

$$\text{Change in trim aft} = -\frac{53\cdot74}{96} \times 45\cdot6$$
$$= -25\cdot52 \text{ cm}$$
$$\text{New draught forward} = 5\cdot60 + 0\cdot076 + 0\cdot282$$
$$= 5\cdot958 \text{ m}$$
$$\text{New draught aft} = 6\cdot30 + 0\cdot076 - 0\cdot255$$
$$= 6\cdot121 \text{ m}$$

If a number of items are added to the ship at different positions along its length, the total mass and nett trimming moment may be used to determine the final draughts.

Example. A ship 150 m long has draughts of 7·70 m forward and 8·25 m aft, MCT I cm 250 tonne m, TPC 26 and LCF 1·8 m forward of midships. Calculate the new draughts after the following masses have been added:

50 tonne, 70 m aft of midships
170 tonne, 36 m aft of midships
100 tonne, 5 m aft of midships
130 tonne, 4 m forward of midships
40 tonne, 63 m forward of midships

Mass (tonne)	Distance from F (m)	moment forward (tonne m)	moment aft (tonne m)
50	71·8A	–	3590
170	37·8A	–	6426
100	6·8A	–	680
130	2·2F	286	–
40	61·2F	2448	
Total 490		2734	10 696

$$\text{Excess moment aft} = 10\,696 - 2734$$
$$= 7962 \text{ tonne m}$$

$$\text{Change in trim} = \frac{7962}{250}$$
$$= 31\cdot85 \text{ cm by the stern}$$

$$\text{Change in trim forward} = -\frac{31\cdot85}{150}\left(\frac{150}{2} - 1\cdot8\right)$$
$$= -15\cdot54 \text{ cm}$$

$$\text{Change in trim aft} = +\frac{31\cdot85}{150}\left(\frac{150}{2} + 1\cdot8\right)$$
$$= +16\cdot31 \text{ cm}$$

$$\text{Bodily sinkage} = \frac{490}{26}$$
$$= 18\cdot85 \text{ cm}$$

$$\text{New draught forward} = 7\cdot70 + 0\cdot189 - 0\cdot155$$
$$= 7\cdot734 \text{ m}$$

$$\text{New draught aft} = 8\cdot25 + 0\cdot189 + 0\cdot163$$
$$= 8\cdot602 \text{ m}$$

DETERMINATION OF DRAUGHTS AFTER THE ADDITION OF LARGE MASSES

When a large mass is added to a ship the resultant increase in draught is sufficient to cause changes in all the hydrostatic details. It then becomes necessary to calculate the final draughts from first principles. Such a problem exists every time a ship loads or discharges the major part of its deadweight.

The underlying principle is that after loading or discharging the vessel is in equilibrium and hence the final centre of gravity is in the same vertical line as the final centre of buoyancy.

For any given condition of loading it is possible to calculate the displacement $\triangle$ and the longitudinal position of the centre of gravity G relative to midships.

From the hydrostatic curves or data, the mean draught may be obtained at this displacement, and hence the value of MCTI cm and the distance of the LCB and LCF from midships. These values are calculated for the level keel condition and it is unlikely that the LCB will be in the same vertical line as G. Thus a trimming moment acts on the ship. This trimming moment is the displacement multiplied by the longitudinal distance between B and G, known as the *trimming lever*.

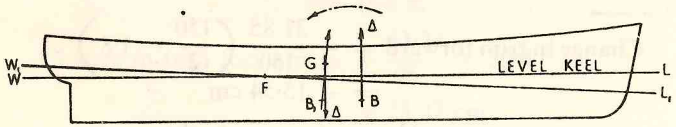


Fig. 64

The trimming moment, divided by the MCTI cm, gives the change in trim from the level keel condition, i.e. the total trim of the vessel. The vessel changes trim about the LCF and hence it is possible to calculate the end draughts. When the vessel has changed trim in this manner, the new centre of buoyancy B_1 lies in the same vertical line as G.

Example. A ship 125 m long has a light displacement of 4000 tonne with LCG 1·60 m aft of midships. The following items are now added:

Cargo 8500 tonne Lcg 3·9 m forward of midships
Fuel 1200 tonne Lcg 3·1 m aft of midships
Water 200 tonne Lcg 7·6 m aft of midships
Stores 100 tonne Lcg 30·5 m forward of midships.

At 14 000 tonne displacement the mean draught is 7·80 m, MCTI cm 160 tonne m, LCB 2·00 m forward of midships and LCF 1·5 m aft of midships.

Calculate the final draughts.

Item	mass(t)	Lcg(m)	moment forward	moment aft
Cargo	8500	3·9F	33 150	—
Fuel	1200	3·1A	—	3720
Water	200	7·6A	—	1520
Stores	100	30·5F	3050	—
Lightweight	4000	1·6A	—	6400
Displacement	14 000		36 200	11 640

$$\text{Excess moment forward} = 36\,200 - 11\,640 = 24\,560 \text{ tonne m}$$

$$\text{LCG from midships} = \frac{24\,560}{14\,000} = 1{\cdot}754 \text{ m forward}$$

$$\text{LCB from midships} = 2{\cdot}000 \text{ m forward}$$

$$\text{trimming lever} = 1{\cdot}754 - 2{\cdot}000 = 0{\cdot}245 \text{ m aft}$$

$$\text{trimming moment} = 14\,000 \times 0{\cdot}246 \text{ tonne m}$$

$$\text{trim} = \frac{14\,000 \times 0{\cdot}246}{160} = 21{\cdot}5 \text{ cm by the stern}$$

$$\text{Change in draught forward} = -\frac{21{\cdot}5}{125}\left(\frac{125}{2} + 1{\cdot}5\right) = -11{\cdot}0 \text{ cm}$$

$$\text{Change in draught aft} = +\frac{21{\cdot}5}{125}\left(\frac{125}{2} - 1{\cdot}5\right) = +10{\cdot}5 \text{ cm}$$

$$\text{Draught forward} = 7{\cdot}80 - 0{\cdot}110 = 7{\cdot}690 \text{ m}$$

$$\text{Draught aft} = 7{\cdot}80 + 0{\cdot}105 = 7{\cdot}905 \text{ m}$$

CHANGE IN MEAN DRAUGHT DUE TO CHANGE IN DENSITY

The displacement of a ship floating freely at rest is equal to the mass of the volume of water which it displaces. For any given displacement, the volume of water displaced must depend upon the density of the water. When a ship moves from sea water into river water without change in displacement, there is a slight increase in draught.

Consider a ship of displacement $\triangle$ tonne, waterplane area A_w m², which moves from sea water of ρ_S t/m³ into river water of ρ_R t/m³ without change in displacement.

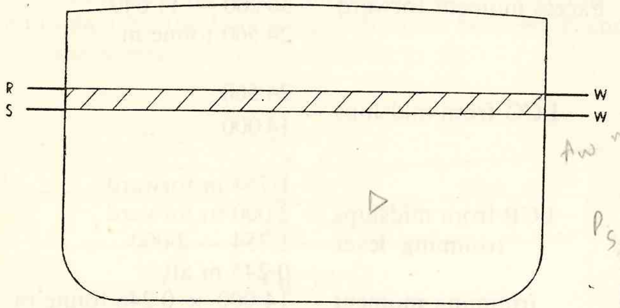


Fig. 65

Volume of displacement in sea water

$$\nabla_S = \frac{\triangle}{\rho_S} \text{ m}^3$$

Volume of displacement in river water

$$\nabla_R = \frac{\triangle}{\rho_R} \text{ m}^3$$

Change in volume of displacement

$$v = \nabla_R - \nabla_S$$

$$= \frac{\triangle}{\rho_R} - \frac{\triangle}{\rho_S}$$

$$= \triangle\left(\frac{1}{\rho_R} - \frac{1}{\rho_S}\right) \text{ m}^3$$

This change in volume causes an increase in draught. Since the increase is small, the waterplane area may be assumed to remain constant and the increase in mean draught may therefore be found by dividing the change in volume by the waterplane area.

$$\text{Increase in draught} = \frac{\triangle}{A_w}\left(\frac{1}{\rho_R} - \frac{1}{\rho_S}\right) \text{ m}$$

$$= \frac{100\,\triangle}{A_w}\left(\frac{\rho_S - \rho_R}{\rho_R \times \rho_S}\right) \text{ cm}$$

The tonne per cm immersion for sea water is given by

$$\text{TPC} = \frac{A_w}{100} \times \rho_S$$

$$\therefore\ A_w = \frac{100\ \text{TPC}}{\rho_S} \text{ m}^2$$

Substituting for A_w in the formula for increase in draught:

$$\text{Increase in draught} = \frac{100\,\triangle\,\rho_S}{100\ \text{TPC}}\left(\frac{\rho_S - \rho_R}{\rho_R \times \rho_S}\right)$$

$$= \frac{\triangle}{\text{TPC}}\left(\frac{\rho_S - \rho_R}{\rho_R}\right) \text{ cm}$$

A particular case occurs when a ship moves from sea water of 1·025 t/m³ into fresh water of 1·000 t/m³, the TPC being given in the sea water.

$$\text{Increase in draught} = \frac{\triangle}{\text{TPC}}\left(\frac{1{\cdot}025 - 1{\cdot}000}{1{\cdot}000}\right)$$

$$= \frac{\triangle}{40\ \text{TPC}} \text{ cm}$$

This is known as the *fresh water allowance*, used when computing the freeboard of a ship and is the difference between the S line and the F line on the freeboard markings.

Example. A ship of 10 000 tonne displacement has a waterplane area of 1300 m². The ship loads in water of 1·010 t/m³ and moves into water of 1·026 t/m³. Find the change in mean draught.

Since the vessel moves into water of a greater density there will be a reduction in mean draught.

$$\text{Reduction in mean draught} = \frac{100\,\triangle}{A_w}\left(\frac{\rho_S - \rho_R}{\rho_R \times \rho_S}\right) \text{ cm}$$

$$= \frac{100 \times 10\,000}{1300}\left(\frac{1{\cdot}026 - 1{\cdot}010}{1{\cdot}010 \times 1{\cdot}026}\right)$$

$$= 11{\cdot}88 \text{ cm}$$

When a vessel moves from water of one density to water of a different density, there may be a change in displacement due to the consumption of fuel and stores, causing an additional change in mean draught. If the vessel moves from sea water into river water, it is possible in certain circumstances for the increase in draught due to change in density to be equal to the reduction in draught due to the removed mass. In such a case there will be no change in mean draught.

Example. 215 tonne of oil fuel and stores are used in a ship while passing from sea water of 1·026 t/m³ into river water of 1·002 t/m³. If the mean draught remains unchanged, calculate the displacement in the river water.

Let $\triangle$ = displacement in river water

Then $\triangle + 215$ = displacement in sea water

Since the draught remains unaltered, the volume of displacement in the river water must be equal to the volume of displacement in the sea water.

$$\nabla_R = \frac{\triangle}{\rho_R}$$

$$= \frac{\triangle}{1{\cdot}002} \text{ m}^3$$

$$\nabla_S = \frac{\triangle + 215}{\rho_S}$$

$$= \frac{\triangle + 215}{1{\cdot}026} \text{ m}^3$$

Hence

$$\nabla_R = \nabla_S$$

$$\frac{\triangle}{1{\cdot}002} = \frac{\triangle + 215}{1{\cdot}026}$$

$$1{\cdot}026\,\triangle = 1{\cdot}002\,\triangle + 1{\cdot}002 \times 215$$

$$0{\cdot}024\,\triangle = 1{\cdot}002 \times 215$$

$$\triangle = \frac{1{\cdot}002 \times 215}{0{\cdot}024}$$

$$= 8976 \text{ tonne}$$

f CHANGE IN TRIM DUE TO CHANGE IN DENSITY

When a ship passes from sea water into river water, or vice versa, without change in displacement, there is a change in trim in addition to the change in mean draught. This change in trim is always small.

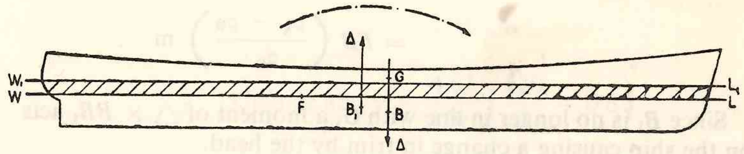


Fig. 66

Consider a ship of displacement $\triangle$ lying at waterline WL in sea water of density ρ_S t/m³. The centre of gravity *G* and the centre of buoyancy *B* are in the same vertical line.

If the vessel now moves into river water of ρ_R t/m³, there is a bodily increase in draught and the vessel lies at waterline W_1L_1. The volume of displacement has been increased by a layer of volume *v* whose centre of gravity is at the centre of flotation *F*. This causes the centre of buoyancy to move from *B* to B_1, the centre of gravity remaining at *G*.

Volume of displacement in sea water

$$\nabla_S = \frac{\triangle}{\rho_S} \text{ m}^3$$

Volume of displacement in river water

$$\nabla_R = \frac{\triangle}{\rho_R}\ \text{m}^3$$

Change in volume of displacement

$$v = \nabla_R - \nabla_S$$

$$= \triangle\left(\frac{1}{\rho_R} - \frac{1}{\rho_S}\right)$$

$$= \triangle\left(\frac{\rho_S - \rho_R}{\rho_R \times \rho_S}\right)\ \text{m}^3$$

Shift in centre of buoyancy

$$BB_1 = \frac{v \times FB}{\nabla_R}$$

$$= \triangle\left(\frac{\rho_S - \rho_R}{\rho_R \times \rho_S}\right) FB \times \frac{\rho_R}{\triangle}$$

$$= FB\left(\frac{\rho_S - \rho_R}{\rho_S}\right)\ \text{m}$$

Since B_1 is no longer in line with G, a moment of $\triangle \times BB_1$ acts on the ship causing a change in trim by the head.

$$\text{Change in trim} = \frac{\triangle \times BB_1}{\text{MCTI cm}}\ \text{cm}$$

$$= \frac{\triangle\ FB}{\text{MCTI cm}}\left(\frac{\rho_S - \rho_R}{\rho_S}\right)\ \text{cm by the head}$$

Note: If the ship moves from the river water into sea water, it will change trim by the stern, and:

$$\text{Change in trim} = \frac{\triangle\ FB}{\text{MCTI cm}}\left(\frac{\rho_S - \rho_R}{\rho_R}\right)\ \text{cm by the stern}$$

Example. A ship 120 m long and 9100 tonne displacement floats at a level keel draught of 6·50 m in fresh water of 1·000 t/m³. MCTI cm 130 tonne m, TPC in sea water 16·5, LCB 2·30 m forward of midships. LCF 0·6 m aft of midships.

Calculate the new draughts if the vessel moves into sea water of 1·024 t/m³ without change in displacement.

$$\text{Reduction in mean draught} = \frac{\triangle}{\text{TPC}}\left(\frac{\rho_S - \rho_R}{\rho_R}\right)$$

$$= \frac{9100}{16\cdot5}\left(\frac{1\cdot024 - 1\cdot000}{1\cdot000}\right)$$

$$= 13\cdot24\ \text{cm}$$

$$\text{Change in trim} = \frac{\triangle\ FB}{\text{MCTI cm}}\left(\frac{\rho_S - \rho_R}{\rho_R}\right)$$

$$= \frac{9100 \times (2\cdot30 + 0\cdot60)}{130}\left(\frac{1\cdot024 - 1\cdot000}{1\cdot000}\right)$$

$$= 4\cdot87\ \text{cm by the stern}$$

$$\text{Change forward} = -\frac{4\cdot87}{120}\left(\frac{120}{2} + 0\cdot6\right)$$

$$= -\ 2\cdot46\ \text{cm}$$

$$\text{Change aft} = +\frac{4\cdot87}{120}\left(\frac{120}{2} - 0\cdot6\right)$$

$$= +\ 2\cdot41\ \text{cm}$$

$$\text{New draught forward} = 6\cdot50 - 0\cdot132 - 0\cdot025$$

$$= 6\cdot343\ \text{m}$$

$$\text{New draught aft} = 6\cdot50 - 0\cdot132 + 0\cdot024$$

$$= 6\cdot392\ \text{m}$$

CHANGE IN MEAN DRAUGHT DUE TO BILGING

BUOYANCY is the upthrust exerted by the water on the ship and depends upon the volume of water displaced by the ship up to the waterline.

RESERVE BUOYANCY is the potential buoyancy of a ship and depends upon the intact, watertight volume above the waterline. When a mass is added to a ship, or buoyancy is lost due to bilging, the reserve buoyancy is converted into buoyancy by increasing the draught. If the loss in buoyancy exceeds the reserve buoyancy the vessel will sink.

Permeability μ is the volume of a compartment into which water may flow if the compartment is laid open to the sea, expressed as a ratio or percentage of the total volume of the compartment. Thus if a compartment is completely empty, the permeability is 100 per cent. The permeability of a machinery space is about 85 per cent and accommodation about 95 per cent. The permeability of a cargo hold varies considerably with the type of cargo, but an average value may be taken as 60 per cent.

The effects of bilging a mid-length compartment may be shown most simply by considering a box barge of length L, breadth B and draught d having a mid-length compartment of length l, permeability μ.

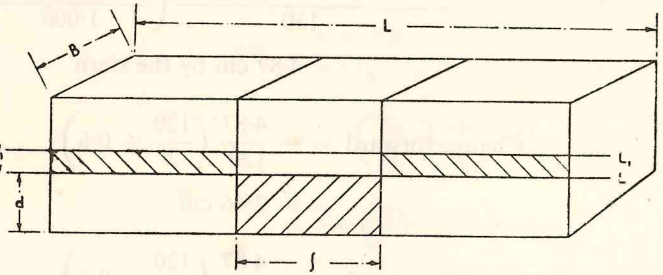


Fig. 67

If this compartment is bilged, buoyancy is lost and must be replaced by increasing the draught. The volume of buoyancy lost is the volume of the compartment up to waterline WL, less the volume of water excluded by the cargo in the compartment.

$$\text{Volume of lost buoyancy} = \mu\, lBd$$

This is replaced by the increase in draught multiplied by the area of the intact part of the waterplane, i.e. the area of waterplane on each side of the bilged compartment plus the area of cargo which projects through the waterplane in the bilged compartment.

$$\begin{aligned}\text{Area of intact waterplane} &= (L-l)B + lB(1-\mu)\\ &= LB - lB + lB - \mu lB\\ &= (L-\mu l)B\end{aligned}$$

$$\begin{aligned}\text{Increase in draught} &= \frac{\text{volume of lost buoyancy}}{\text{area of intact waterplane}}\\ &= \frac{\mu\, lBd}{(L-\mu l)B}\\ &= \frac{\mu ld}{L-\mu l}\end{aligned}$$

μl may be regarded as the *effective length* of the bilged compartment.

Example. A box barge 30 m long and 8 m beam floats at a level keel draught of 3 m and has a mid-length compartment 6 m long. Calculate the new draught if this compartment is bilged:
(a) with $\mu = 100\%$
(b) with $\mu = 75\%$

(a) Volume of lost buoyancy $= 6 \times 8 \times 3\ \text{m}^3$
Area of intact waterplane $= (30-6) \times 8\ \text{m}^2$

$$\begin{aligned}\text{Increase in draught} &= \frac{6 \times 8 \times 3}{24 \times 8}\\ &= 0{\cdot}75\ \text{m}\\ \text{new draught} &= 3 + 0{\cdot}75\\ &= 3{\cdot}75\ \text{m}\end{aligned}$$

(b) Volume of lost buoyancy $= 0{\cdot}75 \times 6 \times 8 \times 3\ \text{m}^3$
Area of intact waterplane $= (30 - 0{\cdot}75 \times 6) \times 8\ \text{m}^2$

$$\begin{aligned}\text{Increase in draught} &= \frac{0{\cdot}75 \times 6 \times 8 \times 3}{25{\cdot}5 \times 8}\\ &= 0{\cdot}529\ \text{m}\\ \text{New draught} &= 3 + 0{\cdot}529\\ &= 3{\cdot}529\ \text{m}\end{aligned}$$

CHANGE IN DRAUGHTS DUE TO BILGING AN END COMPARTMENT

If a bilged compartment does not lie at the mid-length, then there is a change in trim in addition to the change in mean draught.

Consider a box barge of length L, breadth B and draught d having an empty compartment of length l at the extreme fore end.

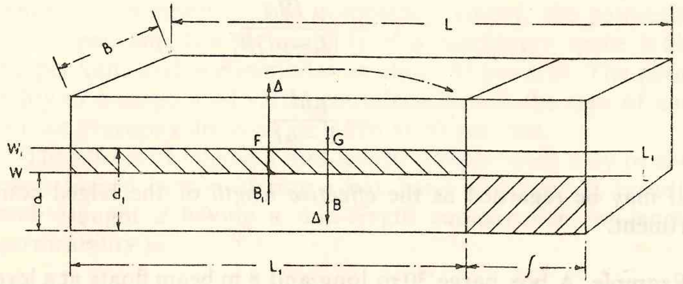


Fig. 68

Before bilging the vessel lies at waterline WL, the centre of gravity G and the centre of buoyancy B lying in the same vertical line.

After bilging the end compartment, the vessel lies initially at waterline W_1L_1. The new mean draught d_1 may be calculated as shown previously assuming that the compartment is amidships.

The volume of lost buoyancy has been replaced by a layer whose centre is at the middle of the length L_1. This causes the centre of buoyancy to move aft from B to B_1, a distance of $\frac{1}{2}l$. Thus a moment of $\triangle \times BB_1$ acts on the ship causing a *considerable* change in trim by the head. The vessel changes trim about the centre of flotation F which is the centroid of the *intact* waterplane i.e. the mid-point of L_1.

$$\text{Trimming moment} = \triangle \times BB_1$$

$$\text{Change in trim} = \frac{\triangle \times BB_1}{\text{MCTI cm}} \text{ cm by the head}$$

$$\text{MCTI cm} = \frac{\triangle \times GM_L}{100\,L} \text{ tonne m}$$

GM_L must be calculated for the *intact* waterplane

$$KB_1 = \frac{d_1}{2}$$

$$B_1M_L = \frac{L_1{}^3\,B}{12\,\nabla}$$

$$\text{where } \nabla = L\,B\,d = L_1\,Bd_1$$

$$GM_L = KB_1 + B_1M_L - KG$$

$$\text{Change in trim} = \frac{\triangle \times \frac{1}{2}l}{\triangle \times GM_L} \times 100\,L$$

$$= \frac{50\,L\,l}{GM_L} \text{ cm by the head.}$$

Example. A box barge 120 m long and 8 m beam floats at an even keel draught of 3 m and has an empty compartment 6 m long at the extreme fore end. The centre of gravity is 2·8 m above the keel. Calculate the final draughts if this compartment is bilged.

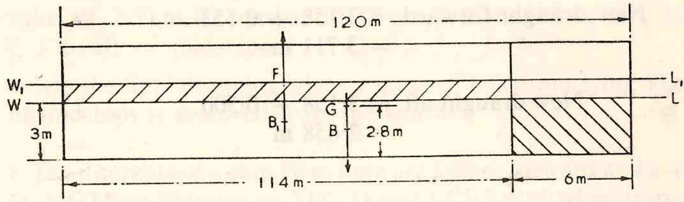


Fig. 69

$$\text{Increase in mean draught} = \frac{6 \times 8 \times 3}{(120 - 6) \times 8}$$

$$= 0{\cdot}158 \text{ m}$$

$$\text{New draught } d_1 = 3{\cdot}158 \text{ m}$$

$$KB_1 = \frac{d_1}{2}$$

$$= 1{\cdot}579 \text{ m}$$

$$B_1M_L = \frac{114^3 \times 8}{12 \times 120 \times 8 \times 3}$$

$$= 342{\cdot}94 \text{ m}$$

$$GM_L = 1{\cdot}58 + 342{\cdot}94 - 2{\cdot}80$$

$$= 341{\cdot}72 \text{ m}$$

$$\text{Change in trim} = \frac{50 \times 120 \times 6}{341{\cdot}72}$$

$$= 105{\cdot}3 \text{ cm by the head}$$

$$\text{Change forward} = + \frac{105{\cdot}3}{120} \times \left(\frac{120}{2} + 3\right)$$

$$= + 55{\cdot}3 \text{ cm}$$

$$\text{Change aft} = - \frac{105{\cdot}3}{120} \times \left(\frac{120}{2} - 3\right)$$

$$= - 50{\cdot}0 \text{ cm}$$

$$\text{New draught forward} = 3{\cdot}158 + 0{\cdot}553 = 3{\cdot}711 \text{ m}$$

$$\text{New draught aft} = 3{\cdot}158 - 0{\cdot}500 = 2{\cdot}658 \text{ m}$$

TEST EXAMPLES 6

*f*1. A ship 125 m long displaces 12 000 tonne. When a mass of 100 tonne is moved 75 m from forward to aft there is a change in trim of 65 cm by the stern. Calculate:

(a) MCTI cm
(b) the longitudinal metacentric height
(c) the distance moved by the centre of gravity of the ship.

*f*2. A ship 120 m long floats at draughts of 5·50 m forward and 5·80 m aft; MCTI cm 80 tonne m, TPC 13, LCF 2·5 m forward of midships. Calculate the new draughts when a mass of 110 tonne is added 24 m aft of midships.

*f*3. A ship 130 m long displaces 14 000 tonne when floating at draughts of 7·50 m forward and 8·10 m aft. GM_L 125 m, TPC 18, LCF 3 m aft of midships.

Calculate the final draughts when a mass of 180 tonne lying 40 m aft of midships is removed from the ship.

*f*4. The draughts of a ship 90 m long are 5·80 m forward and 6·40 m aft. MCTI cm 50 tonne m; TPC 11 and LCF 2 m aft of midships. Determine the point at which a mass of 180 tonne should be placed so that the after draught remains unaltered, and calculate the final draught forward.

*f*5. A ship 150 m long floats at draughts of 8·20 m forward and 8·90 m aft. MCTI cm 260 tonne m, TPC 28 and LCF 1·5 m aft of midships. It is necessary to bring the vessel to an even keel and a double bottom tank 60 m forward of midships is available.

Calculate the mass of water required and the final draught.

*f*6. A ship whose length is 110 m has MCTI cm 55 tonne m; TPC 9; LCF 1·5 m forward of midships and floats at draughts of 4·20 m forward and 4·45 m aft.

Calculate the new draughts after the following masses have been added:

20 tonne 40 m aft of midships
50 tonne 23 m aft of midships
30 tonne 2 m aft of midships
70 tonne 6 m forward of midships
15 tonne 30 m forward of midships.

*f*7. The draughts of a ship 170 m long are 6·85 m forward and 7·50 m aft. MCTI cm 300 tonne m; TPC 28; LCF 3·5 m forward of midships.

Calculate the new draughts after the following changes in loading have taken place;

160 tonne added 63 m aft of midships
200 tonne added 27 m forward of midships
120 tonne removed 75 m aft of midships
70 tonne removed 16 m aft of midships.

*f*8. A ship 80 m long has a light displacement of 1050 tonne and LCG 4·64 m aft of midships.

The following items are then added:

Cargo	2150 tonne,	Lcg 4·71 m forward of midships
Fuel	80 tonne,	Lcg 32·55 m aft of midships
Water	15 tonne,	Lcg 32·90 m aft of midships
Stores	5 tonne,	Lcg 33·60 m forward of midships.

The following hydrostatic particulars are available.

Draught m	Displacement tonne	MCTI cm tonne m	LCB from midships m	LCF from midships m
5·00	3533	43·10	1·00F	1·27A
4·50	3172	41·26	1·24F	0·84A

Calculate the final draughts of the loaded vessel.

9. A ship of 15 000 tonne displacement has a waterplane area of 1950 m². It is loaded in river water of 1·005 t/m³ and proceeds to sea where the density is 1·022 t/m³.

Calculate the change in mean draught.

10. A ship of 7000 tonne displacement has a waterplane area of 1500 m². In passing from sea water into river water of 1005 kg/m³ there is an increase in draught of 10 cm. Find the density of the sea water.

11. The ½ ordinates of the waterplane of a ship of 8200 tonne displacement, 90 m long, are 0, 2·61, 3·68, 4·74, 5·84, 7·00, 7·30, 6·47, 5·35, 4·26, 3·16, 1·88 and 0 m respectively. It floats in sea water of 1·024 t/m³. Calculate:

(a) TPC
(b) mass necessary to increase the mean draught by 12 cm
(c) change in mean draught when moving into water of 1·005 t/m³.

12. A ship consumes 360 tonne of fuel, stores and water when moving from sea water of 1·025 t/m³ into fresh water of 1·000 t/m³ and on arrival it is found that the draught has remained constant.

Calculate the displacement in the sea water.

*f*13. A ship 90 m long displaces 5200 tonne and floats at draughts of 4·95 m forward and 5·35 m aft when in sea water of 1023 kg/m³. The waterplane area is 1100 m², GM_L 95 m, LCB 0·6 m forward of midships and LCF 2·2 m aft of midships.

Calculate the new draughts when the vessel moves into fresh water of 1002 kg/m³.

*f*14. A ship of 22 000 tonne displacement is 160 m long, MCTI cm 280 tonne m, waterplane area 3060 m², centre of buoyancy 1 m aft of midships and centre of flotation 4 m aft of midships. It floats in water of 1·007 t/m³ at draughts of 8·15 m forward and 8·75 m aft.

Calculate the new draughts if the vessel moves into sea water of 1·026 t/m³.

15. A box barge 60 m long and 10 m wide floats at an even keel draught of 4 m. It has a compartment amidships 12 m long.

Calculate the new draught if this compartment is laid open to the sea when:

(a) μ is 100%
(b) μ is 85%
(c) μ is 60%.

16. A box barge 50 m long and 8 m wide floats at a draught of 3 m and has a mid-length compartment 9 m long containing coal (rd 1·28) which stows at 1·22 m³/t.

Calculate the new draught if this compartment is bilged.

17. A vessel of constant rectangular cross-section is 60 m long, 12 m beam and floats at a draught of 4·5 m. It has a mid-length compartment 9 m long which extends right across the ship and up to the deck, but is sub-divided by a horizontal watertight flat 3 m above the keel.

Find the new draught if this compartment is bilged:

(a) below the flat
(b) above the flat.

18. A box barge 25 m long and 4 m wide floats in fresh water at a draught of 1·2 m and has an empty mid-length compartment 5 m long. The bottom of the barge is lined with teak (rd 0·805) 120 mm thick. After grounding all the teak is torn off and the centre compartment laid open to the sea. Calculate the final draught.

f 19. A box barge 100 m long, 12 m beam and 4 m draught has a compartment at the extreme fore end 8 m long, sub-divided by a horizontal watertight flat 2 m above the keel. The centre of gravity is 3 m above the keel.

Calculate the end draughts if the compartment is bilged.

(a) at the flat, water flowing into both compartments
(b) below the flat
(c) above the flat.

CHAPTER 7

RESISTANCE

When a ship moves through the water at any speed, a force or resistance is exerted by the water on the ship. The ship must therefore exert an equal thrust to overcome the resistance and travel at that speed. If, for example, the resistance of the water on the ship at 17 knots is 800 kN, and the ship provides a thrust of 800 kN, then the vessel will travel at 17 knots.

The total resistance or tow-rope resistance R_t of a ship may be divided into two main sections:

(a) frictional resistance R_f
(b) residuary resistances R_r

Hence $$R_t = R_f + R_r$$

Frictional resistance R_f

As the ship moves through the water, friction between the hull and the water causes a belt of eddying water adjacent to the hull to be drawn along with the ship, although at a reduced speed. The belt moves aft and new particles of water are continually set in motion, the force required to produce this motion being provided by the ship.

The frictional resistance of a ship depends upon:

(i) the speed of the ship
(ii) the wetted surface area
(iii) the length of the ship
(iv) the roughness of the hull
(v) the density of the water.

Wm Froude developed the formula:

$$R_f = f\,S\,V^n \text{ N}$$

where f is a coefficient which depends upon the length of the ship L, the roughness of the hull and the density of the water.
S is the wetted surface area in m^2
V is the ship speed in knots.
n is an index of about 1·825

The value of f for a mild steel hull in sea water is given by :

$$f = 0{\cdot}417 + \frac{0{\cdot}773}{L + 2{\cdot}862}$$

Thus f is reduced as the length of the ship is increased.

In a slow or medium-speed ship the frictional resistance forms the major part of the total resistance, and may be as much as 75% of R_t. The importance of surface roughness may be seen when a ship is badly fouled with marine growth or heavily corroded, when the speed of the ship may be considerably reduced.

1 knot = 1·852 km/h

Example. A ship whose wetted surface area is 5150 m² travels at 15 knots. Calculate the frictional resistance and the power required to overcome this resistance.

$f = 0{\cdot}422, \qquad n = 1{\cdot}825$

$$\begin{aligned} R_f &= f\,S\,V^n \\ &= 0{\cdot}422 \times 5150 \times 15^{18{\cdot}25} \\ &= 303\,700\ N \\ \text{Power} &= R_f\,(\text{N}) \times v\,(\text{m/s}) \\ &= 303\,700 \times 15 \times \frac{1852}{3600}\ \text{W} \\ &= 2344\ \text{kW} \end{aligned}$$

Example. A plate drawn through fresh water at 3m/s has a frictional resistance of 12 N/m². Estimate the power required to overcome the frictional resistance of a ship at 12 knots if the wetted surface area is 3300 m² and the index of speed is 1·9.

$$\begin{aligned} 12\ \text{knots} &= 12 \times \frac{1852}{3600} \\ &= 6{\cdot}175\ \text{m/s} \end{aligned}$$

At 3 m/s

$$\begin{aligned} R_f &= 12 \times 3300 \\ &= 39\,600\ \text{N in FW} \\ &= 39\,600 \times 1{\cdot}025\ \text{N in SW} \end{aligned}$$

At 12 knots.

$$\begin{aligned} R_f &= 39\,600 \times 1{\cdot}025 \times \left(\frac{6{\cdot}175}{3}\right)^{1{\cdot}9} \\ &= 160\,000\ \text{N} \\ \text{Power} &= 160\,000 \times 6{\cdot}175 \\ &= 988\,000\ \text{W} \\ &= 988{\cdot}0\ \text{kW} \end{aligned}$$

Residuary resistances R_r

The residuary resistances of a ship may be divided into:

i) Resistance caused by the formation of streamlines round the ship, i.e. due to the change in the direction of the water. If the water changes direction abruptly, such as round a box barge, the resistance may be considerable, but in modern, well-designed ships should be very small.

ii) Eddy resistance caused by sudden changes in form. This resistance will be small in a ship where careful attention is paid to detail. The eddy resistance due to fitting rectangular sternframe and single plate rudder may be as much as 5% of the total resistance of the ship. By streamlining the sternframe and fitting a double plate rudder, eddy resistance is practically negligible.

iii) Resistance caused by the formation of waves as the ship passes through the water. In slow or medium-speed ships the wavemaking resistance is small compared with the frictional resistance. At high speeds, however, the wave making resistance is considerably increased and may be 50% or 60% of the total resistance.

Several attempts have been made to reduce the wave making resistance of ships, with varying degrees of success. One method which has proved to be successful is the use of the bulbous bow. The wave produced by the bulb interferes with the wave produced by the stem, resulting in a reduced height of bow wave and consequent reduction in the energy required to produce the wave.

The relation between the frictional resistance and the residuary resistances is shown in Fig. 70.

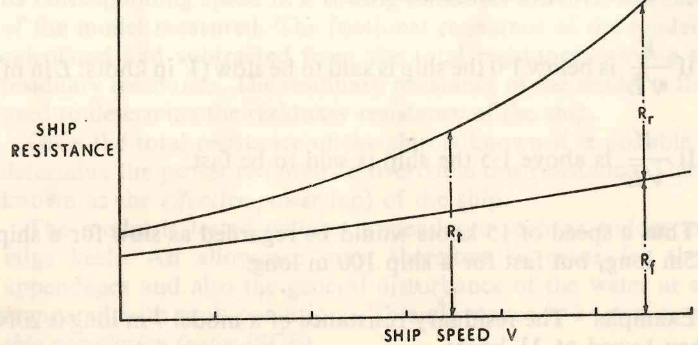


Fig. 70

Residuary Resistances follow Froude's Law of Comparison:

The residuary resistances of similar ships are in the ratio of the cube of their linear dimensions if their speeds are in the ratio of the square root of their linear dimensions.

$$\text{Thus} \quad \frac{R_{r1}}{R_{r2}} = \left(\frac{L_1}{L_2}\right)^3 \quad \text{if} \quad \frac{V_1}{V_2} = \sqrt{\frac{L_1}{L_2}}$$

$$\text{or} \quad \frac{R_{r1}}{R_{r2}} = \frac{\triangle_1}{\triangle_2} \quad \text{if} \quad \frac{V_1}{V_2} = \left(\frac{\triangle_1}{\triangle_2}\right)^{\frac{1}{6}}$$

V_1 and V_2 are termed *corresponding speeds.*

Thus at corresponding speeds:

$$\frac{V_1}{\sqrt{L_1}} = \frac{V_2}{\sqrt{L_2}}$$

$\dfrac{V}{\sqrt{L}}$ is known as the *speed-length ratio.*

It may therefore be seen that at corresponding speeds the wave making characteristics of similar ships are the same. At high speeds the speed-length ratio is high and the wave making resistance is large. To give the same wave making characteristics, the corresponding speed of a much smaller, similar ship will be greatly reduced and may not be what is popularly regarded to be a high speed. A ship is therefore considered slow or fast in relation to its speed-length ratio.

If $\dfrac{V}{\sqrt{L}}$ is below 1·0 the ship is said to be slow (V in knots: L in m)

If $\dfrac{V}{\sqrt{L}}$ is above 1·5 the ship is said to be fast.

Thus a speed of 15 knots would be regarded as slow for a ship 225m long, but fast for a ship 100 m long.

Example. The residuary resistance of a model 7 m long is 20N when towed at 3½ knots.

Calculate the power required to overcome the residuary resistance of a similar ship 140 m long at its corresponding speed.

$$V_2 = V_1 \sqrt{\frac{L_2}{L_1}}$$

$$= 3{\cdot}5 \sqrt{\frac{140}{7}}$$

$$= 15{\cdot}65 \text{ knots.}$$

$$R_{r2} = R_{r1} \left(\frac{L_2}{L_1}\right)^3$$

$$= 20 \left(\frac{140}{7}\right)^3$$

$$= 160\,000 \text{ N}$$

$$\text{Power} = R_r \times v$$

$$160\,000 \times 15{\cdot}65 \times \frac{1852}{3600}$$

$$= 1288 \text{ kW}$$

The calculation of residuary resistance is usually based on the results of model experiments. A wax model of the ship is towed at its corresponding speed in a towing tank and the total resistance of the model measured. The frictional resistance of the model is calculated and subtracted from the total resistance, leaving the residuary resistance. The residuary resistance of the model is then used to determine the residuary resistance of the ship.

Once the total resistance of the ship is known it is possible to determine the power required to overcome this resistance. This is known as the *effective power* (ep) of the ship.

The model is tested without appendages such as rudder and bilge keels. An allowance must therefore be made for these appendages and also the general disturbance of the water at sea compared with tank conditions, This allowance is known as the *ship correlation factor* (SCF).

The power obtained directly from the model tests is known as the *effective power* (*naked*) (ep_n). The true effective power is the ep_n multiplied by the ship correlation factor.

Example. A 6 m model of a ship has a wetted surface area of 8 m². When towed at a speed of 3 knots in fresh water the total resistance is found to be 38N.

If the ship is 130 m long, calculate the effective power at the corresponding speed.

Take $n = 1{\cdot}825$ and calculate f from the formula. SCF 1·15

Model

$$R_t = 38 \text{ N in fresh water}$$
$$= 38 \times 1{\cdot}025$$
$$= 38{\cdot}95 \text{ N in sea water}$$

$$f = 0{\cdot}417 + \frac{0{\cdot}773}{L + 2{\cdot}862}$$
$$= 0{\cdot}417 + \frac{0{\cdot}773}{8{\cdot}862}$$
$$= 0{\cdot}504$$
$$R_f = 0{\cdot}504 \times 8 \times 3^{1{\cdot}825}$$
$$= 29{\cdot}94 \text{ N}$$
$$R_r = R_t - R_f$$
$$= 38{\cdot}95 - 29{\cdot}94$$
$$= 9{\cdot}01 \text{ N}$$

Ship

$$R_r \propto L^3$$
$$\therefore \quad R_r = 9{\cdot}01 \times \left(\frac{130}{6}\right)^3$$
$$= 91\,600 \text{ N}$$
$$S \propto L^2$$
$$\therefore \quad S = 8 \times \left(\frac{130}{6}\right)^2$$
$$= 3755 \text{ m}^2$$

$$V \propto \sqrt{L}$$
$$\therefore \quad V = 3\sqrt{\frac{130}{6}}$$
$$= 13{\cdot}96 \text{ knots}$$
$$f = 0{\cdot}417 + \frac{0{\cdot}773}{132{\cdot}862}$$
$$= 0{\cdot}4228$$
$$R_f = 0{\cdot}4228 \times 3755 \times 13{\cdot}96^{1{\cdot}825}$$
$$= 195\,000 \text{ N}$$
$$R_t = 195\,000 + 91\,600$$
$$= 286\,600 \text{ N}$$
$$ep_n = 286\,600 \times 13{\cdot}96 \times \frac{1852}{3600}$$
$$= 2059 \text{ kW}$$

Effective power

$$ep = 2059 \times 1{\cdot}15$$
$$= 2368 \text{ kW}$$

ADMIRALTY COEFFICIENT

It is sometimes necessary to obtain an approximation to the power of a ship without resorting to model experiments, and several methods are available. One system which has been in use for several years is the Admiralty Coefficient method. This is based on the assumption that for small variations in speed the total resistance may be expressed in the form:

$$Rt \propto \rho S V^n$$

It was seen earlier that

$$S \propto \Delta^{\frac{2}{3}}$$

Hence with constant density

$$R_t \propto \Delta^{\frac{2}{3}} V^n$$

But

$$\text{power} \propto R_t \times V$$
$$\propto \Delta^{\frac{2}{3}} V^{n+1}$$

or

$$\text{power} = \frac{\Delta^{\frac{2}{3}} V^{n+1}}{\text{a coefficient}}$$

The coefficient is known as the Admiralty Coefficient.

Originally this method was used to determine the power supplied by the engine. Since types of machinery vary considerably it is now considered that the relation between displacement, speed and shaft power (sp) is of more practical value.

Most merchant ships may be classed as slow or medium-speed, and for such vessels the index n may be taken as 2. Thus

$$\text{Admiralty Coefficient } C = \frac{\triangle^{\frac{2}{3}} V^3}{\text{sp}}$$

where $\triangle$ = displacement in tonne
V = ship speed in knots
sp = shaft power in kW

The Admiralty Coefficient may be regarded as constant for similar ships at their corresponding speeds. Values of C vary between about 350 and 600 for different ships, the higher values indicating more efficient ships.

For small changes in speed, the value of C may be regarded as constant for any ship at constant displacement.

At corresponding speeds $V \propto \triangle^{\frac{1}{6}}$

$$\therefore \quad V^3 \propto \triangle^{\frac{3}{6}}$$

$$\text{sp} \propto \triangle^{\frac{2}{3}} V^3$$

$$\propto \triangle^{\frac{2}{3}} \times \triangle^{\frac{3}{6}}$$

$$\propto \triangle^{\frac{7}{6}}$$

$$\text{ie} \quad \frac{\text{sp}_1}{\text{sp}_2} = \left(\frac{\triangle_1}{\triangle_2}\right)^{\frac{7}{6}}$$

Thus if the shaft power of one ship is known, the shaft power for a similar ship may be obtained at the corresponding speed.

Example. A ship of 14 000 tonne displacement has an Admiralty Coefficient of 450.
Calculate the shaft power required at 16 knots.

$$\text{sp} = \frac{\triangle^{\frac{2}{3}} V^3}{C}$$

$$= \frac{14\,000^{\frac{2}{3}} \times 16^3}{450}$$

$$= 5286 \text{ kW}$$

Example. A ship of 15 000 tonne displacement requires 3500 kW at a particular speed.
Calculate the shaft power required by a similar ship of 18 000 tonne displacement at its corresponding speed.

$$\text{sp} \propto \triangle^{7/6}$$

$$\therefore \quad \text{sp} = 3500 \times \left(\frac{18\,000}{15\,000}\right)^{7/6}$$

$$= 4330 \text{ kW}$$

The index of speed n for high speed ships may be considerably more than 2 and thus the shaft power may vary as the speed to some index greater than 3 (e.g. 4). This higher index, however, is only applicable within the high speed range.

$$\text{i.e.} \quad \frac{\text{sp}_1}{\text{sp}_2} = \left(\frac{V_1}{V_2}\right)^4$$

where both V_1 and V_2 are within the high speed range.

Example. A ship travelling at 20 knots requires 12 000 kW shaft power.
Calculate the shaft power at 22 knots if, within this speed range, the index of speed is 4.

$$\frac{\text{sp}_1}{\text{sp}_2} = \left(\frac{V_1}{V_2}\right)^4$$

$$\text{sp}_2 = 12\,000 \times \left(\frac{22}{20}\right)^4$$

$$= 17\,570 \text{ kW}$$

FUEL COEFFICIENT AND FUEL CONSUMPTION

The fuel consumption of a ship depends upon the power developed, indeed the overall efficiency of power plant is often measured in terms of the *specific fuel consumption* which is the consumption per unit of power, expressed in kg/h. Efficient diesel engines may have a specific fuel consumption of about 0·20 kg/kW h, while that for a steam turbine may be about 0·30 kg/kW h. The specific fuel consumption of a ship at different speeds follows the form shown in Fig. 71.

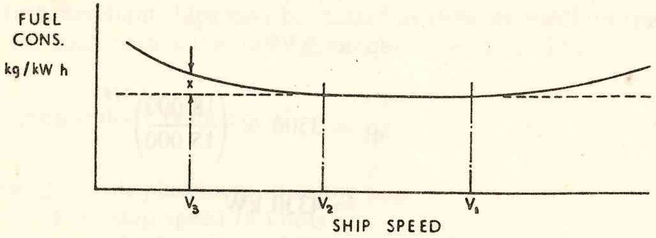


Fig. 71

Between V_1 and V_2 the specific consumption may be regarded as constant for practical purposes, and if the ship speed varies only between these limits, then:

Fuel consumption / unit time $\propto$ power developed

$$\propto sp$$

and since $sp \propto \Delta^{\frac{2}{3}} V^3$

Fuel consumption/unit time $\propto \Delta^{\frac{2}{3}} V^3$

$$\text{or} \quad \text{Fuel consumption / day} = \frac{\Delta^{\frac{2}{3}} V^3}{\text{fuel coefficient}} \text{ tonne}$$

Values of fuel coefficient vary between about 40 000 and 120 000, the higher values indicating more efficient ships.

Example. The fuel coefficient of a ship of 14 000 tonne displacement is 75 000. Calculate the fuel consumption per day if the vessel travels at 12½ knots.

$$\text{Fuel consumption per day} = \frac{14\,000^{2/3} \times 12{\cdot}5^3}{75\,000}$$

$$= 15{\cdot}12 \text{ tonne}$$

If the displacement and fuel coefficient remain constant, i.e. between V_1 and V_2 Fig. 71:

Fuel consumption / unit time $\propto$ speed^3

$$\text{Hence} \quad \frac{\text{cons}_1}{\text{cons}_2} = \left(\frac{V_1}{V_2}\right)^3$$

Example. A ship uses 20 tonne of fuel per day at 13 knots. Calculate the daily consumption at 11 knots.

$$\text{New daily consumption} = 20 \times \left(\frac{11}{13}\right)^3$$

$$= 12{\cdot}11 \text{ tonne.}$$

The total fuel consumption for any voyage may be found by multiplying the daily consumption by the number of days required to complete the voyage.

If D is the distance travelled at V knots, then:

$$\text{Number of days} \propto \frac{D}{V}$$

$$\text{But} \quad \text{daily consumption} \propto V^3$$

$$\therefore \text{ total voyage consumption} \propto V^3 \times \frac{D}{V}$$

$$\propto V^2 D$$

$$\text{i.e.} \quad \frac{\text{voy. cons}_1}{\text{voy. cons}_2} = \left(\frac{V_1}{V_2}\right)^2 \times \frac{D_1}{D_2}$$

Hence for any given distance travelled the voyage consumption varies as the speed *squared.*

Example. A vessel uses 125 tonne of fuel on a voyage when travelling at 16 knots. Calculate the mass of fuel saved if, on the return voyage, the speed is reduced to 15 knots, the displacement of the ship remaining constant.

$$\text{New voyage consumption} = 125 \left(\frac{15}{16}\right)^2$$

$$= 110 \text{ tonne}$$

$$\therefore \text{ Saving in fuel} = 125 - 110$$

$$= 15 \text{ tonne}$$

A general expression for voyage consumption is:

$$\frac{\text{new voy. cons.}}{\text{old voy. cons.}} = \left(\frac{\text{new displ.}}{\text{old displ.}}\right)^{\frac{2}{3}} \times \left(\frac{\text{new speed}}{\text{old speed}}\right)^2 \times \frac{\text{new dist.}}{\text{old dist.}}$$

All of the above calculations are based on the assumption that the ship speed lies between V_1 and V_2 Fig. 71. If the speed is reduced to V_3, however, the specific consumption may be increased by $x\%$. In this case the daily consumption and voyage consumption are also increased by $x\%$.

Example. A ship has a daily fuel consumption of 30 tonne at 15 knots. The speed is reduced to 12 knots and at this speed the consumption per unit power is 8% more than at 15 knots. Calculate the new consumption per day.

$$\text{New daily consumption} = 1{\cdot}08 \times 30 \times \left(\frac{12}{15}\right)^3$$

$$= 16{\cdot}6 \text{ tonne}$$

It should be noted that if a formula for fuel consumption is given in any question, the formula must be used for the *complete question.*

TEST EXAMPLES 7

1. A ship has a wetted surface area of 3200 m². Calculate the power required to overcome frictional resistance at 17 knots if $n = 1{\cdot}825$ and $f = 0{\cdot}424$.

2. A plate towed edgewise in sea water has a resistance of 13 N/m² at 3 m/s.

A ship travels at 15 knots and has a wetted surface area of 3800 m². If the frictional resistance varies as speed $^{1 \cdot 97}$ calculate the power required to overcome frictional resistance.

3. The frictional resistance per square metre of a ship is 12 N at 180 m/min. The ship has a wetted surface area of 4000 m² and travels at 14 knots. Frictional resistance varies as speed $^{1 \cdot 9}$. If frictional resistance is 70% of the total resistance, calculate the effective power.

4. A ship is 125 m long, 15 m beam and floats at a draught of 7·8 m. Its block coefficient is 0·72. Calculate the power required to overcome frictional resistance at 17·5 knots if $n = 1{\cdot}825$ and $f = 0{\cdot}423$. Use Taylor's formula for wetted surface, with $c = 2{\cdot}55$.

5. The residuary resistance of a one-twentieth scale model of a ship in sea water is 36 N when towed at 3 knots. Calculate the residuary resistance of the ship at its corresponding speed and the power required to overcome residuary resistance at this speed.

6. A ship of 14 000 tonne displacement has a residuary resistance of 113 kN at 16 knots. Calculate the corresponding speed of a similar ship of 24 000 tonne displacement and the residuary resistance at this speed.

*f*7. The frictional resistance of a ship in fresh water at 3 m/s is 11 N/m². The ship has a wetted surface area of 2500 m² and the frictional resistance is 72% of the total resistance and varies as speed $^{1 \cdot 92}$. If the effective power is 1100 kW, calculate the speed of the ship.

*f*8. A 6 m model of a ship has a wetted surface area of 7 m², and when towed in fresh water at 3 knots, has a total resistance of 35 N. Calculate the effective power of the ship, 120 m long, at its corresponding speed.

$n = 1{\cdot}825$: f from formula: SCF $= 1{\cdot}15$

9. A ship of 12 000 tonne displacement has an Admiralty Coefficient of 550. Calculate the shaft power at 16 knots.

10. A ship requires a shaft power of 2800 kW at 14 knots, and the Admiralty Coefficient is 520. Calculate:

(a) the displacement

(b) the shaft power if the speed is reduced by 15%.

11. A ship of 8000 tonne displacement has an Admiralty Coefficient of 470. Calculate its speed if the shaft power provided is 2100 kW.

*f*12. A ship 150 m long and 19 m beam floats at a draught of 8 m and has a block coefficient of 0·68.

(a) If the Admiralty Coefficient is 600, calculate the shaft power required at 18 knots.

(b) If the speed is now increased to 21 knots, and within this speed range resistance varies as speed^3, find the new shaft power.

13. A ship of 15 000 tonne displacement has a fuel coefficient of 62 500. Calculate the fuel consumption per day at 14½ knots.

14. A ship of 9000 tonne displacement has a fuel coefficient of 53 500. Calculate the speed at which it must travel to use 25 tonne of fuel per day.

15. A ship travels 2000 nautical miles at 16 knots and returns with the same displacement at 14 knots. Find the saving in fuel on the return voyage if the consumption per day at 16 knots is 28 tonne.

16. The daily fuel consumption of a ship at 15 knots is 40 tonne. 1100 nautical miles from port it is found that the bunkers are reduced to 115 tonne. If the ship reaches port with 20 tonne of fuel on board, calculate the reduced speed and the time taken in hours to complete the voyage.

17. A ship uses 23 tonne of fuel per day at 14 knots. Calculate the speed if the consumption per day is:

(a) increased by 15%

(b) reduced by 12%

(c) reduced to 18 tonne.

18. The normal speed of a ship is 14 knots and the fuel consumption per hour is given by $0{\cdot}12 + 0{\cdot}001\ V^3$ tonne, with V in knots, Calculate:

(a) the total fuel consumption over a voyage of 1700 nautical miles

(b) the speed at which the vessel must travel to save 10 tonne of fuel per day.

19. A ship's speed is increased by 20% above normal for 8 hours, reduced by 10% below normal for 10 hours and for the remaining 6 hours of the day the speed is normal. Calculate the precentage variation in fuel consumption in that day from normal.

20. A ship's speed was 18 knots. A reduction of 3·5 knots gave a saving in fuel consumption of 22 tonne per day. Calculate the consumption per day at 18 knots..

*f*21. The daily fuel consumption of a ship at 17 knots is 42 tonne. Calculate the speed of the ship if the consumption is reduced to 28 tonne per day, and the specific consumption at the reduced speed is 18% more than at 17 knots.

CHAPTER 8

PROPELLERS

The after side of a marine propeller is the driving face and is in the form of a helical screw. This screw is formed by a number of blades, from 3 to 7, set at an angle to the plane of rotation.

DIAMETER D The diameter of the propeller is the diameter of the circle or disc cut out by the blade tips.

PITCH P If the propeller is assumed to work in an unyielding fluid, then in one revolution of the shaft the propeller will move forward a distance which is known as the pitch.

PITCH RATIO p, or face pitch ratio is the face pitch divided by the diameter. Thus

$$p = \frac{P}{D}$$

THEORETICAL SPEED V_T is the distance the propeller would advance in unit time if working in an unyielding fluid. Thus if the propeller turns at N rev/min,

$$V_T = P \times N \text{ m/min}$$
$$= \frac{P \times N \times 60}{1852} \text{ knots.}$$

APPARENT SLIP Since the propeller works in water, the ship speed V will normally be less than the theoretical speed. The difference between the two speeds is known as the apparent slip and is usually expressed as a ratio or percentage of the theoretical speed.

$$\text{Apparent slip speed} = V_T - V \text{ knots}$$

$$\text{Apparent slip} = \frac{V_T - V}{V_T} \times 100\%.$$

If the ship speed is measured relative to the surrounding water i.e. by means of a log line, the theoretical speed will invariably exceed the ship speed, giving a *positive* apparent slip. If, however the ship speed is measured relative to the land, then any movement of water will affect the apparent slip, and should the vessel be travelling in a following current the ship speed may exceed the theoretical speed, resulting in a *negative* apparent slip.

WAKE In its passage through the water the ship sets in motion particles of water in its neighbourhood, caused, as mentioned earlier, by friction between the hull and the water. This moving water is known as the wake and is important in propeller calculations since the propeller works in wake water. The speed of the ship relative to the wake is termed the SPEED OF ADVANCE V_a. The wake speed is often expressed as a fraction of the ship speed.

$$\text{Wake fraction } w = \frac{V - V_a}{V}$$

The wake fraction may be obtained approximately from the expression $w = 0{\cdot}5C_b - 0{\cdot}05$

where C_b is the block coefficient.

REAL SLIP OF TRUE SLIP is the difference between the theoretical speed and the speed of advance, expressed as a ratio or percentage of the theoretical speed.

$$\text{Real slip speed} = V_T - V_a \text{ knots}$$

$$\text{Real slip} = \frac{V_T - V_a}{V_T} \times 100\%$$

The real slip is always positive and is independent of current.

Example. A propeller of 4·5 m pitch turns at 120 rev/min and drives the ship at 15·5 knots. If the wake fraction is 0·30 calculate the apparent slip and the real slip.

$$\text{Theoretical speed } V_T = \frac{4{\cdot}5 \times 120 \times 60}{1852}$$
$$= 17{\cdot}49 \text{ knots.}$$

$$\text{Apparent slip} = \frac{17{\cdot}49 - 15{\cdot}5}{17{\cdot}49} \times 100$$
$$= 11{\cdot}38\%$$

$$\text{Wake fraction } w = \frac{V - V_a}{V}$$

$$\therefore \quad wV = V - V_a$$
$$V_a = V - wV$$
$$= V(1 - w)$$
$$= 15{\cdot}5 \times 0{\cdot}7$$
$$= 10{\cdot}85 \text{ knots}$$

$$\text{Real ship} = \frac{17{\cdot}49 - 10{\cdot}85}{17{\cdot}49} \times 100$$
$$= 37{\cdot}96\%$$

The relation between the different speeds may be shown clearly by the following line diagram.

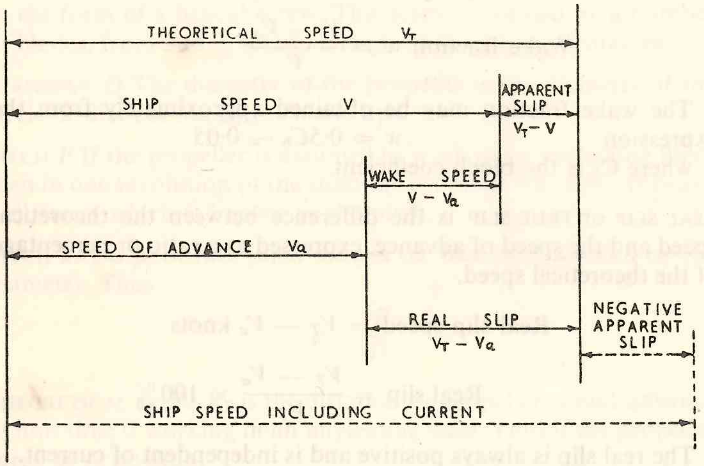


Fig. 72

PROJECTED AREA A_p is the sum of the blade areas projected onto a plane which is perpendicular to the axis of the screw.

DEVELOPED AREA is the actual area of the driving faces
(a) clear of the boss A_d
(b) including the boss area A_b

BLADE AREA RATIO BAR is the developed area excluding boss divided by the area of the circle cut out by the blade tips

$$\text{BAR} = \frac{A_d}{\frac{\pi}{4} D^2}$$

DISC AREA RATIO D.A.R. is the developed area including boss divided by the area of the circle cut out by the blade tips.

$$\text{D.A.R.} = \frac{A_b}{\frac{\pi}{4} D^2}$$

THRUST

The thrust exerted by a propeller may be calculated approximately by regarding the propeller as a reaction machine. Water is received into the propeller disc at the speed of advance and projected aft at the theoretical speed.

Consider a time interval of one second.

Let A = effective disc area in m²
= disc area—boss area
ρ = density of water in kg/m³
P = pitch of propeller in m
n = rev/s
v_a = speed of advance in m/s

Mass of water passing through disc in one second.

$$M = \rho\, APn \text{ kg}$$

$$\text{Change in velocity} = Pn - v_a \quad \text{m/s}$$

Since this change occurs in one second,

$$\text{Acceleration } a = Pn - v_a \quad \text{m/s}^2$$

But
$$\text{real slip } s = \frac{Pn - v_a}{Pn}$$

$$Pn - v_a = sPn$$
and
$$a = sPn$$

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$\therefore \quad \text{Thrust T} = M \times a$$
$$= \rho APn \times sPn$$
$$= \rho AP^2n^2 \text{ N}$$

It is interesting to note that increased slip leads to increased thrust and that the propeller will not exert a thrust with zero slip. The power produced by the propeller is known as the *thrust power* tp.

$$\text{tp} = \text{thrust } (N) \times \text{speed of advance (m/s) W}$$

$$= T \times v_a \quad \text{W}$$

Hence $$\frac{\text{tp}_1}{\text{tp}_2} = \frac{T_1\, v_{a1}}{T_2\, v_{a2}}$$

If the power remains constant, but the external conditions vary, then

$$T_1\, v_{a1} = T_2\, v_{a2}$$

and since the speed of advance depends upon rev/min,

$$T_1\, N_1 = T_2\, N_2$$

Now the thrust is absorbed by the thrust collars and hence the thrust varies directly as the pressure t on the thrust collars.

$$\therefore \qquad t_1\, N_1 = t_2\, N_2$$

This indicates that if, with constant power, the ship meets a head wind, the speed will reduce but the pressure on the thrust collars will increase.

Example. The tp of a ship is 2000 kW and the pressure on the thrust 20 bar at 120 rev/min.

Calculate the pressure on the thrust when the tp is 1800 kW at 95 rev/min.

$$\frac{20 \times 120}{2000} = \frac{t_2 \times 95}{1800}$$

$$t_2 = \frac{1800 \times 20 \times 120}{2000 \times 95}$$

$$= 22{\cdot}74 \text{ bar}$$

RELATION BETWEEN POWERS

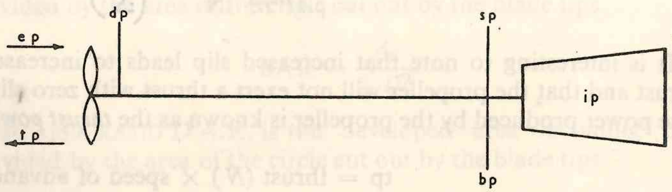


Fig. 73

The power produced by the engine is the indicated power ip. The mechanical efficiency of the engine is usually between about 80% and 90% and therefore only this percentage of the ip is transmitted to the shaft, giving the shaft power sp or brake power bp.

$$\text{sp or bp} = \text{ip} \times \text{mechanical efficiency}$$

Shaft losses vary between about 3% and 5% and therefore the power delivered to the propeller, the delivered power dp, is almost 95% of the sp.

$$\text{dp} = \text{sp} \times \text{transmission efficiency}$$

The delivered power may be calculated from the torque on the shaft

$$\text{dp} = \text{torque} \times 2\pi n$$

The propeller has an efficiency of 60% to 70% and hence the thrust power tp is given by:

$$\text{tp} = \text{dp} \times \text{propeller efficiency}$$

The action of the propeller in accelerating the water creates a suction on the after end of the ship. The thrust exerted by the propeller must exceed the total resistance by this amount. The relation between thrust and resistance may be expressed in the form

$$R_t = T\,(1 - t)$$

where t is the thrust deduction factor.

The thrust power will therefore differ from the effective power. The ratio of ep to tp is known as the hull efficiency which is a little more than unity for single screw ships and about unity for twin screw ships.

$$\text{ep} = \text{tp} \times \text{hull efficiency}$$

In an attempt to estimate the power required by the machinery from the calculation of ep, a quasi propulsive coefficient QPC is introduced. This is the ratio of ep to dp and obviates the use of hull efficiency and propeller efficiency. The prefix quasi is used to show that the mechanical efficiency of the machinery and the transmission losses have not been taken into account.

$$\text{ep} = \text{dp} \times \text{QPC}$$

The true propulsive coefficient is the relation between the ep and the ip, although in many cases sp is used in place of ip

i.e. ep = ip × propulsive coefficient
or ep = sp × propulsive coefficient

Example. The total resistance of a ship at 13 knots is 180 kN, the QPC is 0·70, shaft losses 5% and the mechanical efficiency of the machinery 87%.
Calculate the indicated power.

$$ep = R_t \times v$$

$$= 180 \times 10^3 \times 13 \times \frac{1852}{3600} \text{ W}$$

$$= 1204 \text{ kW}$$

$$dp = \frac{ep}{QPC}$$

$$= \frac{1204}{0{\cdot}7}$$

$$= 1720 \text{ kW}$$

$$sp = \frac{dp}{\text{transmission efficiency}}$$

$$= \frac{1720}{0{\cdot}95}$$

$$= 1810 \text{ kW}$$

$$ip = \frac{sp}{\text{mechanical efficiency}}$$

$$= \frac{1810}{0{\cdot}87}$$

$$\text{indicated power} = 2080 \text{ kW}$$

ƒRELATION BETWEEN MEAN PRESSURE AND SPEED

The indicated power of a steam reciprocating engine is given by:

$$ip \propto pm\ ALN$$

where p_m = mean effective pressure (m.e.p.)
A = area of piston
L = length of stroke
N = number of strokes per minute or revolutions per minute

Since A and L remain constant,

$$\text{power} \propto p_m \times N$$

But it was shown earlier, that with constant displacement

$$\text{power} \propto \text{speed}^3$$

and since

$$\text{speed} \propto \text{propeller pitch} \times \text{revs/min.}$$
$$\propto P \times N$$
$$\text{power} \propto P^3 N^3$$
$$\therefore \quad p_m \times N = P^3 N^3$$
$$p_m \propto P^3 N^2$$

or

$$\frac{p_{m1}}{p_{m2}} = \left(\frac{P_1}{P_2}\right)^3 \times \left(\frac{N_1}{N_2}\right)^2$$

If the pitch of the propeller remains constant, then

$$\frac{p_{m1}}{p_{m2}} = \left(\frac{N_1}{N_2}\right)^2$$

Example. The mean effective pressure of an engine is 5·5 bar at 115 rev/min. Calculate the mep if the revs are reduced to 95 per minute.

$$p_{m2} = p_{m1}\left(\frac{N_2}{N_1}\right)^2$$

$$= 5{\cdot}5\left(\frac{95}{115}\right)^2$$

$$= 3{\cdot}753 \text{ bar}$$

MEASUREMENT OF PITCH

If the propeller is assumed to have no forward motion, then a point on the blade, distance R from the centre of boss will move a distance of $2\pi R$ in one revolution. If the propeller is now assumed to work in an unyielding fluid, then in one revolution it will advance a distance of P, the pitch. The pitch angle θ may be defined as

$$\tan \theta = \frac{P}{2\pi R}$$

$$\therefore \quad \text{Pitch} = \tan \theta \times 2\pi R$$

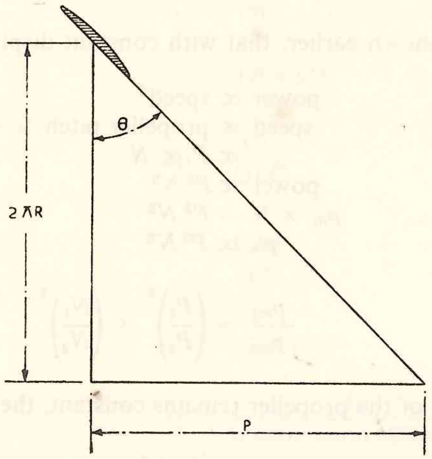
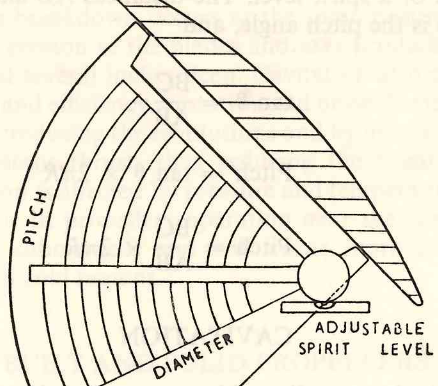


Fig. 74

The pitch of a propeller may be measured without removing the propeller from the ship, by means of a simple instrument known as a pitchometer. One form of this instrument consists of a protractor with an adjustable arm. The face of the boss is used as a datum, and a spirit level is set horizontal when the pitchometer is set on the datum. The instrument is then set on the propeller blade at the required distance from the boss and the arm containing the level moved until it is horizontal, a reading of pitch angle or pitch may then be read from the protractor at the required radius (Fig 75).



PITCHOMETER

Fig. 75

An alternative method is to turn the propeller until one blade is horizontal. A weighted cord is draped over the blade at any given radius as shown in Fig. 76.

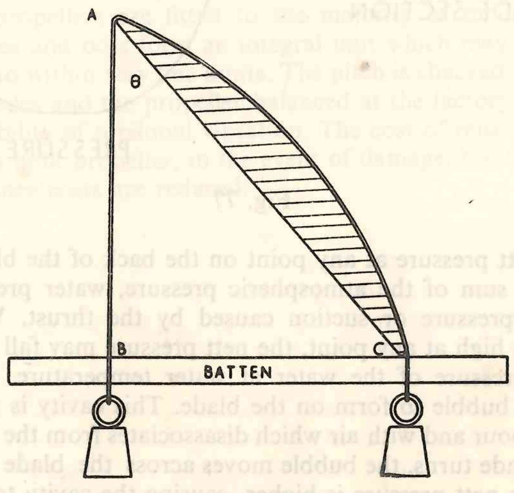


Fig. 76

A batten is placed horizontally at the lower edge of the blade with the aid of a spirit level. The distances AB and BC are then measured. θ is the pitch angle, and

$$\tan\theta = \frac{BC}{AB}$$

But $$\text{Pitch} = \tan\theta \times 2\pi R$$

$\therefore$ $$\text{Pitch} = \frac{BC}{AB} \times 2\pi R$$

CAVITATION

The thrust of a propeller varies approximately as the square of the revolutions. Thus as the speed of rotation is increased there is a considerable increase in thrust. The distribution of pressure due to thrust over the blade section is approximately as shown in Fig. 77.

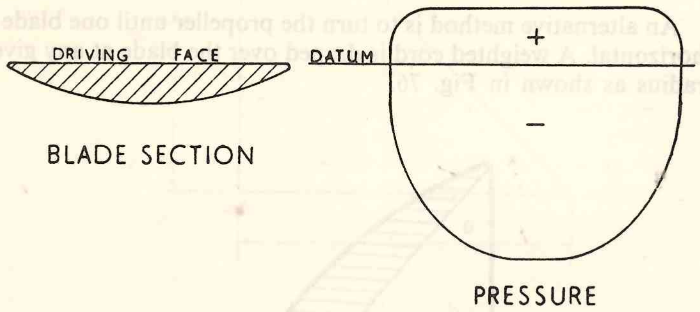


Fig. 77

The nett pressure at any point on the back of the blade is the algebraic sum of the atmospheric pressure, water pressure and negative pressure or suction caused by the thrust. When this suction is high at any point, the nett pressure may fall below the vapour pressure of the water at water temperature, causing a cavity or bubble to form on the blade. This cavity is filled with water vapour and with air which disassociates from the sea water. As the blade turns, the bubble moves across the blade to a point where the nett pressure is higher, causing the cavity to collapse. The forming and collapsing of these cavities is known as *cavitation.*

When the cavity collapses, the water pounds the blade material, and since the breakdown occurs at the same position each time, causes severe erosion of the blades and may produce holes in the blade material several inches deep. Cavitation also causes reduction in thrust and efficiency, vibration and noise. It may be reduced or avoided by reducing the revolutions and by increasing the blade area for constant thrust, thus reducing the negative pressure. Since cavitation is affected by pressure and temperature, it is more likely to occur in propellers operating near the surface than in those deeply submerged, and will occur more readily in the tropics than in cold regions.

BUILT AND SOLID PROPELLERS

Propellers may be either cast in a solid unit or built up from a boss to which are bolted the separate blades. Built propellers have the advantage that if a blade is damaged or lost it may be replaced cheaply without removing the propeller. The bolt holes in the boss are slotted to allow alterations in pitch. Unfortunately the diameter of boss is increased and leads to a reduction in efficiency. Blades have been known to slacken and it is therefore necessary to check the propeller both for slackness and pitch when the ship is in dry dock. Variations in pitch from blade to blade lead to hull vibration.

Solid propellers are fitted to the majority of modern ships. The blades and boss form an integral unit which may be manufactured to within very fine limits. The pitch is checked accurately on all blades and the propeller balanced at the factory to reduce the possibility of torsional vibration. The cost of repair is higher than with built propeller, in the event of damage, but the overall maintenance costs are reduced.

TEST EXAMPLES 8

1. A ship travels at 14 knots when the propeller, 5 m pitch, turns at 105 rev/min. If the wake fraction is 0·35, calculate the apparent and real slip.

2. A propeller of 5·5 m diameter has a pitch ratio of 0·8. When turning at 120 rev/min, the wake fraction is found to be 0·32 and the real slip 35%

Calculate the ship speed, speed of advance and apparent slip.

3. A ship of 12 400 tonne displacement is 120 m long, 17·5 m beam and floats at a draught of 7·5 m. The propeller has a face pitch ratio of 0·75 and, when turning at 100 rev/min, produces a ship speed of 12 knots with a real slip of 30%. Calculate the apparent slip, pitch and diameter of the propeller. The wake fraction w may be found from the expression:

$$w = 0{\cdot}5C_b - 0{\cdot}05$$

*f*4. When a propeller of 4·8 m pitch turns at 110 rev/min, the apparent slip is found to be $-s\%$ and the real slip $+1{\cdot}5s\%$. If the wake speed is 25% of the ship speed, calculate the ship speed, the apparent slip and the real slip.

*f*5. A propeller 4·6 m diameter has a pitch of 4·3 m and boss diameter of 0·75 m. The real slip is 28% at 95 rev/min.

Calculate the speed of advance, thrust and thrust power.

6. The pressure exerted on the thrust is 17·5 b at 115 rev/min. Calculate the thrust pressure at 90 rev/min.

7. The power required to drive a ship at a given speed was 3400 kW and the pressure on the thrust 19·5 b.

Calculate the new thrust pressure if the speed is reduced by 12% and the corresponding power is 2900 kW.

*f*8. A ship of 15 000 tonne displacement has an Admiralty Coefficient, based on shaft power, of 420. The mechanical efficiency of the machinery is 83%, shaft losses 6%, propeller efficiency 65% and QPC 0·71. At a particular speed the thrust power is 2550 kW. Calculate: (a) indicated power,
(b) effective power,
(c) ship speed.

*f*9. A propeller of 4 m pitch has an efficiency of 67%. When turning at 125 rev/min the real slip is 36% and the delivered power 2800 kW.

Calculate the thrust of the propeller.

*f*10. The mean referred pressure was found to be 4·5 b at 130 rev/min. Calculate the pressure at 105 rev/min.

*f*11. At 120 rev/min the m e p is 5·5 b.

Calculate the m e p and the rev/min if the power is reduced by 20%

12 The pitch angle, measured at a distance of 2 m from the centre of the boss, was found to be 21·5°.

Calculate the pitch of the propeller.

13. The pitch of a propeller is measured by means of a batten and cord. The horizontal ordinate is found to be 40 cm while the vertical ordinate is 1·15 m at a distance of 2·6 m from the centre of the boss. Calculate the pitch of the propeller and the blade width at that point.

CHAPTER 9

RUDDER THEORY

When a rudder is turned from the centreline plane to any angle, the water flows round the rudder and creates an additional resistance on that side of the centreline. The force F which acts on the rudder parallel to the centreline has two components:

(a) the force created by the formation of streamlines round the rudder, i.e. due to the change in direction of the water.

(b) the suction on the after side of the rudder caused by eddying.

This force F follows the laws of fluid friction and may be determined from the expression.

$$F = k\,A\,v^2 \quad \text{N}$$

where k = a coefficient which depends upon the shape of the rudder, the rudder angle and the density of the water. When the ship speed is expressed in m/s, average values of k for sea water vary between about 570 and 610.

A = rudder area

v = ship speed.

The area of rudder is not specified by Classification Societies, but experience has shown that the area should be related to the area of the middle-line plane (i.e. length of ship × draught), and values of one sixtieth for fast ships and one seventieth for slow ships have been found successful.

$$\text{i.e. Area of rudder} = \frac{L \times d}{60} \text{ for fast ships}$$

$$= \frac{L \times d}{70} \text{ for slow ships.}$$

If the rudder is turned to an angle α, then the component of force acting normal to the plane of the rudder F_n is given by:

$$F_n = F \sin \alpha$$
$$= k\,A\,v^2 \sin \alpha$$

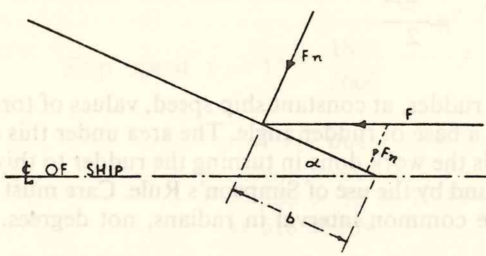


Fig. 78

This force F_n acts at the centre of effort of the rudder. The position of the centre of effort varies with the shape of the rudder and the rudder angle. For rectangular rudders the centre of effort is between 20% and 38% of the width of the rudder from the leading edge. The effect of the normal force is to tend to push the rudder back to its centreline position. Such movement is resisted by the rudder stock and the steering gear. It is therefore possible to calculate the turning moment or torque on the rudder stock.

If the centre of effort is b m from the centre of the rudder stock, then at any angle α

$$\text{Torque on stock } T = F_n \times b$$
$$= k\,A\,v^2\,b \sin \alpha \quad \text{N m}$$

From the basic torsion equation the diameter of the stock may be found for any given allowable stress.

$$\frac{T}{J} = \frac{q}{r}$$

where q = allowable stress in N/m^2
r = radius of stock in m
J = second moment of area about a polar axis in m^4

$$= \frac{\pi d^4}{32}$$

$$= \frac{\pi r^4}{2}$$

For any rudder, at constant ship speed, values of torque may be plotted on a base of rudder angle. The area under this curve up to any angle is the work done in turning the rudder to this angle, and may be found by the use of Simpson's Rule. Care must be taken to express the common interval in radians, not degrees.

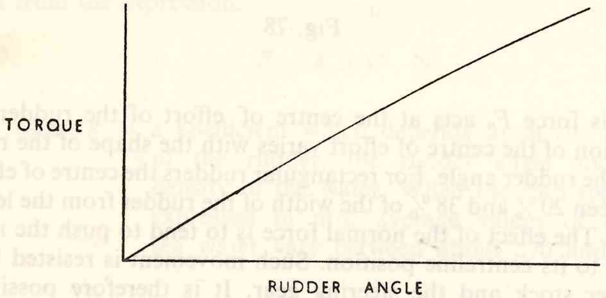


Fig. 79

If the centre of the rudder stock is between 20 % and 38 % of the width of the rudder from the leading edge, then at a given angle the centre of stock will coincide with the centre of effort and thus there will be no torque. The rudder is then said to be *balanced*. At any other rudder angle the centres of stock and effort will not coincide and there will be a torque of reduced magnitude. Thus it may be seen that the diameter of stock and power of the steering gear may be reduced if a balanced rudder is fitted.

It is usual to limit the rudder angle to 35° on each side of the centreline, since, if this angle is exceeded, the diameter of the turning circle is increased.

Example. A rudder has an area of 15 m^2 with its centre of effort 0·9 m from the centre of stock. The maximum rudder angle is 35° and it is designed for a service speed of 15 knots. Calculate the diameter of the rudder stock if the maximum allowable stress in the stock is 55 MN/m^2 and the rudder force parallel to the centreline of the ship is given by:

$$F = 580\ Av^2 \text{ N with } v \text{ in m/s}$$

$$\text{Ship speed } v = 15 \times \frac{1852}{3600}$$

$$= 7{\cdot}717 \text{ m/s}$$

$$F = 580 \times 15 \times 7{\cdot}717^2$$
$$= 518\ 060 \text{ N}$$

$$\text{Torque } T = F_n\, b$$
$$= F \sin\alpha \times b$$
$$= 518\ 060 \times 0{\cdot}5736 \times 0{\cdot}9$$
$$= 267\ 440 \text{ N m}$$

$$\frac{T}{J} = \frac{q}{r}$$

$$J = \frac{Tr}{q}$$

$$\frac{\pi r^4}{2} = \frac{Tr}{q}$$

$$r^3 = \frac{2T}{\pi q}$$

$$= \frac{2 \times 267\ 440}{3{\cdot}412 \times 55 \times 10^6}$$

$$= 0{\cdot}003\ 095 \text{ m}^3$$

$$r = 0{\cdot}145 \text{ m}$$

$$\text{Diameter of stock} = 0{\cdot}29 \text{ m}$$

ANGLE OF HEEL DUE TO FORCE ON RUDDER

When the rudder is turned from its central position, a transverse component of the normal rudder force acts on the rudder.

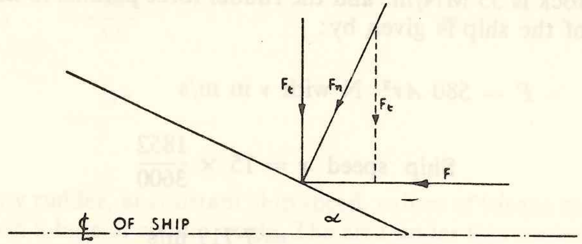


Fig. 80

Let F_n = rudder force normal to the plane of the rudder
F_t = transverse rudder force
α = rudder angle

Then $F_t = F_n \cos \alpha$
$= F \sin \alpha \cos \alpha$
$= k\, A\, v^2 \sin \alpha \cos \alpha$

This transverse force acts at the centre of the rudder N, and tends to push the ship sideways. A resistance R is exerted by the water on the ship, and acts at the *centre of lateral resistance L* which is the centroid of the projected, immersed plane of the ship (sometimes taken as the centre of buoyancy). This resistance is increased as the ship moves, until it reaches its maximum value when it is equal to the transverse force. At this point a moment acts on the ship causing it to heel to an angle θ when the heeling moment is equal to the righting moment.

Heeling moment $= F_t \times NL \cos \theta$
Righting moment $= \triangle g \times GZ$
$= \triangle g \times GM \sin \theta$ if θ is small

For equilibrium:

Righting moment = heeling moment

$$\triangle g \times GM \sin \theta = F_t \times NL \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = \frac{F_t \times NL}{\triangle g \times GM}$$

$$\tan \theta = \frac{F_t \times NL}{\triangle g \times GM}$$

From this the angle of heel may be obtained.

The angle of heel due to the force on the rudder is small unless the speed is excessive or the metacentric height small. In most merchant ships this angle is hardly noticeable.

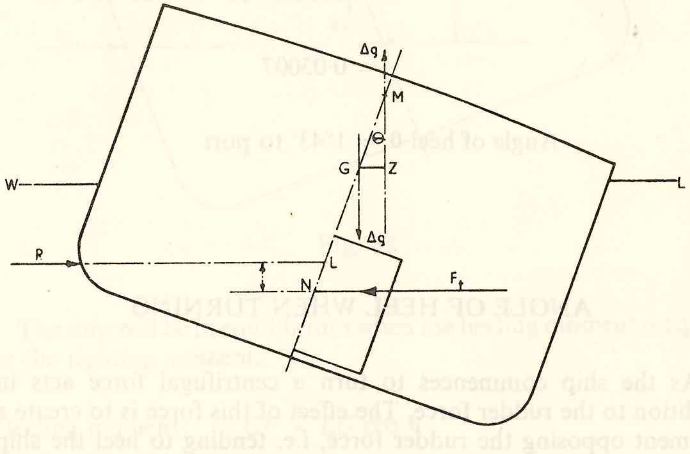


Fig. 81

Example. A ship of 8000 tonne displacement has a rudder of area 18m². The centre of lateral resistance is 4 m above the keel while the centroid of the rudder is 2·35 m above the keel. The maximum rudder angle is 35°. Calculate the angle of heel due to the force on the rudder if the latter is put hard over to port when travelling at 21 knots with a metacentric height of 0·4 m.

Given $F = 580\, Av^2$ N

$$\text{Ship speed } v = 21 \times \frac{1852}{3600}$$

$$= 10{\cdot}80 \text{ m/s}$$

$$\text{Transverse force } F_t = 580\, A\, v^2 \sin \alpha \cos \alpha$$
$$= 580 \times 18 \times 10{\cdot}80^2 \times 0{\cdot}5736 \times 0{\cdot}8192$$

$$= 572\ 200 \text{ N}$$

$$\tan\theta = \frac{F_t \times NL}{\triangle g \times GM}$$

$$= \frac{572\ 200 \times (4{\cdot}0 - 2{\cdot}35)}{8000 \times 10^3 \times 9{\cdot}81 \times 0{\cdot}4}$$

$$= 0{\cdot}03007$$

$$\text{Angle of heel } \theta = 1°43' \text{ to port}$$

ANGLE OF HEEL WHEN TURNING

As the ship commences to turn a centrifugal force acts in addition to the rudder force. The effect of this force is to create a moment opposing the rudder force, i.e. tending to heel the ship in the opposite direction.

It is convenient to ignore the rudder force and consider only the centrifugal force. This force acts at the centre of gravity of the ship and may be calculated from the formula:

$$\text{Centrifugal force } CF = \frac{\triangle v^2}{\rho} \text{ kN}$$

where v is the ship speed in m/s

ρ is the radius of the turning circle in m

A resistance R is again exerted by the water on the ship due to the transverse movement, and has its maximum value when it is equal to the centrifugal force. This resistance is known as the centripetal force. A moment then acts on the ship causing it to heel.

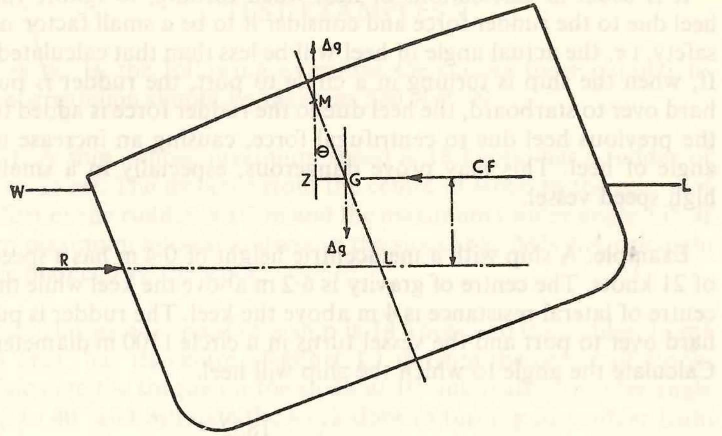


Fig. 82

The ship will be in equilibrium when the heeling moment is equal to the righting moment.

$$\text{Heeling moment} = CF \times LG \cos\theta$$

$$= \frac{\triangle v^2}{\rho} \times LG \cos\theta$$

$$\text{Righting moment} = \triangle g \times GZ$$

$$= \triangle g \times GM \sin\theta \quad \text{if } \theta \text{ is small}$$

$$\therefore \triangle g \times GM \sin\theta = \frac{\triangle v^2}{\rho} \times LG \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\triangle v^2}{\rho} \times \frac{LG}{\triangle g \times GM}$$

$$\tan\theta = \frac{v^2 \times LG}{g \times \rho \times GM}$$

From this expression the angle of heel may be calculated.

It is usual in calculations of heel when turning, to ignore the heel due to the rudder force and consider it to be a small factor of safety, i.e. the actual angle of heel will be less than that calculated. If, when the ship is turning in a circle to port, the rudder is put hard over to starboard, the heel due to the rudder force is added to the previous heel due to centrifugal force, causing an increase in angle of heel. This may prove dangerous, especially in a small, high speed vessel.

Example. A ship with a metacentric height of 0·4 m has a speed of 21 knots. The centre of gravity is 6·2 m above the keel while the centre of lateral resistance is 4 m above the keel. The rudder is put hard over to port and the vessel turns in a circle 1100 m diameter. Calculate the angle to which the ship will heel.

$$\text{Ship speed } v = 21 \times \frac{1852}{3600}$$

$$= 10{\cdot}80 \text{ m/s}$$

$$\tan\theta = \frac{v^2 \times LG}{g \times \rho \times GM}$$

$$= \frac{10{\cdot}80^2 \times (6{\cdot}2 - 4{\cdot}0)}{9{\cdot}81 \times 1100 \times 0{\cdot}4}$$

$$= 0{\cdot}1189$$

$$\text{Angle of heel } \theta = 6°47' \text{ to starboard}$$

Using the details from the previous example, it may be seen that if θ_1 is the final angle of heel:

$$\tan\theta_1 = 0{\cdot}1189 - 0{\cdot}03007$$

$$= 0{\cdot}08883$$

$$\text{Final angle of heel } \theta_1 = 5°\,5' \text{ to starboard.}$$

TEST EXAMPLES 9

NOTE: In the following questions the rudder force parallel to the streamline should be taken as 580 Av^2 N.

1. A ship, whose maximum speed is 18 knots, has a rudder of area 25 m². The distance from the centre of stock to the centre of effort of the rudder is 1·2 m and the maximum rudder angle 35°. If the maximum allowable stress in the stock is 85 MN/m², calculate the diameter of the stock.

2. The service speed of a ship is 14 knots and the rudder, 13 m² in area, has its centre of effort 1·1 m from the centre of stock. Calculate the torque on the stock at 10° intervals of rudder angle up to 40° and estimate the work done in turning the rudder from the centreline to 40°.

3. A ship 150 m long and 8·5 m draught has a rudder whose area is one sixtieth of the middle-line plane and diameter of stock 320 mm. Calculate the maximum speed at which the vessel may travel if the maximum allowable stress is 70 MN/m², the centre of stock 0·9 m from the centre of effort and the maximum rudder angle is 35°.

4. A ship displaces 5000 tonne and has a rudder of area 12 m². The distance between the centre of lateral resistance and the centre of the rudder is 1·6 m and the metacentric height 0·24 m. Calculate the initial angle of heel if the rudder is put over to 35° when travelling at 16 knots.

5. A vessel travelling at 17 knots turns with a radius of 450 m when the rudder is put hard over. The centre of gravity is 7 m above the keel, the transverse metacentre 7·45 m above the keel and the centre of buoyancy 4 m above the keel. If the centripetal force is assumed to act at the centre of buoyancy, calculate the angle of heel when turning. The rudder force may be ignored.

SOLUTIONS TO TEST EXAMPLES 1

1. (a) Mass of aluminium $= 300$ g

$= 0{\cdot}300$ kg

Volume of aluminium $= 42\ \text{cm}^3$

$= 42 \times 10^{-6}\ \text{m}^3$

Density of aluminium $= \dfrac{0{\cdot}300}{42 \times 10^{-6}}$

$= \dfrac{0{\cdot}300 \times 10^6}{42}$

$= 7143\ \text{kg/m}^3$

(b)
Mass of equal volume of water $= 42 \times 10^{-3}$ kg

Relative density of aluminium $= \dfrac{0{\cdot}300}{42 \times 10^{-3}}$

$= 7{\cdot}143$

Alternatively,
Density of water $= 1000\ \text{kg/m}^3$

Relative density of aluminium $= \dfrac{\text{density of aluminium}}{\text{density of water}}$

$= \dfrac{7143}{1000}$

$= 7{\cdot}143$

(c) Volume of aluminium $= 100\ \text{cm}^3$

$= 100 \times 10^{-6}\ \text{m}^3$

Mass of aluminium $= 7143 \times 100 \times 10^{-6}\ \text{kg/m}^3 \times \text{m}^3$

$= 0{\cdot}7143$ kg

2. Load on tank top $= \rho g A h$

$9{\cdot}6 \times 10^6 = 1025 \times 9{\cdot}81 \times 12 \times 10 \times h$

$h = \dfrac{9{\cdot}6 \times 10^6}{1025 \times 9{\cdot}81 \times 12 \times 10}$

$= 7{\cdot}96$ m

3.

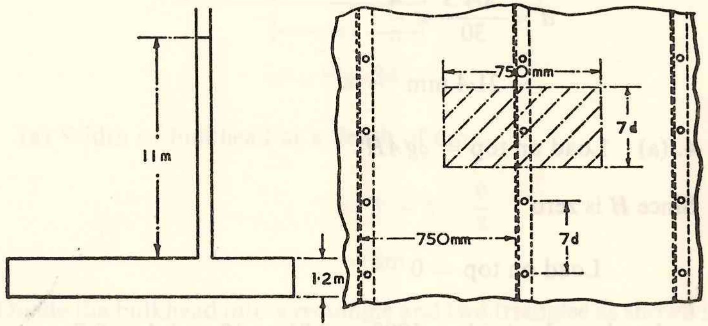


Fig. 83

Pressure on outer bottom $= \rho g h$

$= 0{\cdot}89 \times 10^3 \times 9{\cdot}81 \times (11 + 1{\cdot}2)$

$= 106{\cdot}5 \times 10^3\ \text{N/m}^2$

$= 106{\cdot}5\ \text{kN/m}^2$

Let d = diameter of rivets in mm.

Maximum stress in rivets

$= \dfrac{\text{load on one rivet}}{\text{area of one rivet}}$

$\therefore$ load on one rivet

$= 30 \times 10^6 \times \dfrac{\pi}{4} \times d^2 \times 10^{-6}$

$= 30 \times \dfrac{\pi}{4} d^2$ N

But load on one rivet

= load on area of tank top supported by one rivet

$= \rho g A h$

$= 0{\cdot}89 \times 10^3 \times 9{\cdot}81 \times 0{\cdot}75 \times 7d \times 10^{-3} \times 11$

$= 504{\cdot}3d$ N

$$30 \times \frac{\pi}{4} d^2 = 504{\cdot}3d$$

$$d = \frac{504{\cdot}3}{30} \times \frac{4}{\pi}$$

$= 21{\cdot}4$ mm

4. (a) Load on top $= \rho g A H$

Since H is zero

Load on top = 0

Load on short side $= 1000 \times 9{\cdot}81 \times 12 \times 1{\cdot}4 \times 0{\cdot}7$

$= 115{\cdot}3 \times 10^3$ N

$= 115{\cdot}3$ kN

(b) Load on top $= 1000 \times 9{\cdot}81 \times 15 \times 12 \times 7$

$= 12{\cdot}36 \times 10^6$ N

$= 12{\cdot}36$ MN

Load on short side $= 1000 \times 9{\cdot}81 \times 12 \times 1{\cdot}4 \times (7 + 0{\cdot}7)$

$= 1{\cdot}268 \times 10^6$ N

$= 1{\cdot}268$ MN

5. Pressure at bottom $= \rho g h$

$= 1025 \times 9{\cdot}81 \times 6$

$= 60{\cdot}34 \times 10^3$ N/m²

$= 60{\cdot}34$ kN/m²

Load on bulkhead $= \rho g A H$

$= 1025 \times 9{\cdot}81 \times 9 \times 6 \times 3$

$= 1{\cdot}629 \times 10^6$ N

$= 1{\cdot}629$ MN

6.

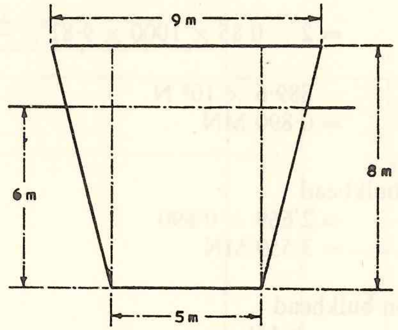


Fig. 84

(a) Width of bulkhead at a depth of 6m

$= 5 + 4 \times \frac{6}{8}$

= 8m

Divide the bulkhead into a rectangle and two triangles as shown in Fig. 84.

Load on rectangle $= 0{\cdot}85 \times 1000 \times 9{\cdot}81 \times 5 \times 6 \times 3$

$= 750{\cdot}4 \times 10^3$ N

$= 750{\cdot}4$ kN

Load on two triangles

$$= 2 \times 0{\cdot}85 \times 1000 \times 9{\cdot}81 \times \frac{1{\cdot}5 \times 6}{2} \times \frac{6}{3}$$

$= 150{\cdot}1 \times 10^3$ N

$= 150{\cdot}1$ kN

Total load on bulkhead

$= 750{\cdot}4 + 150{\cdot}1$

$= 900{\cdot}5$ kN

(b) Load on rectangle

$$= 0{\cdot}85 \times 1000 \times 9{\cdot}81 \times 5 \times 8 \times \left(\frac{8}{2} + 4\right)$$

$= 2{\cdot}669 \times 10^6$ N

$= 2{\cdot}669$ MN

Load on two triangles

$$= 2 \times 0{\cdot}85 \times 1000 \times 9{\cdot}81 \times \frac{2 \times 8}{2} \times \left(\frac{8}{3} + 4\right)$$

$$= 889{\cdot}6 \times 10^3 \text{ N}$$
$$= 0{\cdot}890 \text{ MN}$$

Total load on bulkhead

$$= 2{\cdot}669 + 0{\cdot}890$$
$$= 3{\cdot}559 \text{ MN}$$

7. (a)Load on bulkhead

$$= \rho g A H$$
$$= 0{\cdot}9 \times 1000 \times 9{\cdot}81 \times 10 \times 12 \times 6$$
$$= 6{\cdot}356 \times 10^6 \text{ N}$$
$$= 6{\cdot}356 \text{ MN}$$

Distance of centre of pressure from top of bulkhead

$$= \tfrac{2}{3} D$$
$$= \tfrac{2}{3} \times 12$$
$$= 8 \text{ m}$$

(b) Load on bulkhead

$$= 0{\cdot}9 \times 1000 \times 9{\cdot}81 \times 10 \times 12 \times \left(\frac{12}{2} + 3\right)$$

$$= 9{\cdot}534 \times 10^6 \text{ N}$$
$$= 9{\cdot}534 \text{ MN}$$

Distance of centre of pressure from surface of oil

$$= \frac{I_{NA}}{AH} + H$$

$$= \frac{\frac{1}{12} 10 \times 12^3}{10 \times 12 \times 9} + 9$$

$$= 1{\cdot}333 + 9$$
$$= 10{\cdot}333 \text{ m}$$

Distance of centre of pressure from top of bulkhead

$$= 10{\cdot}333 - 3$$
$$= 7{\cdot}333 \text{ m}$$

8.

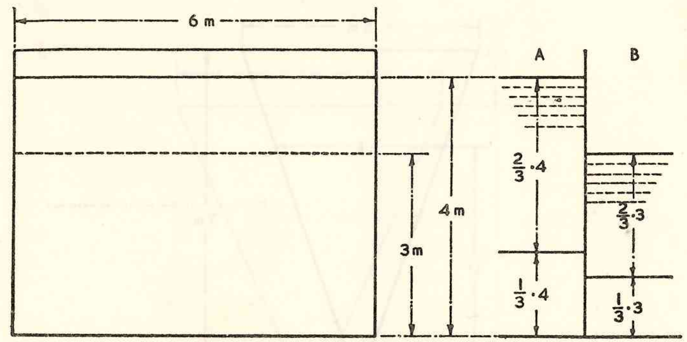


Fig 85

$$\text{Load on side A} = 1025 \times 9{\cdot}81 \times 6 \times 4 \times 2$$
$$= 482{\cdot}6 \times 10^3 \text{ N}$$
$$= 482{\cdot}6 \text{ kN}$$

$$\text{Centre of pressure on side A} = \tfrac{2}{3} \times 4$$
$$= 2{\cdot}667 \text{ m from top}$$
$$= 1{\cdot}333 \text{ m from bottom}$$

$$\text{Load on side B} = 1000 \times 9{\cdot}81 \times 6 \times 3 \times 1{\cdot}5$$
$$= 265{\cdot}0 \times 10^3 \text{ N}$$
$$= 265{\cdot}0 \text{ kN}$$

$$\text{Centre of pressure on side B} = \tfrac{2}{3} \times 3$$
$$= 2 \text{ m from top}$$
$$= 1 \text{ m from bottom}$$

Taking moments of load about bottom of gate:

Resultant centre of pressure from bottom

$$= \frac{482{\cdot}6 \times 1{\cdot}333 - 265{\cdot}0 \times 1}{482{\cdot}6 - 265{\cdot}0}$$

$$= \frac{643{\cdot}47 - 265{\cdot}0}{217{\cdot}6}$$

$$= 1{\cdot}740 \text{ m}$$

$$\text{Resultant load} = 217{\cdot}6 \text{ kN}$$

9.

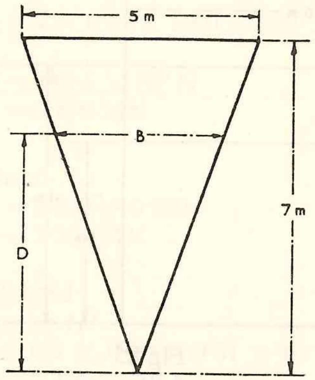


Fig. 86

There are two types of solution possible, one if the water is above the top of the bulkhead and one if the water is below the top of the bulkhead.

Assume water *at* top edge:

$$\text{Load on bulkhead} = 1025 \times 9{\cdot}81 \times \frac{5 \times 7}{2} \times \frac{7}{3}$$
$$= 410{\cdot}6 \times 10^3 \text{ N}$$
$$= 410{\cdot}6 \text{ kN}$$

Since the load on the bulkhead is only 190 kN, the water must be *below* the top of the bulkhead.

$$\text{Width at water level } B = \frac{5}{7} \times D$$

$$190 \times 10^3 = 1025 \times 9{\cdot}81 \times \frac{D}{2} \times \frac{5D}{7} \times \frac{D}{3}$$

$$D^3 = \frac{190 \times 10^3 \times 2 \times 7 \times 3}{1025 \times 9{\cdot}81 \times 5}$$

from which $D = 5{\cdot}414$ m

Centre of pressure below surface of water

$$= \tfrac{1}{2} \times D$$
$$= 2{\cdot}707 \text{ m}$$

Centre of pressure below top of bulkhead

$$= 2{\cdot}707 + (7{\cdot}00 - 5{\cdot}414)$$
$$= 4{\cdot}293 \text{ m}$$

10.

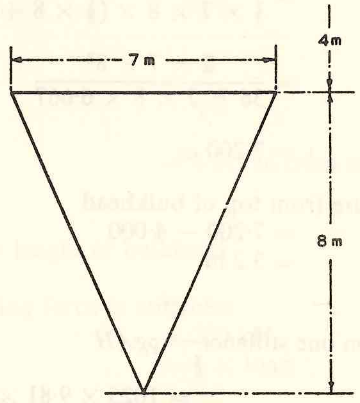


Fig. 87

(a)

$$\text{Load on bulkhead} = 1025 \times 9{\cdot}81 \times \frac{7 \times 8}{2} \times \frac{8}{3}$$
$$= 750{\cdot}8 \times 10^3 \text{ N}$$
$$= 750{\cdot}8 \text{ kN}$$

Centre of pressure from top

$$= \tfrac{1}{2}D$$
$$= 4\text{m}$$

(b)

$$\text{Load on bulkhead} = 1025 \times 9{\cdot}81 \times \frac{7 \times 8}{2} \times \left(\frac{8}{3} + 4\right)$$
$$= 1{\cdot}877 \times 10^6 \text{ N}$$
$$= 1{\cdot}877 \text{ MN}$$

$$\text{For triangle } I_{NA} = \frac{1}{36} BD^3$$

Centre of pressure from surface of water

$$= \frac{I_{NA}}{AH} + H$$

$$= \frac{\frac{1}{36} \times 7 \times 8^3}{\frac{1}{2} \times 7 \times 8 \times (\frac{1}{3} \times 8 + 4)} + (\tfrac{1}{3} \times 8 + 4)$$

$$= \frac{2 \times 7 \times 8^3}{36 \times 7 \times 8 \times 6.667} + 6.667$$

$$= 7.200 \text{ m}$$

Centre of pressure from top of bulkhead

$$= 7.200 - 4.000$$
$$= 3.2 \text{ m}$$

11.

Load on one stiffener $= \rho g A H$

$$= 1025 \times 9.81 \times 8 \times 0.700 \times \frac{8}{2}$$
$$= 225.3 \times 10^3 \text{ N}$$
$$= 225.3 \text{ kN}$$

(a) Shearing force at top of stiffener

$$= \tfrac{1}{3} \times \text{load}$$
$$= \tfrac{1}{3} \times 225.3$$
$$= 75.1 \text{ kN}$$

(b) Shearing force at bottom of stiffener

$$= \tfrac{2}{3} \times \text{load}$$
$$= \tfrac{2}{3} \times 225.3$$
$$= 150.2 \text{ kN}$$

$\therefore$ Shear force in rivets $= 150.2$ kN

Cross-sectional area of rivets $= 10 \times \frac{\pi}{4} \times 20^2 \times 10^{-6}$

$$= 3.142 \times 10^{-3} \text{ m}^2$$

Shear stress in rivets $= \dfrac{\text{shear force}}{\text{area}}$

$$= \frac{150.2}{3.142 \times 10^{-3}}$$

$$= 47.75 \times 10^3 \text{ kN/m}^2$$
$$= 47.75 \text{ MN/m}^2$$

(c) Position of zero shear $= \dfrac{l}{\sqrt{3}}$

$$= \frac{8}{\sqrt{3}}$$

$$= 4.619 \text{ m from top}$$

12. (a) Let l = height of bulkhead

Maximum shearing force in stiffeners

$$= 200 \text{ kN}$$
$$= \tfrac{2}{3} \times \text{load}$$

$$\therefore \text{load} = 200 \times \frac{3}{2}$$

$$= 300 \text{ kN}$$

But load on stiffener $= 1025 \times 9.81 \times l \times \dfrac{l}{9} \times \dfrac{l}{2}$

$$l^3 = \frac{300 \times 10^3 \times 9 \times 2}{1025 \times 9.81}$$

from which $l = 8.128$ m

(b) Shearing force at top $= \tfrac{1}{3} \times$ load

$$= \tfrac{1}{3} \times 300$$
$$= 100 \text{ kN}$$

(c) Position of zero shear $= \dfrac{l}{\sqrt{3}}$

$$= \frac{8.128}{\sqrt{3}}$$

$$= 4.693 \text{ m from top}$$

SOLUTIONS TO TEST EXAMPLES 2

1. Volume of wood and metal immersed

$$= 3{\cdot}5 \times 10^3 + 250 - 100$$
$$= 3650 \text{ cm}^3$$

$$\text{Mass of wood and metal} = 1{\cdot}000 \times 3650 \text{ g/cm}^3 \times \text{cm}^3$$
$$= 3650 \text{ g}$$

$$\text{Mass of wood} = 3{\cdot}5 \times 10^3 \times 1{\cdot}000 \times 0{\cdot}60$$
$$= 2100 \text{ g}$$

$$\therefore \text{Mass of metal} = 3650 - 2100$$
$$= 1550 \text{ g}$$

Mass of equal volume of fresh water

$$= 250 \times 1{\cdot}000$$
$$= 250 \text{ g}$$

$$\text{Relative density} = \frac{1550}{250}$$
$$= 6{\cdot}20$$

2. $$\text{Mass of raft} = 1000 \times 0{\cdot}7 \times 3 \times 2 \times 0{\cdot}25$$
$$= 1{\cdot}05 \times 10^3 \text{ kg}$$

Mass of raft when completely submerged

$$= 1018 \times 3 \times 2 \times 0{\cdot}25$$
$$= 1{\cdot}527 \times 10^3 \text{ kg}$$

$\therefore$ mass required to submerge raft

$$= 1{\cdot}527 \times 10^3 - 1{\cdot}050 \times 10^3$$
$$= 0{\cdot}477 \times 10^3 \text{ kg}$$
$$= 477 \text{ kg}$$

3. (a) $$\text{Displacement of barge} = 1025 \times 65 \times 12 \times 5{\cdot}5$$
$$= 4397 \times 10^3 \text{ kg}$$
$$= 4397 \text{ tonne}$$

(b) $$\text{Draught in fresh water} = 5{\cdot}5 \times \frac{1{\cdot}025}{1{\cdot}000}$$
$$= 5{\cdot}637 \text{ m}$$

4.

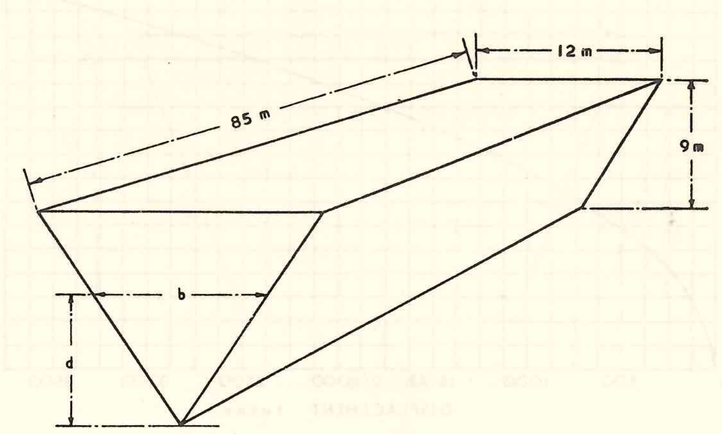


Fig. 88

Let d = draught
b = breadth at waterline

$$\text{By similar triangles } b = \frac{12}{9}d$$
$$= \frac{4}{3}d$$

$$\text{At draught } d\text{, displacement} = 1{\cdot}025 \times 85 \times \frac{b \times d}{2}$$
$$= 1{\cdot}025 \times 85 \times \tfrac{4}{3}d \times \tfrac{1}{2}d$$
$$= 58{\cdot}08d^2 \text{ tonne}$$

Tabulating:

draught d	d^2	displacement tone
0	0	0
1·25	1·563	91
2·50	6·250	363
3·75	14·062	817
5·00	25·000	1452
6·25	39·062	2269
7·50	56·250	3267

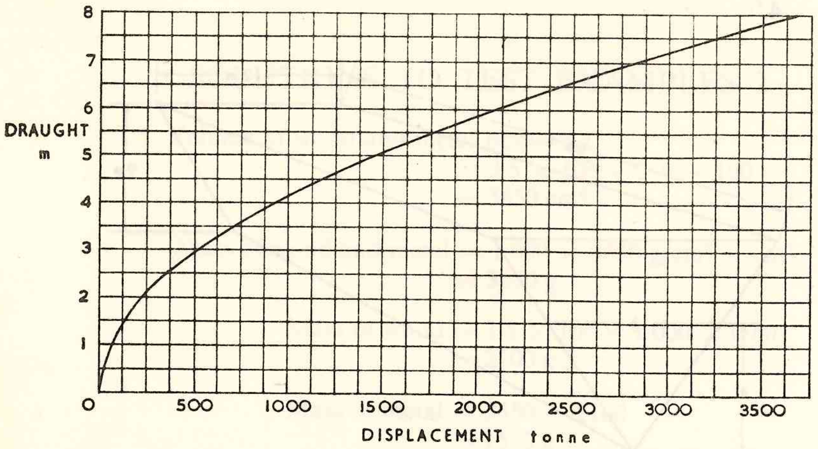


Fig. 89

At 6·50m draught, displacement in sea water is 2450 tonne.

$$\therefore \text{Displacement in fresh water} = 2450 \times \frac{1{\cdot}000}{1{\cdot}025}$$

$$= 2390 \text{ tonne}$$

5. $$\text{Immersed volume of cylinder} = \tfrac{1}{2} \times 15 \times \frac{\pi}{4} \times 4^2$$

$$= 30\,\pi \text{ m}^3$$

$$\text{Mass of cylinder} = 1{\cdot}025 \times 30\pi$$
$$= 96{\cdot}62 \text{ tonne}$$

6. $$\text{Mass of water displaced} = 1{\cdot}025 \times 22$$
$$= 22{\cdot}55 \text{ tonne}$$

$$\therefore \text{apparent mass of bilge keels} = 36 - 22{\cdot}55$$
$$= 13{\cdot}45 \text{ tonne}$$

$$\text{Increase in draught} = \frac{13{\cdot}45}{20}$$

$$= 0{\cdot}673 \text{ cm}$$

7.

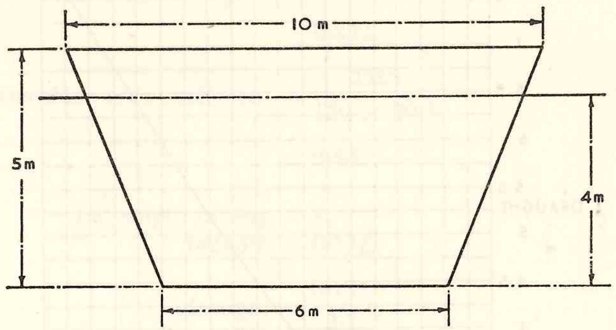


Fig. 90

$$\text{Breadth at waterline} = 6 + \frac{4}{5} \times 4$$

$$= 9{\cdot}2 \text{ m}$$

$$\text{Displacement} = 1{\cdot}025 \times 40 \times \frac{6 + 9{\cdot}2}{2} \times 4$$

$$= 1246 \text{ tonne}$$

8. $$\text{TPC} = 0{\cdot}01025 \times Aw$$

draught	waterplane area	TPC
7·5	1845	18·91
6·25	1690	17·32
5·00	1535	15·73
3·75	1355	13·89
2·50	1120	11·48

$$\text{Increase in draught required} = 0{\cdot}20 \text{ m}$$
$$= 20 \text{ cm}$$

$$\text{Mean TPC at } 6{\cdot}20 \text{ m} = 17{\cdot}25$$

$$\text{Mass required} = 17{\cdot}25 \times 20$$
$$= 345 \text{ tonne}$$

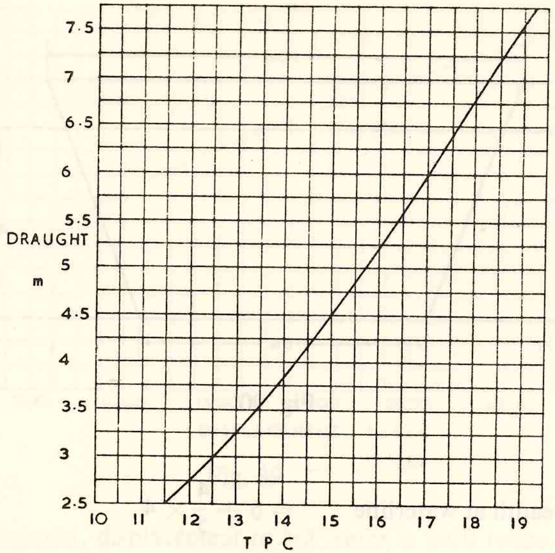


Fig. 91

9. Volume of displacement $= \dfrac{19\,500}{1 \cdot 025}$

$= 19\,024 \text{ m}^3$

Waterplane area $= \dfrac{\text{TPC}}{0 \cdot 01025}$

$= \dfrac{26 \cdot 5}{0 \cdot 01025}$

$= 2585 \text{ m}^3$

Block coefficient $= \dfrac{19\,024}{150 \times 20 \cdot 5 \times 8}$

$= 0 \cdot 773$

Prismatic coefficient $= \dfrac{C_b}{C_m}$

$= \dfrac{0 \cdot 773}{0 \cdot 94}$

$= 0 \cdot 822$

Waterplane area coefficient $= \dfrac{2585}{150 \times 20 \cdot 5}$

$= 0 \cdot 841$

10. Let length of ship $= L$

Then breadth $= 0 \cdot 13L$

and draught $= \dfrac{0 \cdot 13L}{2 \cdot 1}$

$= 0 \cdot 0619L$

Volume of displacement $= \dfrac{9450}{1 \cdot 025}$

$= 9219 \cdot 5 \text{ m}^3$

$$C_b = \frac{\nabla}{L \times B \times d}$$

$$0 \cdot 7 = \frac{9219 \cdot 5}{L \times 0 \cdot 13L \times 0 \cdot 0619L}$$

$$L^3 = \frac{9219 \cdot 5}{0 \cdot 7 \times 0 \cdot 13 \times 0 \cdot 0619}$$

Length of ship $L = 117 \cdot 9$ m

$$C_p = \frac{9219 \cdot 5}{117 \cdot 9 \times 106}$$

Prismatic coefficient $= 0 \cdot 738$

11. Let length of ship $= L$

Then draught $= \dfrac{L}{18}$

and breadth $= 2 \cdot 1 \times$ draught

$= \dfrac{2 \cdot 1}{18} L$

TPC sea water $= 0 \cdot 01025\, A_w$
TPC fresh water $= 0 \cdot 0100\, A_w$

TPC sea water − TPC fresh water

$$= 0{\cdot}7$$

$$\therefore\ 0{\cdot}01025\,A_w - 0{\cdot}0100\,A_w = 0{\cdot}7$$

$$0{\cdot}00025\,A_w = 0{\cdot}7$$

$$A_w = \frac{0{\cdot}7}{0{\cdot}00025}$$

$$= 2800 \text{ m}^2$$

But $A_w = C_w \times L \times B$

$$2800 = 0{\cdot}83 \times L \times \frac{2{\cdot}1}{18}L$$

$$L^2 = \frac{2800 \times 18}{0{\cdot}83 \times 2{\cdot}1}$$

From which $L = 170$ m

TPC fresh water $= 0{\cdot}0100 \times 2800$

$$= 28$$

12.

½ girth	SM	product
2·1	1	2·1
6·6	4	26·4
9·3	2	18·6
10·5	4	42·0
11·0	2	22·0
11·0	4	44·0
11·0	2	22·0
9·9	4	39·6
7·5	2	15·0
3·9	4	15·6
0	1	0
		247·3

Common interval = 9 m

Wetted surface area $= \frac{2}{3} \times 9 \times 247{\cdot}3$

$= 1483{\cdot}8 \text{ m}^2$

$\frac{1}{2}\% = 7{\cdot}42 \text{ m}^2$

Appendages $= 30{\cdot}00 \text{ m}^2$

Total wetted surface area $= 1521{\cdot}22 \text{ m}^2$

13. (a)

Volume of displacement $= \dfrac{14\,000}{1{\cdot}025}$

$= 13\,658 \text{ m}^3$

$$S = 1{\cdot}7Ld + \frac{\nabla}{d}$$

$$= 1{\cdot}7 \times 130 \times 8 + \frac{13\,658}{8}$$

$$= 1768{\cdot}0 + 1707{\cdot}3$$

$$= 3475{\cdot}3 \text{ m}^2$$

(b)

$$S = c\sqrt{\triangle L}$$

$$= 2{\cdot}58\sqrt{14\,000 \times 130}$$

$$= 3480 \text{ m}^2$$

14. Volume of existing barge $= 75 \times 9 \times 6$

$= 4050 \text{ m}^3$

Volumes of similar ships $\propto$ length³

i.e. $$\frac{V_1}{V_2} = \left(\frac{L_1}{L_2}\right)^3$$

$$L_2 = L_1 \sqrt[3]{\frac{V_2}{V_1}}$$

$$= 75 \sqrt[3]{\frac{3200}{4050}}$$

New length $L_2 = 69{\cdot}34$ m

new beam $$B_2 = 9 \sqrt[3]{\frac{3200}{4050}}$$

$= 8{\cdot}32$ m

New depth $$D_2 = 6 \sqrt[3]{\frac{3200}{4050}}$$

$= 5{\cdot}55$ m

15. Let S = wetted surface area of small ship

$2S$ = wetted surface area of large ship

$\triangle$ = displacement of small ship

$\triangle + 2000$ = displacement of large ship

It was shown that for similar ships:

$$\triangle \propto S^{\frac{3}{2}}$$

Hence

$$\frac{\triangle}{\triangle + 2000} = \left(\frac{S}{2S}\right)^{\frac{3}{2}}$$

$$= (\tfrac{1}{2})^{\frac{3}{2}}$$

$$2^{\frac{3}{2}} \times \triangle = \triangle + 2000$$

$$2{\cdot}828\triangle = \triangle + 2000$$

$$1{\cdot}828\triangle = 2000$$

$$\triangle = \frac{2000}{1{\cdot}828}$$

Displacement of smaller ship $\triangle$

$$= 1094 \text{ tonne}$$

16. For similar ships:

$$\triangle \propto L^3$$

$$\therefore \frac{\triangle_1}{\triangle_2} = \left(\frac{L_1}{L_2}\right)^3$$

Displacement of model $\triangle_2 = 11\,000 \times \left(\frac{6}{120}\right)^3$

$$= 1{\cdot}375 \text{ tonne}$$

$$S \propto L^2$$

$$\therefore \frac{S_1}{S_2} = \left(\frac{L_1}{L_2}\right)^2$$

Wetted surface area of model

$$S_2 = 2500 \times \left(\frac{6}{120}\right)^2$$

$$= 6{\cdot}25 \text{ m}^2$$

SOLUTIONS TO TEST EXAMPLES 3

1.

½ width	SM	Product for area
1	1	1
7·5	4	30
12	2	24
13·5	4	54
14	2	28
14	4	56
14	2	28
13·5	4	54
12	2	24
7	4	28
0	1	0
		327 = ΣA

Common interval $h = \frac{180}{10}$

$$= 18 \text{ m}$$

(a) Waterplane area $= \frac{h}{3} \Sigma A \times 2$

$$= \frac{18}{3} \times 327 \times 2$$

$$= 3924 \text{ m}^2$$

(b) $\text{TPC} = \frac{\text{waterplane area} \times \text{density}}{100}$

$$= \frac{3924 \times 1{\cdot}025}{100}$$

$$= 40{\cdot}22$$

(c)

Waterplane area coefficient $= \frac{\text{waterplane area}}{\text{length} \times \text{breadth}}$

$$= \frac{3924}{180 \times 28}$$

$$= 0{\cdot}778$$

2.

Waterplane area	SM	Product for volume
865	1	865
1735	4	6940
1965	2	3930
2040	4	8160
2100	2	4200
2145	4	8580
2215	1	2215
		$34\,890 = \Sigma\nabla$

$$\text{Volume of displacement} = \frac{h}{3}\Sigma\nabla$$

$$\text{Displacement} = \frac{h}{3}\Sigma\nabla \times \rho$$

$$= \frac{1{\cdot}5}{3} \times 34\,890 \times 1{\cdot}025$$

$$= 17\,881 \text{ tonne}$$

3.

Cross-sectional area	SM	Product for volume
5	1	5
60	4	240
116	2	232
145	4	580
152	2	304
153	4	612
153	2	306
151	4	604
142	2	284
85	4	340
0	1	0
		$3507 = \Sigma\nabla$

$$\text{Common interval} = \frac{140}{10}$$

$$= 14 \text{ m}$$

(a)

$$\text{Volume of displacement} = \frac{h}{3}\Sigma\nabla$$

$$= \frac{14}{3} \times 3507$$

$$= 16\,366 \text{ m}^3$$

$$\text{Displacement} = \text{volume} \times \text{density}$$

$$= 16\,366 \times 1{\cdot}025$$

$$= 16\,775 \text{ tonne}$$

(b)

$$\text{Block coefficient} = \frac{\text{volume of displacement}}{\text{length} \times \text{breadth} \times \text{draught}}$$

$$= \frac{16\,366}{140 \times 18 \times 9}$$

$$= 0{\cdot}722$$

(c)

$$\text{Midship section area coefficient} = \frac{\text{midship section area}}{\text{breadth} \times \text{draught}}$$

$$= \frac{153}{18 \times 9}$$

$$= 0{\cdot}944$$

(d)

$$\text{Prismatic coefficient} = \frac{\text{volume of displacement}}{\text{length} \times \text{midship section area}}$$

$$= \frac{16\,366}{140 \times 153}$$

$$= 0{\cdot}764$$

Alternatively

$$\text{Prismatic coefficient} = \frac{\text{block coefficient}}{\text{midship section area coefficient}}$$

$$= \frac{0{\cdot}722}{0{\cdot}944}$$

$$= 0{\cdot}764$$

4.

Section	½ ordinate	SM	Product for area	Lever	Product for 1st moment
AP	1·2	½	0·6	+5	+ 3·0
½	3·5	2	7·0	+4½	+31·5
1	5·3	1	5·3	+4	+21·2
1½	6·8	2	13·6	+3½	+47·6
2	8·0	1½	12·0	+3	+36·0
3	8·3	4	33·2	+2	+66·4
4	8·5	2	17·0	+1	+17·0
5	8·5	4	34·0	0	+222·7 = ΣMA
6	8·5	2	17·0	−1	−17·0
7	8·4	4	33·6	−2	−67·2
8	8·2	1½	12·3	−3	−36·9
8½	7·9	2	15·8	−3½	−55·3
9	6·2	1	6·2	−4	−24·8
9½	3·5	2	7·0	−4½	−31·5
FP	0	½	0	−5	− 0
			214·6 = ΣA		−232·7 = ΣMF

$$\text{Common interval} = \frac{120}{10} = 12 \text{ m}$$

(a)
$$\text{Waterplane area} = \frac{h}{3}\,\Sigma_A \times 2 = \frac{12}{3} \times 214{\cdot}6 \times 2 = 1716{\cdot}8 \text{ m}^2$$

Since ΣMF exceeds ΣMA the centroid will be *forward* of midships.

$$\text{Distance of centroid from midships} = \frac{h(\Sigma_{MA} + \Sigma_{MF})}{\Sigma_A} = \frac{12(222{\cdot}7 - 232{\cdot}7)}{214{\cdot}6} = 0{\cdot}559 \text{ m forward}$$

5.

TPC	SM	Product for displacement	Lever	Product for 1st moment
4·0	1	4·0	0	0
6·1	4	24·4	1	24·4
7·8	2	15·6	2	31·2
9·1	4	36·4	3	109·2
10·3	2	20·6	4	82·4
11·4	4	45·6	5	228·0
12·0	1	12·0	6	72·0
		158·6 = ΣΔ		547·2 = ΣM

Common interval = 1·5 m
= 150 cm

(a)
$$\text{Displacement} = \frac{h}{3}\,\Sigma\Delta \quad (h \text{ in cm})$$
$$= \frac{150}{3} \times 158{\cdot}6$$
$$= 7930 \text{ tonne}$$

(b)
$$KB = \frac{h\,\Sigma_M}{\Sigma\Delta} \quad (h \text{ in m})$$
$$= \frac{1{\cdot}5 \times 547{\cdot}2}{158{\cdot}6}$$
$$= 5{\cdot}175 \text{ m}$$

6.

½ breadths	SM	Product for area	Lever	Product for 1st moment	Lever	Product for 2nd moment
0·3	1	0·3	+5	+ 1·5	+5	+ 7·5
3·8	4	15·2	+4	+ 60·8	+4	+ 243·2
6·0	2	12·0	+3	+ 36·0	+3	+ 108·0
7·7	4	30·8	+2	+ 61·6	+2	+ 123·2
8·3	2	16·6	+1	+ 16·6	+1	+ 16·6
9·0	4	36·0	0	+176·5 = ΣMA	0	0

8·4	2	16·8	−1	−16·8	−1	+16·8
7·8	4	31·2	−2	−62·4	−2	+124·8
6·9	2	13·8	−3	−41·4	−3	+124·2
4·7	4	18·8	−4	−75·2	−4	+300·8
0	1	0	−5	0	−5	0
		191·5 = Σ_A		−195·8 = Σ_{MF}		+1065·1 = Σ_I

Common interval = 15 m

(a) Waterplane area $= \frac{h}{3} \Sigma_A \times 2$

$$= \frac{15}{3} \times 191{\cdot}5 \times 2$$

$$= 1915 \text{ m}^2$$

(b)
Distance of centroid from midships

$$= \frac{h(\Sigma_{MA} + \Sigma_{MF})}{\Sigma_A}$$

$$= \frac{15(176{\cdot}5 - 195{\cdot}8)}{191{\cdot}5}$$

$$= 1{\cdot}512 \text{ m forward}$$

Second moment of area about midships

$$= \frac{h^3}{3} \Sigma_I \times 2$$

$$= \frac{15^3}{3} \times 1065{\cdot}1 \times 2$$

$$= 2\,396\,475 \text{ m}^4$$

Second moment of area about centroid

$$= 2\,396\,475 - 1915 \times 1{\cdot}512^2$$
$$= 2\,396\,475 - 4378$$
$$= 2\,392\,097 \text{ m}^4$$

7.

Draught	Displacement	SM	Product for moment
0	0	1	0
1	189	4	756
2	430	2	860
3	692	4	2768
4	977	1	977
			5361 = Σ_M

Common interval = 1 m

Area of curve $= \frac{h}{3} \Sigma_M$

$$= \tfrac{1}{3} \times 5361$$
$$= 1787 \text{ tonne m}$$

VCB below waterline $= \dfrac{\text{area of curve}}{\text{displacement}}$

$$= \frac{1787}{977}$$

$$= 1{\cdot}829 \text{ m}$$

$$KB = 4 - 1{\cdot}829$$
$$= 2{\cdot}171 \text{ m}$$

8.

Width	SM	Product for area	Lever	Product for 1st moment	Lever	Product for 2nd moment
8·0	1	8·0	0	0	0	0
7·5	4	30·0	1	30·0	1	30·0
6·5	2	13·0	2	26·0	2	52·0
5·7	4	22·8	3	68·4	3	205·2
4·7	2	9·4	4	37·6	4	150·4
3·8	4	15·2	5	76·0	5	380·0
3·0	1	3·0	6	18·0	6	108·0
				256·0 = Σ_M		925·6 = Σ_I

Common interval = 1·2 m

Load on bulkhead = density × g × 1st moment

$$= \rho g \times \frac{h^2}{3} \Sigma M$$

$$= 1{\cdot}025 \times 9{\cdot}81 \times \frac{1{\cdot}2^2}{3} \times 256{\cdot}0$$

$$= 1235 \text{ kN}$$

$$\text{Centre of pressure from top} = \frac{\text{2nd moment about top}}{\text{1st moment about top}}$$

$$= \frac{h\,\Sigma I}{\Sigma M}$$

$$= \frac{1{\cdot}2 \times 925{\cdot}6}{256{\cdot}0}$$

$$= 4{\cdot}339 \text{ m}$$

9.

Width	SM	Product for area	(Width)²	SM	Product for 1st moment	(Width)³	SM	Product for 2nd moment
10	1	10	100	1	100	1000	1	1000
9	4	36	81	4	324	729	4	2916
7	2	14	49	2	98	343	2	686
4	4	16	16	4	64	64	4	256
1	1	1	1	1	1	1	1	1
		77 = Σa			587 = Σm			4859 = Σi

$$\text{Common interval} = \frac{12}{4}$$

$$= 3 \text{ m}$$

Area of surface

$$a = \frac{h}{3} \Sigma a$$

$$= \frac{3}{3} \times 77$$

$$= 77 \text{ m}^2$$

Distance of centroid from longitudinal bulkhead

$$= \bar{y} \frac{\Sigma m}{2\Sigma a}$$

$$= \frac{587}{2 \times 77}$$

$$= 3{\cdot}812 \text{ m}$$

Second moment of area about longitudinal bulkhead

$$i_b = \frac{h}{9} \Sigma i$$

$$= \frac{3}{9} \times 4859$$

$$= 1619{\cdot}7 \text{ m}^4$$

Second moment of area about centroid

$$i_g = i_b - ay^2$$
$$= 1619{\cdot}7 - 77 \times 3{\cdot}812^2$$
$$= 1619{\cdot}7 - 1118{\cdot}7$$
$$= 501{\cdot}0 \text{ m}^4$$

10.

½ ordinate	(½ ordinate)³	SM	Product for 2nd moment
1·6	4·1	1	4·1
5·7	185·2	4	740·8
8·8	681·5	2	1363·0
10·2	1061·2	4	4244·8
10·5	1157·6	2	2315·2
10·5	1157·6	4	4630·4
10·5	1157·6	2	2315·2
10·0	1000·0	4	4000·0
8·0	512·0	2	1024·0
5·0	125·0	4	500·0
0	0	1	0
			21 137·5 = ΣI

Common interval = 16 m
Second moment of area about centreline

$$= \frac{h}{9} \times \Sigma I \times 2$$

$$= \frac{16}{9} \times 21\,137{\cdot}5 \times 2$$

$$= 75\,155 \text{ m}^4$$

11.

Cross-sectional area	SM	Product for volume	Lever	Product for 1st moment
2	1	2	+5	+ 10
40	4	160	+4	+ 640
79	2	158	+3	+ 474
100	4	400	+2	+ 800
103	2	206	+1	+ 206
104	4	416	0	+2130 = ΣMA
104	2	208	−1	− 208
103	4	412	−2	− 824
97	2	194	−3	− 582
58	4	232	−4	− 928
0	1	0	−5	0
		2388 = Σ∇		−2542 = ΣMF

Common interval = 12 m

(a) Displacement $= \rho \times \frac{h}{3}\Sigma\nabla$

$$= 1{\cdot}025 \times \frac{12}{3} \times 2388$$

$$= 9790{\cdot}8 \text{ tonne}$$

(b) Centre of buoyancy from midships

$$= \frac{h(\Sigma MA + \Sigma MF)}{\Sigma\nabla}$$

$$= \frac{12(2130 - 2542)}{2388}$$

$$= 2{\cdot}070 \text{ m forward}$$

12.

½ ordinate	SM	Product for area
1·6	1	1·6
6·0	3	18·0
9·2	3	27·6
10·5	2	21·0
11·0	3	33·0
11·0	3	33·0
10·2	2	20·4
8·3	3	24·9
5·1	3	15·3
0	1	0
		194·8 = ΣA

$$\text{Common interval} = \frac{180}{9}$$

$$= 20 \text{ m}$$

$$\text{Waterplane area} = \frac{3}{8} h\, \Sigma A \times 2$$

$$= \frac{3}{8} \times 20 \times 194{\cdot}8 \times 2$$

$$= 2922 \text{ m}^2$$

SOLUTIONS TO TEST EXAMPLES 4

1.

Mass	Kg	Vertical moment
4000	6·0	24 000
1000	0·8	800
200	1·0	200
5000	5·0	25 000
3000	9·5	28 500
13 200		78 500

Thus displacement = 13 200 tonne

$$\text{Centre of gravity above keel} = \frac{78\,500}{13\,200} = 5{\cdot}947 \text{ m}$$

2.

Mass	Kg	Vertical moment	LCG from midships	Longitudinal moment
5000	6·0	30 000	1·5 ford	7 500 ford
500	10·0	5 000	36·0 aft	18 000 aft
5500		35 000		10 500 aft

$$\text{Centre of gravity above keel} = \frac{35\,000}{5500} = 6{\cdot}364 \text{ m}$$

$$\text{Centre of gravity from midships} = \frac{10\,500}{5500} = 1{\cdot}909 \text{ m aft}$$

3. $$\text{Shift in centre of gravity} = \frac{300 \times (24 + 40)}{6000} = 3{\cdot}2 \text{ m aft}$$

New position of centre of gravity = 1·2 − 3·2
= − 2·0 m
or = 2·0 m aft of midships

4. (a) Shift in centre of gravity due to transfer of oil

$$= \frac{250 \times (75 + 50)}{17\,000} = 1{\cdot}839 \text{ m aft}$$

∴ new position of centre of gravity = 1·0 + 1·839
= 2·839 m aft of midships

(b) Taking moments about midships:

$$\text{new position of centre of gravity} = \frac{17\,000 \times 2{\cdot}839 - 200 \times 50}{17\,000 - 200} = \frac{48\,260 - 10\,000}{16\,800} = 2{\cdot}278 \text{ m aft of midships}$$

5. Shift in centre of gravity due to lowered cargo

$$= \frac{500 \times 3}{3000} = 0{\cdot}5 \text{ m down}$$

Taking moments about the new position of the centre of gravity:
Shift in centre of gravity due to added cargo

$$= \frac{3000 \times 0 + 500 \times 3{\cdot}5}{3000 + 500} = \frac{1750}{3500} = 0{\cdot}5 \text{ m up}$$

i.e. the position of the centre of gravity does not change.

6.

	Mass	Kg	Vertical moment
	2000	1·5	3000
	300	4·5	1350
	50	6·0	300
Total removed	2350		4650
Original	10 000	3·0	30 000
Final	7650		25 350