REED'S
NAVAL ARCHITECTURE
FOR
MARINE ENGINEERS

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By

E. A. STOKOE

C.Eng., F.R.I.N.A., F.I.Mar.E., M.N.E.C.Inst.,
Principal Lecturer in Naval Architecture at
South Shields Marine and Technical College

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PREFACE

This book is intended to cover the theoretical work in the Syllabus for Naval Architecture in Part B of the Department of Trade Examinations for Second and First Class Engineers.

In each section the work progresses from an elementary stage to the standard required for First Class Examinations. Parts of the subject matter and the attendant Test Examples are marked with the prefix "f" to indicate that they are normally beyond the syllabus for the Second Class Examination and so can be temporarily disregarded by such candidates. Throughout the book emphasis is placed on basic principles, and the profusely illustrated text, together with the worked examples, assists the student to assimilate these principles more easily.

All students attempting Part B of their certificate will have covered the work required for Part A, and several of the principles of Mathematics and Mechanics are used in this volume. Where a particularly important principle is required, however, it is revised in this book. Fully worked solutions are given for all Test Examples and Examination Questions. In several cases shorter methods are available and acceptable in the examination, but the author has attempted to use a similar method for similar problems, and to avoid methods which may only be used in isolated cases. It should be noted that a large proportion of the worked solutions include diagrams and it is suggested that the students follow this practice. The typical Examination Questions are intended as a revision of the whole of the work, and should be treated as such by attempting them in the order in which they are given. The student should avoid attempting a number of similar types of questions at the same time. A number of Examination Questions have been selected from Department of Trade papers and are reproduced by kind permission of the Controller of Her Majesty's Stationery Office.

An engineer who works systematically through this volume will find that his time is amply repaid when attending a course of study at a college and his chance of success in the Examination will be greatly increased.

CONTENTS

INTRODUCTION TO SI UNITS

SI is the abbreviation for Système International d'Unités, the metric system of measurement now coming into international use and being adopted by British Industry.

BASIC UNITS

There are six basic units in the system:

QUANTITY	UNIT	SYMBOL
length	metre	m
mass	kilogramme	kg
time	second	s
temperature	kelvin	K
electric current	ampere	A
luminous intensity	candela	cd

DERIVED UNITS

It is possible to obtain derived units from these basic units. The system has been designed in such a way that the basic units are used without numerical multipliers to obtain the fundamental derived units. The system is therefore said to be *coherent*.

unit area $= m^2$
unit volume $= m^3$
unit velocity $= m/s$
unit acceleration $= m/s^2$

The unit of *force* is the *newton* N

Now force = mass × acceleration
Hence 1 newton $= 1\ kg \times 1 m/s^2$
N $= kg\ m/s^2$

The unit of *work* is the *joule* J

Now work done = force × distance.
1 joule $= 1\ N \times 1\ m$
J $= N\ m$

The unit of *power* is the *watt* W

Now power = work done per unit time
1 watt $= 1\ J \div 1\ s$
W $= J/s$

The unit of *pressure* is the *pascal* Pa

Now pressure = force per unit area
1 Pa $= 1N \div m^2$
Pa $= N/m^2$

MULTIPLES AND SUB-MULTIPLES

In order to keep the number of names of units to a minimum, multiples and sub-multiples of the fundamental units are used. In each case powers of ten are found to be most convenient and are represented by prefixes which are combined with the symbol of the unit.

MULTIPLICATION FACTOR	STANDARD FORM	PREFIX	SYMBOL
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto	h
10	10^{1}	deca	da
0·1	10^{-1}	deci	d
0·01	10^{-2}	centi	c
0·001	10^{-3}	milli	m
0·000 001	10^{-6}	micro	μ
0·000 000 001	10^{-9}	nano	n
0·000 000 000 001	10^{-12}	pico	p

Only one prefix may be used with each symbol. Thus a thousand kilogrammes would be expressed as a Mg and not kkg. When a prefix is attached to a unit, it becomes a new unit symbol on its own account and this can be raised to positive or negative powers of ten.

Multiples of 10^3 are recommended but others are recognised because of convenient sizes and established usage and custom. A good example of this convenient usage lies in the calculation of volumes. If only metres or millimetres are used for the basic dimensions, the volume is expressed in m^3 or mm^3.

now $\quad 1 m^3 = 10^9 mm^3$

i.e. the gap is too large to be convenient. If, on the other hand, the basic dimensions may be expressed in decimetres or centimetres in addition to metres and millimetres, the units of volume change in 10^3 intervals.

i.e.
$$1 m^3 = 1000 dm^3$$
$$1 dm^3 = 1000 cm^3$$
$$1 cm^3 = 1000 mm^3$$

Several special units are introduced, again because of their convenience. A megagramme, for instance, is termed a *tonne* which is approximately equal to an imperial ton mass. Pressure may be expressed in *bars* (b) of value $10^5 N/m^2$. A bar is approximately equal to one atmosphere. Stresses may be expressed in *hectobars* ($10^7 N/m^2$) of about $\frac{2}{3}$tonf/in².

It is unwise, however, to consider comparisons between imperial and SI units and it is probable that the pressure and stress units will revert to the basic unit and its multiples.

CHAPTER 1

HYDROSTATICS

DENSITY ρ of a substance is the mass of a unit volume of the substance and may be expressed in grammes per millilitre (g/ml), kilogrammes per cubic metre (kg/m³) or tonnes per cubic metre (t/m³). The numerical values of g/ml are the same as t/m³. The density of fresh water may be taken as 1·000 t/m³ or 1000 kg/m³ and the density of sea water 1·025 t/m³ or 1025 kg/m³.

RELATIVE DENSITY or specific gravity of a substance is the density of the substance divided by the density of fresh water, i.e. the ratio of the mass of any volume of the substance to the mass of the same volume of fresh water. Thus the relative density (rd) of fresh water is 1·000 while the relative density of sea water is 1025 ÷ 1000 or 1·025. It is useful to know that the density of a substance expressed in t/m³ is numerically the same as the relative density. If a substance has a relative density of x, then one cubic metre of the substance will have a mass of x tonnes. V cubic metres will have a mass of Vx tonnes or $1000Vx$ kilogrammes.

Thus:

mass of substance = volume × density of substance

Example. If the relative density of lead is 11·2, find
(a) its density
(b) the mass of 0·25 m³ of lead.

Density of lead = relative density of lead × density of fresh water
= 11·2 t/m³

Mass of lead = 0·25 × 11·2
= 2·8 t

Example. A plank 6 m long, 0·3 m wide and 50 mm thick has a mass of 60 kg. Calculate the density of the wood.

$$\text{Volume of wood} = 6{\cdot}0 \times 0{\cdot}3 \times 0{\cdot}050$$
$$= 90 \times 10^{-3}\ \text{m}^3$$

$$\text{Density of wood} = \frac{\text{mass}}{\text{volume}}$$

$$= \frac{60}{90 \times 10^{-3}}\ \frac{\text{kg}}{\text{m}^3}$$

$$= 667\ \text{kg/m}^3$$

PRESSURE EXERTED BY A LIQUID

Liquid pressure is the load per unit area exerted by the liquid and may be expressed in multiples of N/m^2.

e.g. $10^3\ \text{N/m}^2 = 1\ \text{kN/m}^2$
$10^5\ \text{N/m}^2 = 1\ \text{bar}$

This pressure acts equally in all directions and perpendicular to the surface of any immersed plane.

Consider a trough containing liquid of density ρ kg/m³

Let A = cross-sectional area of a cylinder of this liquid in m²

and h = height of cylinder in m (Fig. 1).

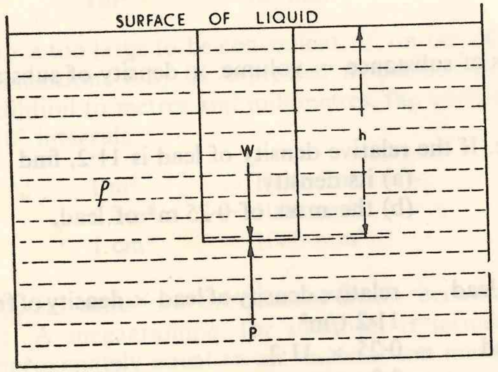


Fig. 1

The cylinder is in equilibrium under the action of two vertical forces:—

(a) the gravitational force W acting vertically down
(b) the upthrust P exerted by the liquid on the cylinder.

Thus $P = W$
but $P = pA$
where p = liquid pressure at a depth h m
and $W = \rho g A h$
$\therefore\ pA = \rho g A h$
$p = \rho g h$

Thus it may be seen that the liquid pressure depends upon the density ρ and the vertical distance h from the point considered to the surface of the liquid. Distance h is known as the *head*.

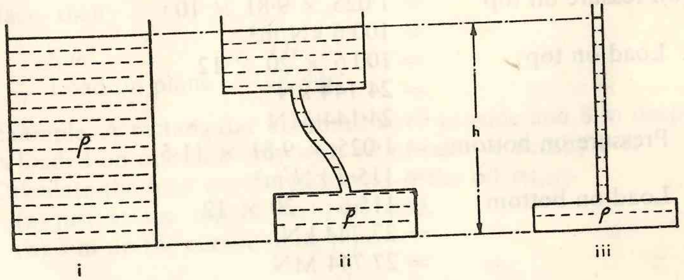


Fig. 2

The pressure at the base of each of the containers shown in Fig. 2 is ρgh although it may be seen that the total mass of the liquid is different in each case. Container (ii) could represent a supply tank and header tank used in most domestic hot water systems. The pressure at the supply tank depends upon the height of the header tank.

Container (iii) could represent a double bottom tank having a vertical overflow pipe. The pressure inside the tank depends upon the height to which the liquid rises in the pipe.

The total load exerted by a liquid on a *horizontal* plane is the product of the pressure and the area of the plane.

$$P = p\,A$$

Example. A rectangular double bottom tank is 20 m long, 12 m wide and 1·5 m deep, and is full of sea water having a density of 1·025 tonne/m³.

Calculate the pressure in kN/m³ and the load in MN on the top and bottom of the tank if the water is:

(a) at the top of the tank
(b) 10 m up the sounding pipe above the tank top.

(a) Pressure on top $= \rho g h$

$= 1{\cdot}025 \times 9{\cdot}81 \times 0$

$= 0$

Load on top $= 0$

Pressure on bottom $= 1{\cdot}025 \times 9{\cdot}81 \times 1{\cdot}5 \quad \frac{\text{Mg}}{\text{m}^3} \times \frac{\text{m}}{\text{s}^2} \times \text{m}$

$= 15{\cdot}09 \text{ kN/m}^2$

Load on bottom $= 15{\cdot}09 \times 20 \times 12$

$= 3622 \text{ kN}$

$= 3{\cdot}622 \text{ MN}$

(b) Pressure on top $= 1{\cdot}025 \times 9{\cdot}81 \times 10$

$= 100{\cdot}6 \text{ kN/m}^2$

Load on top $= 100{\cdot}6 \times 20 \times 12$

$= 24\,144 \text{ kN}$

$= 24{\cdot}144 \text{ MN}$

Pressure on bottom $= 1{\cdot}025 \times 9{\cdot}81 \times 11{\cdot}5$

$= 115{\cdot}6 \text{ kN/m}^2$

Load on bottom $= 115{\cdot}6 \times 20 \times 12$

$= 27\,744 \text{ kN}$

$= 27{\cdot}744 \text{ MN}$

This example shows clearly the effect of a head of liquid. It should be noted that a very small volume of liquid in a vertical pipe may cause a considerable increase in load.

LOAD ON AN IMMERSED PLANE

The pressure on any horizontal plane is constant, but if the plane is inclined to the horizontal there is a variation in pressure over the plane due to the difference in head. The total load on such a plane may be determined as follows.

Consider an irregular plane of area A, totally immersed in a liquid of density ρ and lying at an angle θ to the surface of the liquid as shown in Fig. 3.

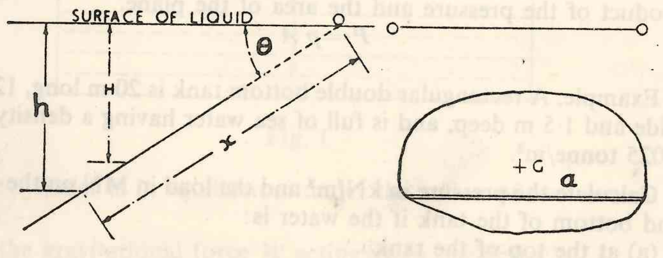


Fig. 3

Divide the plane into thin strips parallel to the surface of the liquid. Let one such strip, distance h below the surface of the liquid, have an area a. Since the strip is thin, any variation in pressure may be ignored.

Load on strip $= \rho g a h$

Load on plane $= \rho g(a_1 h_1 + a_2 h_2 + a_3 h_3 + \ldots)$

$= \rho g \Sigma ah$

But Σah is the first moment of area of the plane about the surface of the liquid.

If H is the distance of the centroid of the plane from the liquid surface, then:

$\Sigma ah = AH$

$\therefore$ Load on plane $= \rho g AH$

Example. A rectangular bulkhead is 10 m wide and 8 m deep. It is loaded on one side only with oil of relative density 0·8. Calculate the load on the bulkhead if the oil is:

(a) just at the top of the bulkhead.

(b) 3 m up the surrounding pipe.

(a) Load on bulkhead $= \rho g AH$

$= 0{\cdot}8 \times 1{\cdot}0 \times 9{\cdot}81 \times 10 \times 8 \times \frac{8}{2}$

$= 2511 \text{ kN}$

(b) Load on bulkhead $= 0{\cdot}8 \times 1{\cdot}0 \times 9{\cdot}81 \times 10 \times 8 \times \left(\frac{8}{2} + 3\right)$

$= 4395 \text{ kN}$

CENTRE OF PRESSURE

The centre of pressure on an immersed plane is the point at which the whole liquid load may be regarded as acting.

Consider again Fig. 3.

Let the strip be distance x from the axis 0-0.

Then $h = x \sin \theta$

Load on strip $= \rho g a h$

$= \rho g a x \sin \theta$

Load on plane $= \rho g \sin \theta \,(a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots\ldots)$

$= \rho g \sin \theta \,\Sigma ax$

Taking moments about axis 0-0:

$$\text{Moment of load on strip} = x \times \rho g a x \sin\theta$$
$$= \rho g a x^2 \sin\theta$$

$$\text{Moment of load on plane} = \rho g \sin\theta\,(a_1 x_1^2 + a_2 x_2^2 + \ldots\ldots)$$
$$= \rho g \sin\theta\,\Sigma a x^2$$

$$\text{Centre of pressure from 0-0} = \frac{\text{moment}}{\text{load}}$$
$$= \frac{\rho g \sin\theta\,\Sigma a x^2}{\rho g \sin\theta\,\Sigma\, a x}$$
$$= \frac{\Sigma a x^2}{\Sigma a x}$$

But Σax is the first moment of area of the plane about 0-0 and Σax^2 is the second moment of area of the plane about 0-0. If the plane is vertical, then 0-0 represents the surface of the liquid, and thus:

Centre of pressure from surface

$$= \frac{\text{second moment of area about surface}}{\text{first moment of area about surface}}$$

The second moment of area may be calculated using the *theroem of parallel axes.*

If I_{NA} is the second moment about an axis through the centroid (the neutral axis), then the second moment about an axis 0-0 parallel to the neutral axis and distance H from it is given by

$$I_{OO} = I_{NA} + AH^2$$

where A is the area of the plane

Thus Centre of pressure from 0-0

$$= \frac{I_{OO}}{AH}$$
$$= \frac{I_{NA} + AH^2}{AH}$$
$$= \frac{I_{NA}}{AH} + H$$

I_{NA} for a rectangle is $\frac{1}{12} BD^3$
I_{NA} for a triangle is $\frac{1}{36} BD^3$
I_{NA} for a circle is $\frac{\pi}{64} D^4$

The following examples show how these principles may be applied.

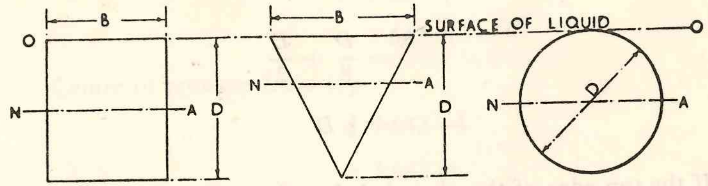


Fig. 4

(a) RECTANGULAR PLANE WITH EDGE IN SURFACE

$$\text{Centre of pressure from 0-0} = \frac{I_{NA}}{AH} + H$$
$$= \frac{\frac{1}{12} BD^3}{BD \times \frac{1}{2}D} + \frac{D}{2}$$
$$= \frac{D}{6} + \frac{D}{2}$$
$$= \tfrac{2}{3} D$$

(b) TRIANGULAR PLANE WITH EDGE IN SURFACE

$$\text{Centre of pressure from 0-0} = \frac{I_{NA}}{AH} + H$$
$$= \frac{\frac{1}{36} BD^3}{\frac{1}{2}BD \times \frac{1}{3}D} + \frac{D}{3}$$
$$\frac{D}{6} + \frac{D}{3}$$
$$= \tfrac{1}{2} D$$

(c) CIRCULAR PLANE WITH EDGE IN SURFACE

$$\text{Centre of pressure from 0-0} = \frac{I_{NA}}{AH} + H$$

$$= \frac{\frac{\pi}{64} D^4}{\frac{\pi}{4} D^2 \times \frac{1}{2} D} + \frac{D}{2}$$

$$= \frac{D}{8} + \frac{D}{2}$$

$$= \tfrac{5}{8} D$$

If the top edge of the plane is below the surface of the liquid, these figures change considerably.

Example. A peak bulkhead is in the form of a triangle, apex down, 6 m wide at the top and 9 m deep. The tank is filled with sea water. Calculate the load on the bulkhead and the position of the centre of pressure relative to the top of the bulkhead if the water is;

(a) at the top of the bulkhead
(b) 4 m up the sounding pipe.

(a) Load on bulkhead $= \rho g A H$

$$= 1{\cdot}025 \times 9{\cdot}81 \times \frac{6 \times 9}{2} \times \frac{9}{3}$$

$$= 814{\cdot}5 \text{ kN}$$

Centre of pressure from top

$$= \frac{D}{2}$$

$$= \frac{9}{2}$$

$$= 4{\cdot}5 \text{ m}$$

(b) Load on bulkhead $= 1{\cdot}025 \times 9{\cdot}81 \times \frac{6 \times 9}{2} \times \left(\frac{9}{3} + 4\right)$

$$= 1901 \text{ kN}$$

Centre of pressure from surface

$$= \frac{I_{NA}}{AH} + H$$

$$= \frac{\frac{1}{36} \times 6 \times 9^3}{\frac{1}{2} \times 6 \times 9 \times 7} + 7$$

$$= 0{\cdot}624 + 7$$

$$= 7{\cdot}642 \text{ m}$$

Centre of pressure from top

$$= 7{\cdot}642 - 4$$

$$= 3{\cdot}642 \text{ m}$$

LOAD DIAGRAM

If the pressure at any point in an immersed plane is multiplied by the width of the plane at this point, the load per unit depth of plane is obtained. If this is repeated at a number of points, the resultant values may be plotted to form the load diagram for the plane.

The *area* of this load diagram represents the *load* on the plane, while its *centroid* represents the position of the *centre of pressure.*

For a rectangular plane with its edge in the surface, the load diagram is in the form of a triangle.

For a rectangular plane with its edge below the surface, the load diagram is in the form of a trapezoid.

The load diagrams for triangular planes are parabolic.

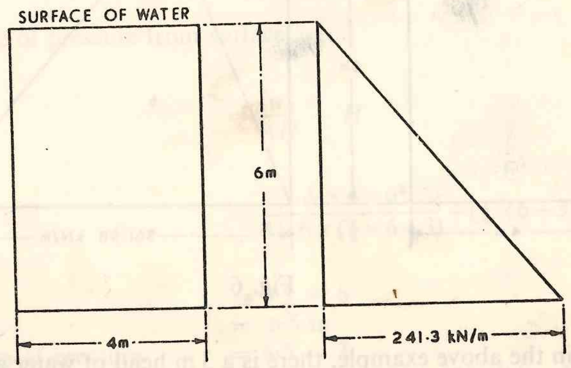


Fig. 5

Consider a rectangular bulkhead 4 m wide and 6 m deep loaded to its top edge with sea water.

Load/m at top of bulkhead $= \rho gh \times$ width
$= 1{\cdot}025 \times 9{\cdot}81 \times 0 \times 4$
$= 0$

Load/m at bottom of bulkhead
$= 1{\cdot}025 \times 9{\cdot}81 \times 6 \times 4$
$= 241{\cdot}3$ kN

Load on bulkhead = area of load diagram
$= \frac{1}{2} \times 6 \times 241{\cdot}3$
$= 723{\cdot}9$ kN

Centre of pressure = centroid of load diagram
$= \frac{2}{3} \times 6$
= 4 m from top

Check: Load on bulkhead $= \rho g A H$
$= 1{\cdot}025 \times 9{\cdot}81 \times 4 \times 6 \times \frac{1}{2} \times 6$
$= 724{\cdot}0$ kN

and Centre of pressure $= \frac{2}{3} D$
= 4 m from top

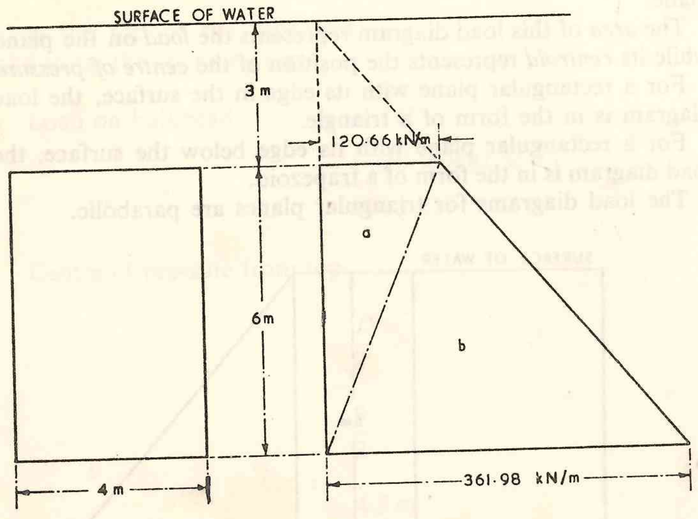


Fig. 6

If, in the above example, there is a 3 m head of water above the bulkhead, then:

Load/m at top of bulkhead $= 1{\cdot}025 \times 9{\cdot}81 \times 3 \times 4$
$= 120{\cdot}66$ kN

Load/m at bottom of bulkhead
$= 1{\cdot}025 \times 9{\cdot}81 \times 9 \times 4$
$= 361{\cdot}98$ kN

The load diagram may be divided into two triangles *a* and *b*

Area *a* $= \frac{1}{2} \times 6 \times 120{\cdot}66$
$= 361{\cdot}98$ kN

Area *b* $= \frac{1}{2} \times 6 \times 361{\cdot}98$
$= 1085{\cdot}94$ kN

Total load $= 361{\cdot}98 + 1085{\cdot}94$
$= 1447{\cdot}92$ kN

Taking moments about the top of a bulkhead

$$\text{Centre of pressure} = \frac{361{\cdot}98 \times \frac{1}{3} \times 6 + 1085{\cdot}94 \times \frac{2}{3} \times 6}{361{\cdot}98 + 1085{\cdot}94}$$

$$= \frac{723{\cdot}96 + 4343{\cdot}76}{1447{\cdot}92}$$

= 3·5 m from top of bulkhead

These results may again be checked by calculation.

Load on bulkhead $= 1{\cdot}025 \times 9{\cdot}81 \times 4 \times 6 \times (\frac{1}{2} \times 6 + 3)$
$= 1448$ kN

Centre of pressure from surface

$$= \frac{I_{NA}}{AH} + H$$

$$= \frac{\frac{1}{12} \times 4 \times 6^3}{4 \times 6 \times (\frac{1}{2} \times 6 + 3)} + (\frac{1}{2} \times 6 + 3)$$

$= 0{\cdot}5 + 6$
$= 6{\cdot}5$ m

Centre of pressure $= 6{\cdot}5 - 3$
= 3·5 m from top of bulkhead.

SHEARING FORCE ON BULKHEAD STIFFENERS

A bulkhead stiffener supports a rectangle of plating equal to the length of the stiffener times the spacing of the stiffeners. If the bulkhead has liquid on one side to the top edge, the stiffener supports a load which increases uniformly from zero at the top to a maximum at the bottom (Fig. 7).

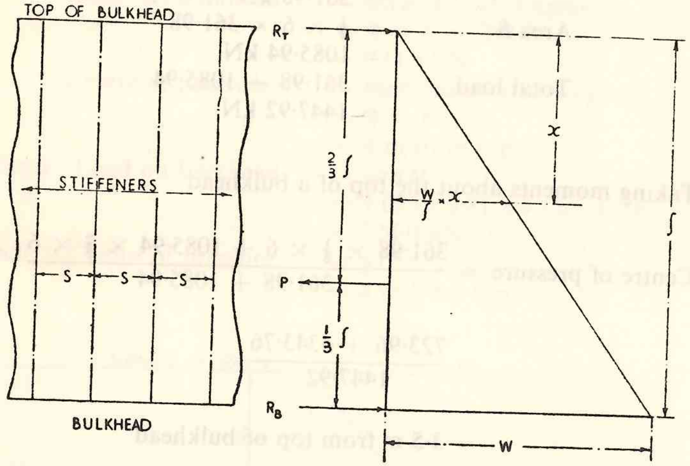


Fig. 7

Let
l = length of stiffener in m
s = spacing of stiffeners in m
ρ = density of liquid in kg/m³
P = load on stiffener
W = load/m at bottom of stiffener

Then

$$P = \rho g l s \times \frac{l}{2}$$

$$= \tfrac{1}{2}\rho g l^2 s$$

$$W = \rho g l s$$

$$\therefore P = W \times \frac{l}{2}$$

The load P acts at the centre of pressure which is $\frac{2}{3}l$ from the top. Reactions are set up by the end connections at the top (R_T) and at the bottom (R_B).

Taking moments about the top.

$$R_B \times l = P \times \tfrac{2}{3}l$$

$$R_B = \tfrac{2}{3}P$$

$$\text{and } R_T = \tfrac{1}{3}P$$

The shearing force at a distance x from the top will be the reaction at the top, less the area of the load diagram from this point to the top.

$$\text{i.e. } SF_x = R_T - \frac{Wx}{l} \times \frac{x}{2}$$

$$= \tfrac{1}{3}P - \frac{Wx^2}{2l}$$

$$= \frac{Wl}{6} - \frac{Wx^2}{2l}$$

Let $x = 0$

$$\text{SF at top} = \frac{Wl}{6}$$

$$= \tfrac{1}{3}P$$

Let $x = l$

$$\text{SF at bottom} = \frac{Wl}{6} - \frac{Wl^2}{2l}$$

$$= \frac{Wl}{6} - \frac{Wl}{2}$$

$$= -\frac{Wl}{3}$$

$$= -\tfrac{2}{3}P$$

Since the shearing force is positive at the top and negative at the bottom, there must be some intermediate point at which the shearing force is zero. This is also the position of the maximum bending moment.

Let SF = 0

$$O = \frac{Wl}{6} - \frac{Wx^2}{2l}$$

$$\frac{Wx^2}{2l} = \frac{Wl}{6}$$

$$x^2 = \frac{l^2}{3}$$

Position of zero shear $x = \frac{l}{\sqrt{3}}$ from the top

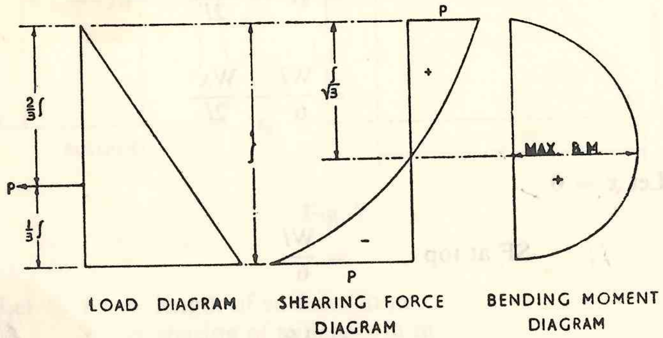


Fig. 8.

Example. A bulkhead 9 m deep is supported by vertical stiffeners 750 mm apart. The bulkhead is flooded to the top edge with sea water on one side only. Calculate:

(a) shearing force at top
(b) shearing force at bottom
(c) position of zero shear.

Load on stiffener $P = \rho g A H$

$$= 1{\cdot}025 \times 9{\cdot}81 \times 9 \times 0{\cdot}75 \times \frac{9}{2}$$

$$= 305{\cdot}4 \text{ kN}$$

(a) Shearing force at top $= \frac{1}{3}P$

$$= \frac{305{\cdot}4}{3}$$

$$= 101{\cdot}8 \text{ kN}$$

(b) Shearing force at bottom $= \frac{2}{3}P$

$$= \frac{2}{3} \times 305{\cdot}4$$

$$= 203{\cdot}6 \text{ kN}$$

(c) Position of zero shear $= \frac{l}{\sqrt{3}}$

$$= \frac{9}{\sqrt{3}}$$

$= 5{\cdot}197$ m from the top

$$\text{mass of wood} = \text{mass of water displaced}$$
$$= 1025 \times 4 \times 0{\cdot}3 \times 0{\cdot}15$$
$$= 184{\cdot}5 \text{ kg}$$

$$\text{mass of equal volume of fresh water} = 1000 \times 4 \times 0{\cdot}3 \times 0{\cdot}25$$
$$= 300 \text{ kg}$$

$$\text{Relative density of wood} = \frac{184{\cdot}5}{300}$$
$$= 0{\cdot}615$$

Example. A box barge 40 m long and 9 m wide floats in sea water at a draught of 3·5 m. Calculate the mass of the barge.

$$\text{mass of barge} = \text{mass of water displaced}$$
$$= 1025 \times 40 \times 9 \times 3{\cdot}5$$
$$= 1292 \times 10^3 \text{ kg}$$
$$= 1292 \text{ tonne}$$

DISPLACEMENT

When a ship is floating freely at rest the mass of the ship is equal to the mass of the volume of water displaced by the ship and is therefore known as the *displacement* of the ship. Thus if the volume of the underwater portion of the ship is known, together with the density of the water, it is possible to obtain the displacement of the ship.

It is usual to assume that a ship floats in sea water of density 1025 kg/m³ or 1·025 t/m³. Corrections may then be made if the

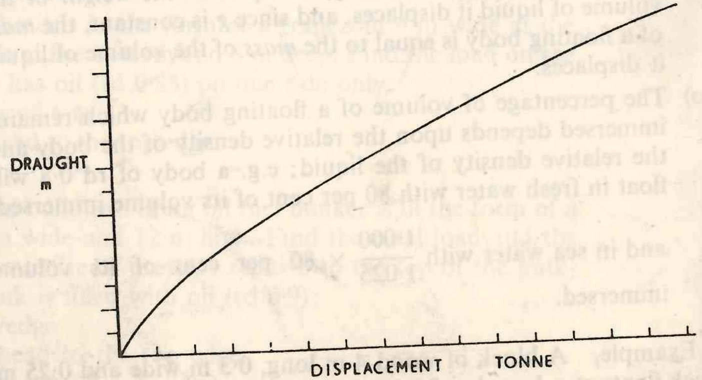


Fig. 9

vessel floats in water of any other density. Since the volume of water displaced depends upon the draught, it is useful to calculate values of displacement for a range of draughts. These values may then be plotted to form a *displacement curve*, from which the displacement may be obtained at any intermediate draught.

The following symbols will be used throughout the text:

$\triangle$ = displacement in tonne

∇ = volume of displacement in m³

Thus for sea water $\triangle = \nabla \times 1{\cdot}025$

Some confusion exists between the *mass* of the ship and the *weight* of the ship. This confusion may be reduced if the displacement is always regarded as a mass. The gravitational force acting on this mass—the weight of the ship—will then be the product of the displacement $\triangle$ and the acceleration due to gravity g.

Hence mass of ship (displacement) = $\triangle$ tonne

weight of ship = $\triangle$ gkN

Example. A ship displaces 12 240 m³ of sea water at a particular draught.

(a) Calculate the displacement of the ship.

(b) How many tonnes of cargo would have to be discharged for the vessel to float at the same draught in fresh water?

(a) Displacement in sea water = 12 240 × 1·025
= 12 546 tonne

(b) Displacement at same draught in fresh water
= 12 240 × 1·000
= 12 240 tonne

∴ Cargo to be discharged = 12 546 — 12 240
= 306 tonne

BUOYANCY

Buoyancy is the term given to the upthrust exerted by the water on the ship. If a ship floats freely, the buoyancy is equal to the weight of the ship.

The force of buoyancy acts at the *centre of buoyancy*, which is the centre of gravity of the underwater volume of the ship.

The *longitudinal* position of the centre of buoyancy (LCB) is usually given as a distance forward or aft of midships and is represented by the longitudinal centroid of the curve of immersed cross-sectional areas (see Chapter 3).

The *vertical* position of the centre of buoyancy (VCB) is usually given as a distance above the keel. This distance is denoted by *KB* and is represented by the vertical centroid of the waterplane area curve (see Chapter 3). The distance from the waterline to the VCB may be found by two other methods:

(a) from the displacement curve (Fig. 10)

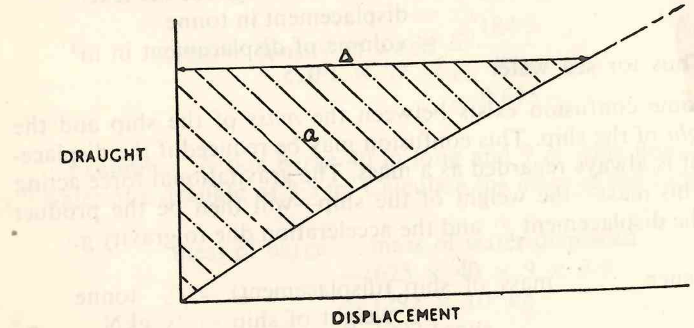


Fig. 10

VCB below waterline

$$= \frac{\text{area between displacement curve and draught axis}}{\text{displacement}}$$

$$= \frac{a}{\Delta}$$

(b) by Morrishes approximate formula

$$\text{VCB below the waterline} = \frac{1}{3}\left(\frac{d}{2} + \frac{\nabla}{A_w}\right)$$

where d = draught in m
∇ = volume of displacement in m^3
A_w = waterplane area in m^2

TONNE PER CENTIMETRE IMMERSION

The tonne per centimetre immersion (TPC) of a ship at any given draught is the mass required to increase the mean draught by 1 cm.

Consider a ship floating in water of density ρ t/m³.
If the mean draught is increased by 1 cm, then:

$$\text{Increase in volume of displacement} = \frac{1}{100} \times \text{waterplane area}$$

$$= \frac{A_w}{100} \text{ m}^3$$

$$\text{Increase in displacement} = \frac{A_w}{100} \times \rho \text{ t}$$

$$\text{Thus} \quad \text{TPC} = \frac{A_w \times \rho}{100}$$

$$\text{For sea water } \rho = 1{\cdot}025 \text{ t/m}^3$$

$$\therefore \text{TPC sw} = 0{\cdot}01025\, Aw$$

At different draughts, variations in waterplane area cause variations in TPC. Values of TPC may be calculated for a range of draughts and plotted to form a TPC curve, from which values of TPC may be obtained at intermediate draughts.

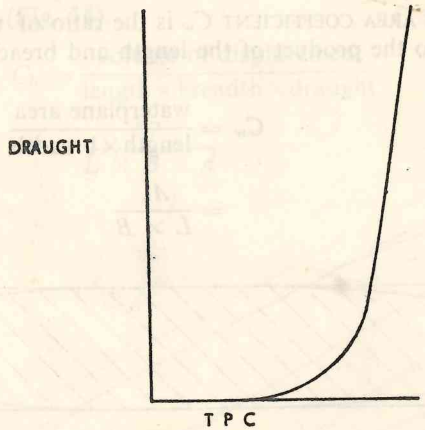


Fig. 11

The area between the TPC curve and the draught axis to any given draught represents the displacement of the ship at that draught, while its centroid represents the vertical position of the centre of buoyancy.

It may be assumed for small alterations in draught, that the ship is wall-sided and therefore TPC remains constant. If the change in draught exceeds about 0·5 m, then a mean TPC value should be used. If the change in draught is excessive, however, it is more accurate to use the area of the relevant part of the TPC curve.

Example. The waterplane area of a ship is 1730 m². Calculate the TPC and the increase in draught if a mass of 270 tonne is added to the ship.

$$\text{TPC} = 0{\cdot}01025 \times 1730$$
$$= 17{\cdot}73$$

$$\text{Increase in draught} = \frac{\text{mass added}}{\text{TPC}}$$
$$= \frac{270}{17{\cdot}73}$$
$$= 15{\cdot}23 \text{ cm}$$

COEFFICIENTS OF FORM

Coefficients of form have been devised to show the relation between the form of the ship and the dimensions of the ship.

WATERPLANE AREA COEFFICIENT C_w is the ratio of the area of the waterplane to the product of the length and breadth of the ship. (Fig. 12).

$$C_w = \frac{\text{waterplane area}}{\text{length} \times \text{breadth}}$$
$$= \frac{A_w}{L \times B}$$

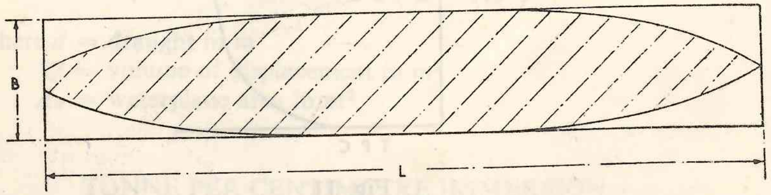


Fig. 12

MIDSHIP SECTION AREA COEFFICIENT C_m is the ratio of the area of the immersed portion of the midship section to the product of the breadth and the draught (Fig. 13).

$$C_m = \frac{\text{area of immersed midship section}}{\text{breadth} \times \text{draught}}$$
$$= \frac{A_m}{B \times d}$$

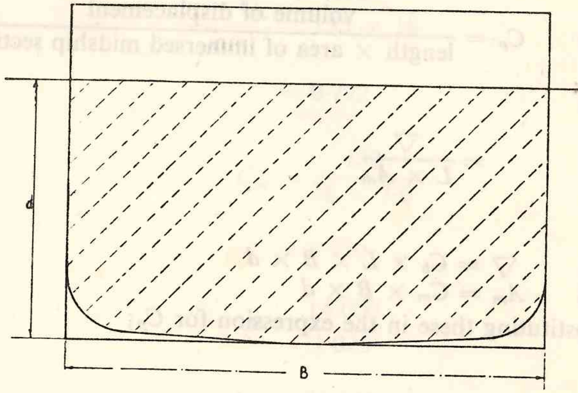


Fig. 13

BLOCK COEFFICIENT OR COEFFICIENT OF FINENESS C_b is the ratio of the volume of displacement to the product of the length, breadth and draught (Fig. 14).

$$C_b = \frac{\text{volume of displacement}}{\text{length} \times \text{breadth} \times \text{draught}}$$
$$= \frac{\nabla}{L \times B \times d}$$

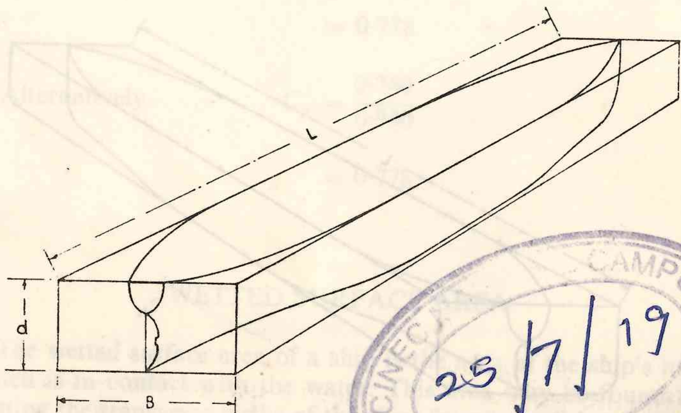


Fig. 14

PRISMATIC COEFFICIENT C_p is the ratio of the volume of displacement to the product of the length and the area of the immersed portion of the midship section (Fig. 15).

$$C_p = \frac{\text{volume of displacement}}{\text{length} \times \text{area of immersed midship section}}$$

$$= \frac{\nabla}{L \times A_m}$$

But $\nabla = C_b \times L \times B \times d$

and $A_m = C_m \times B \times d$

Substituting these in the expression for C_p;

$$C_p = \frac{C_b \times L \times B \times d}{L \times C_m \times B \times d}$$

$$C_p = \frac{C_b}{C_m}$$

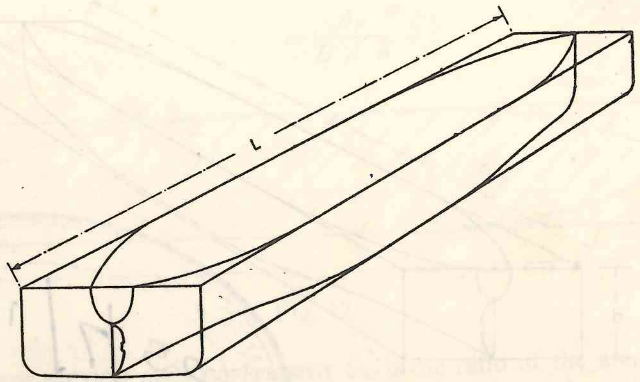


Fig. 15

Example. A ship 135 m long, 18 m beam and 7·6 m draught has a displacement of 14 000 tonne. The area of the load waterplane is 1925 m² and the area of the immersed midship section 130 m². Calculate (a) C_w; (b) C_m; (c) C_b; (d) C_p.

(a)
$$C_w = \frac{1925}{135 \times 18}$$
$$= 0{\cdot}792$$

(b)
$$C_m = \frac{130}{18 \times 7{\cdot}6}$$
$$= 0{\cdot}950$$

(c)
$$\nabla = \frac{14\,000}{1{\cdot}025}$$
$$= 13\,658 \text{ m}^3$$
$$C_b = \frac{13\,658}{135 \times 18 \times 7{\cdot}6}$$
$$= 0{\cdot}740$$

(d)
$$C_p = \frac{13\,658}{135 \times 130}$$
$$= 0{\cdot}778$$

Alternatively
$$C_p = \frac{0{\cdot}740}{0{\cdot}950}$$
$$= 0{\cdot}778$$

WETTED SURFACE AREA

The wetted surface area of a ship is the area of the ship's hull which is in contact with the water. This area may be found by putting the transverse girths of the ship, from waterline to waterline, through Simpson's Rule and adding about ½ per cent to allow for the longitudinal curvature of the shell. To this area should be added the wetted surface area of appendages such as cruiser stern, rudder and bilge keels.

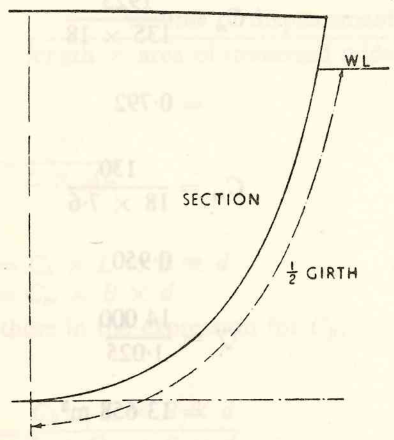


Fig. 16

Several approximate formulae for wetted surface area are available, two of which are:

Denny $$S = 1{\cdot}7Ld + \frac{\nabla}{d}$$

Taylor: $$S = c\sqrt{\triangle L}$$

where S = wetted surface area in m^2
L = length of ship in m
d = draught in m
∇ = volume of displacement in m^3
$\triangle$ = displacement in tonne
c = a coefficient of about 2·6 which depends upon the shape of the ship.

Example. A ship of 5000 tonne displacement, 95 m long, floats at a draught of 5·5 m. Calculate the wetted surface area of the ship:

(a) Using Denny's formula

(b) Using Taylor's formula with c = 2·6

(a) $$S = 1{\cdot}7Ld + \frac{\nabla}{d}$$
$$= 1{\cdot}7 \times 95 \times 5{\cdot}5 + \frac{5000}{1{\cdot}025 \times 5{\cdot}5}$$
$$= 888{\cdot}2 + 886{\cdot}9$$
$$= 1775{\cdot}1 \text{ m}^2$$

(b) $$S = c\sqrt{\triangle L}$$
$$= 2{\cdot}6\sqrt{5000 \times 95}$$
$$= 1793 \text{ m}^2$$

SIMILAR FIGURES

Two planes or bodies are said to be similar when their linear dimensions are in the same ratio. This principle may be seen in a projector where a small image is projected from a slide onto a screen. The height of the image depends upon the distance of the screen from the light source, but the *proportions and shape* of the image remain the same as the image on the slide. Thus the image on the screen is a scaled-up version of the image on the slide.

The *areas* of similar figures vary as the *square* of their corresponding dimensions. This may be shown by comparing two circles having diameters D and d respectively.

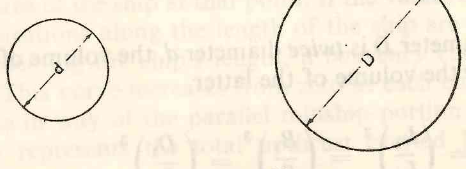


Fig. 17

$$\text{Area of large circle} = \frac{\pi}{4}D^2$$

$$\text{Area of small circle} = \frac{\pi}{4}d^2$$

Since $\frac{\pi}{4}$ is constant:

$$\text{ratio of areas} = \frac{D^2}{d^2}$$

Thus if the diameter D is *twice* diameter d, the area of the former is *four times* the area of the latter.

$$\frac{A_1}{A_2} = \left(\frac{L_1}{L_2}\right)^2 = \left(\frac{B_1}{B_2}\right)^2$$

The *volumes* of similar bodies vary as the *cube* of their corresponding dimensions. This may be shown by comparing two spheres of diameters D and d respectively.

$$\text{Volume of large sphere} = \frac{\pi}{6}D^3$$

$$\text{Volume of small sphere} = \frac{\pi}{6}d^3$$

Since $\frac{\pi}{6}$ is constant:

$$\text{ratio of volumes} = \frac{D^3}{d^3}$$

Thus if diameter D is *twice* diameter d, the volume of the former is *eight times* the volume of the latter.

$$\frac{V_1}{V_2} = \left(\frac{L_1}{L_2}\right)^3 = \left(\frac{B_1}{B_2}\right)^3 = \left(\frac{D_1}{D_2}\right)^3$$

These rules may be applied to any similar bodies no matter what their shape, and in practice are applied to ships.

Thus if L = length of ship
S = wetted surface area
$\triangle$ = displacement,

then $S \propto L^2$
or $S^{\frac{1}{2}} \propto L$
and $\triangle \propto L^3$
or $\triangle^{\frac{1}{3}} \propto L$
$\therefore$ $S^{\frac{1}{2}} \propto \triangle^{\frac{1}{3}}$
and $S \propto \triangle^{\frac{2}{3}}$
or $\triangle \propto S^{\frac{3}{2}}$

Example. A ship 110 m long displaces 9000 tonne and has a wetted surface area of 2205 m². Calculate the displacement and wetted surface area of a 6 m model of the ship.

$$\frac{\triangle_1}{\triangle_2} = \left(\frac{L_1}{L_2}\right)^3$$

$$\triangle_2 = 9000\left(\frac{6}{110}\right)^3$$

Displacement of model = 1·46 tonne

$$\frac{S_1}{S_2} = \left(\frac{L_1}{L_2}\right)^2$$

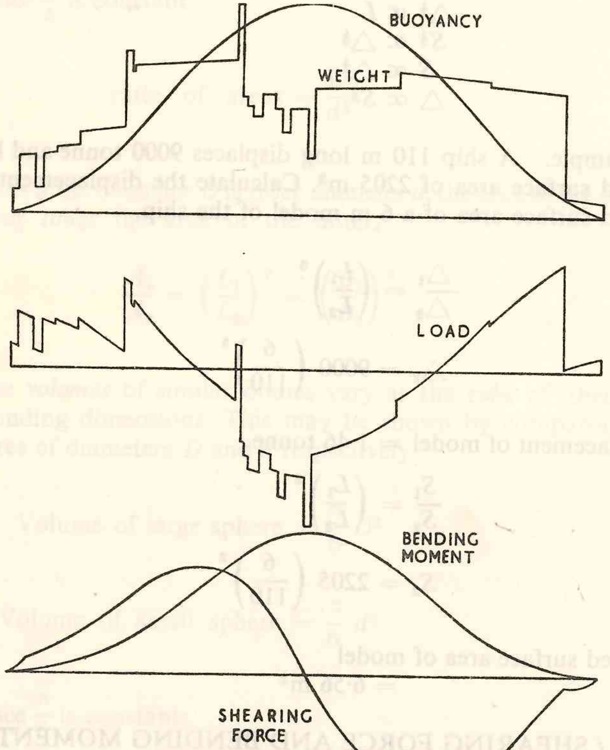
$$S_2 = 2205\left(\frac{6}{110}\right)^2$$

Wetted surface area of model
= 6·56 m²

f SHEARING FORCE AND BENDING MOMENT

Consider a loaded ship lying in still water. The upthrust over any unit length of the ship depends upon the immersed cross-sectional area of the ship at that point. If the values of upthrust at different positions along the length of the ship are plotted on a base representing the ship's length, a *bouyancy curve* is formed (Fig. 18). This curve increases from zero at each end to a maximum value in way of the parallel midship portion. The area of this curve represents the total upthrust exerted by the water on the ship.

The total weight of a ship consists of a number of independent weights concentrated over short lengths of the ship, such as cargo, machinery, accommodation, cargo handling gear, poop and fore-castle, and a number of items which form continuous material over the length of the ship, such as decks, shell and tank top. A *curve of weights* is shown in the diagram.



LOAD DISTRIBUTION

Fig. 18

The difference between the weight and buoyancy at any point is the *load* at that point. In some cases the load is an excess of weight over buoyancy and in other cases an excess of buoyancy

over weight. A load diagram is formed by plotting these differences. Because of this unequal loading, however, shearing forces and bending moments are set up in the ship.

The *shearing force* at any point is represented by the *area* of the load diagram on one side of that point. A *shearing force diagram* may be formed by plotting these areas on a base of the ship's length.

The *bending moment* at any point is represented by the *area* of the shearing force diagram on one side of that point. A *bending moment diagram* may be formed by plotting such areas on a base of the ship's length.

The maximum bending moment occurs where the shearing force is zero and this is usually near amidships.

Example. A box barge 200 m long is divided into five equal compartments. The weight is uniformly distributed along the vessel's length.

500 tonne of cargo are added to each of the end compartments. Sketch the shearing force and bending moment diagrams and state their maximum values.

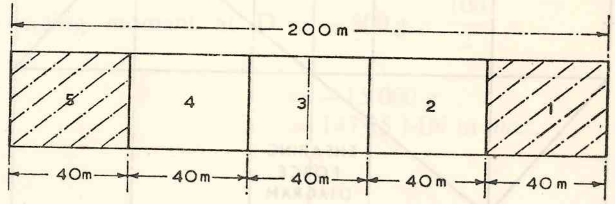


Fig. 19

Before adding the cargo, the buoyancy and weight were equally distributed and produced no shearing force or bending moment. It is therefore only necessary to consider the added cargo and the additional buoyancy required.

$$\text{Additional buoyancy/m} = \frac{1000\,g}{200}$$

$$= 5\,g \text{ kN}$$

Compartments 1 and 5

$$\text{Additional weight/m} = \frac{500g}{40}$$

$$= 12{\cdot}5\,g \text{ kN}$$

$$\text{Load/m} = 12{\cdot}5\,g - 5\,g$$

$$= 7{\cdot}5\,g \text{ kN excess weight}$$

Compartments 2, 3 and 4

$$\text{Load/m} = 5\,g \text{ kN excess buoyancy}$$

These values may be plotted to form a load diagram.

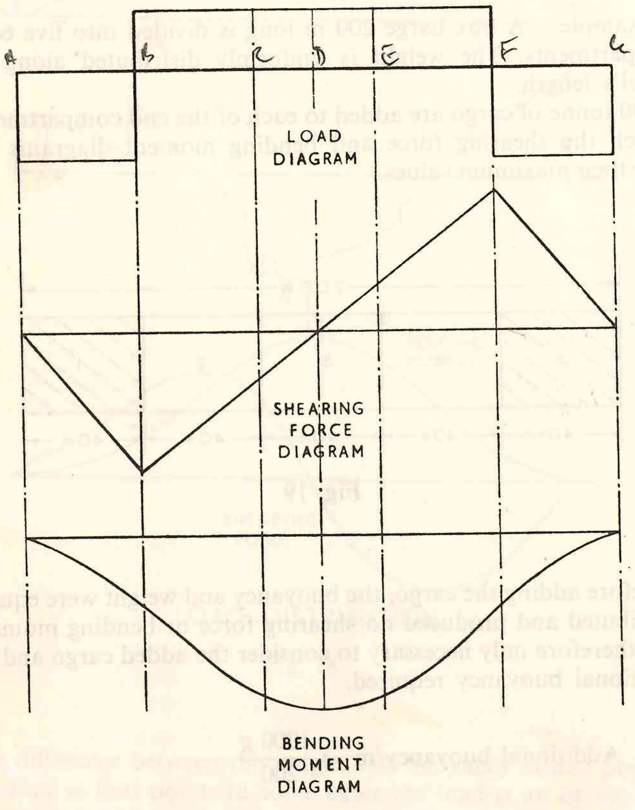


Fig. 20

$$\text{Shearing force at A} = 0$$

$$\text{Shearing force at B} = -7{\cdot}5\,g \times 40$$

$$= -300\,g = 2943 \text{ kN } max.$$

$$\text{Shearing force at C} = -300\,g + 5\,g \times 40$$

$$= -100\,g = 981 \text{ kN}$$

$$\text{Shearing force at D} = -300\,g + 5\,g \times 60$$

$$= 0$$

$$\text{Shearing force at E} = +100\,g$$

$$\text{Shearing force at F} = +300\,g$$

$$\text{Shearing force at G} = 0$$

$$\text{Bending moment at A and G} = 0$$

$$\text{Bending moment at B and F} = -300\,g \times \frac{40}{2}$$

$$= -6000\,g = 58{\cdot}86 \text{ MN m}$$

$$\text{Bending moment at C and E} = -15\,000\,g + 100\,g \times \frac{20}{2}$$

$$= -14\,000\,g = 137{\cdot}34 \text{ MN m}$$

$$\text{Bending moment at D} = -300\,g \times \frac{100}{2}$$

$$= -15\,000\,g$$

$$= 147{\cdot}15 \text{ MN m } max.$$

TEST EXAMPLES 2

1. A piece of metal 250 cm^3 in volume is attached to the bottom of a block of wood 3·5 dm^3 in volume and having a relative density of 0·6. The system floats in fresh water with 100 cm^3 projecting above the water. Calculate the relative density of the metal.

2. A raft 3 m long and 2 m wide is constructed of timber 0·25 m thick having a relative density of 0·7. It floats in water of density 1018 kg/m^3. Calculate the minimum mass which must be placed on top of the raft to sink it.

3. A box barge 65 m long and 12 m wide floats at a draught of 5·5 m in sea water. Calculate:
   (a) the displacement of the barge,
   (b) its draught in fresh water.

4. A ship has a constant cross-section in the form of a triangle which floats apex down in sea water. The ship is 85 m long, 12 m wide at the deck and has a depth from keel to deck of 9 m. Draw the displacement curve using 1·25 m intervals of draught from the keel to the 7·5 m waterline. From this curve obtain the displacement in fresh water at a draught of 6·50 m.

5. A cylinder 15 m long and 4 m outside diameter floats in sea water with its axis in the waterline. Calculate the mass of the cylinder.

6. Bilge keels of mass 36 tonne and having a volume of 22 m^3 are added to a ship. If the TPC is 20, find the change in mean draught.

7. A vessel 40 m long has a constant cross-section in the form of a trapezoid 10 m wide at the top, 6 m wide at the bottom and 5 m deep. It floats in sea water at a draught of 4 m. Calculate its displacement.

8. The waterplane areas of a ship at 1·25 m intervals of draught, commencing at the 7·5 m waterline, are 1845, 1690, 1535, 1355 and 1120 m^2. Draw the curve of tonne per cm immersion and determine the mass which must be added to increase the mean draught from 6·10 m to 6·30 m.

9. A ship 150 m long and 20·5 m beam floats at a draught of 8 m and displaces 19 500 tonne. The TPC is 26·5 and midship section area coefficient 0·94. Calculate the block, prismatic and waterplane area coefficients.

10. A ship displaces 9450 tonne and has a block coefficient of 0·7. The area of immersed midship section is 106 m^2.
    If a beam $= 0{\cdot}13 \times$ length $= 2{\cdot}1 \times$ draught, calculate the length of the ship and the prismatic coefficient.

11. The length of a ship is 18 times the draught, while the breadth is 2·1 times the draught. At the load waterplane, the waterplane area coefficient is 0·83 and the difference between the TPC in sea water and the TPC in fresh water is 0·7. Determine the length of the ship and the TPC in fresh water.

12. The $\frac{1}{2}$ girths of a ship 90 m long are as follows: 2·1, 6·6, 9·3, 10·5, 11·0, 11·0, 11·0, 9·9, 7·5, 3·9 and 0 m respectively. The wetted surface area of the appendages is 30 m^2 and $\frac{1}{2}$% is to be added for longitudinal curvature. Calculate the wetted surface area of the ship.

13. A ship of 14 000 tonne displacement, 130 m long, floats at a draught of 8 m. Calculate the wetted surface area of the ship using:
    (a) Denny's formula
    (b) Taylor's formula with $c = 2{\cdot}58$

14. A box barge is 75 m long, 9 m beam and 6 m deep. A similar barge having a volume of 3200 m^3 is to be constructed. Calculate the length, breadth and depth of the new barge.

15. The wetted surface area of a ship is twice that of a similar ship. The displacement of the latter is 2000 tonne less than the former. Determine the displacement of the latter.

16. A ship 120 m long displaces 11 000 tonne and has a wetted surface area of 2500 m^2. Calculate the displacement and wetted surface area of a 6 m model of the ship.

CHAPTER 3

CALCULATION OF AREA, VOLUME, FIRST AND SECOND MOMENTS

SIMPSON'S FIRST RULE

Simpson's First Rule is based on the assumption that the curved portion of a figure forms part of a parabola ($y = ax^2 + bx + c$), and gives the area contained between *three* consecutive, equally-spaced ordinates.

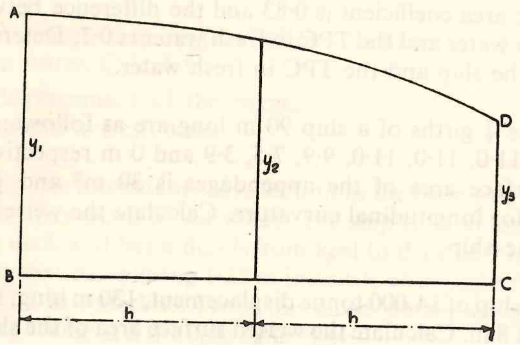


Fig 21.

$$\text{Area ABCD} = \frac{h}{3}(1y_1 + 4y_2 + 1y_3)$$

This rule may be applied repeatedly to determine the area of a larger plane such as EFGH (Fig. 22).

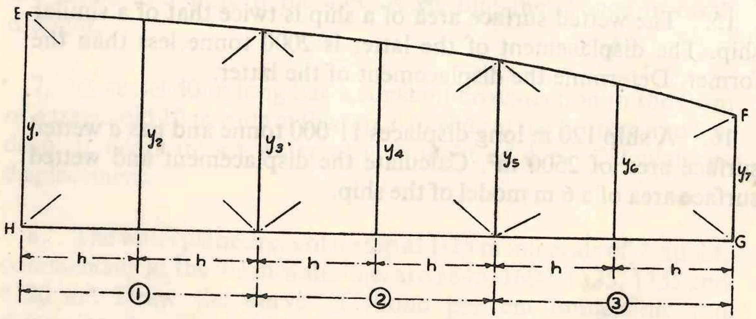


Fig. 22

$$\text{Area 1} = \frac{h}{3}(1y_1 + 4y_2 + 1y_3)$$

$$\text{Area 2} = \frac{h}{3}(1y_3 + 4y_4 + 1y_5)$$

$$\text{Area 3} = \frac{h}{3}(1y_5 + 4y_6 + 1y_7)$$

Area EFGH
= Area 1 + Area 2 + Area 3

$$= \frac{h}{3}[(1y_1+4y_2+1y_3)+(1y_3+4y_4+1y_5)+(1y_5+4y_6+1y_7)]$$

$$= \frac{h}{3}[1y_1+4y_2+2y_3+4y_4+2y_5+4y_6+1y_7]$$

It should be noted at this stage that it is necessary to apply the whole rule and thus an *odd* number of equally-spaced ordinates is necessary. Greater speed and accuracy is obtained if this rule is applied in the form of a table. The distance h is termed the common interval and the numbers 1, 4, 2, 4, etc. are termed Simpson's multipliers.

When calculating the area of a waterplane it is usual to divide the length of the ship into about 10 equal parts, giving 11 sections. These sections are numbered from 0 at the after end to 10 at the fore end. Thus amidships will be section number 5. It is convenient to measure distances from the centreline to the ship side, giving half ordinates. These half ordinates are used in conjunction with Simpson's Rule and the answer multiplied by 2.

Example. The equally-spaced half ordinates of a watertight flat 27 m long are 1·1, 2·7, 4·0, 5·1, 6·1, 6·9 and 7·7 m respectively. Calculate the area of the flat.

½ Ordinate	Simpson's Multipliers	Product for Area
1·1	1	1·1
2·7	4	10·8
4·0	2	8·0
5·1	4	20·4
6·1	2	12·2
6·9	4	27·6
7·7	1	7·7
		$87{\cdot}8 = \Sigma_A$

Since there are 7 ordinates there will be 6 spaces

$$\therefore \text{Common interval} = \frac{27}{6} = 4{\cdot}5 \text{ m}$$

$$\text{Area} = \frac{h}{3} \Sigma_A \times 2 = \frac{4{\cdot}5}{3} \times 87{\cdot}8 \times 2$$

$$= 263{\cdot}4 \text{ m}^2$$

APPLICATION TO VOLUMES

Simpson's Rule is a mathematical rule which will give the area under any continuous curve, no matter what the ordinates represent. If the immersed cross-sectional areas of a ship at a number of positions along the length of the ship are plotted on a base representing the ship's length (Fig. 23), the area under the resulting curve will represent the volume of water displaced by the ship and may be found by putting the cross-sectional areas through Simpson's Rule. Hence the displacement of the ship at any given draught may be calculated. The longitudinal centroid of this figure represents the longitudinal centre of buoyancy of the ship.

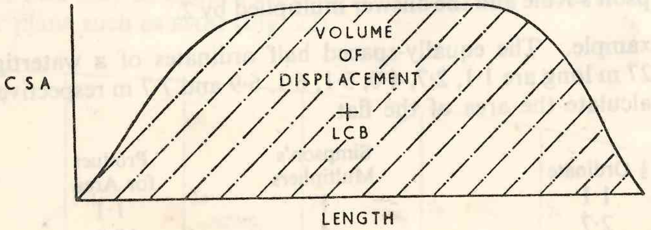


Fig. 23

It is also possible to calculate the displacement by using ordinates of waterplane area or tonne per cm immersion, with a

common interval of draught (Fig. 24). The vertical centroids of these two curves represent the vertical centre of buoyancy of the ship.

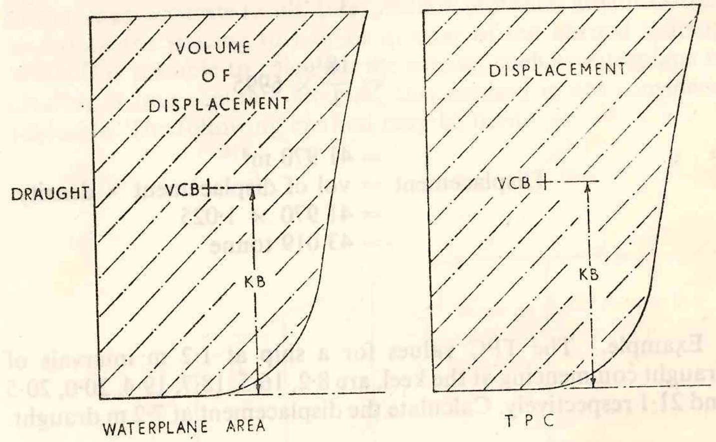


Fig. 24

Similar methods are used to determine hold and tank capacities.

Example. The immersed cross-sectional areas through a ship 180 m long, at equal intervals, are 5, 118, 233, 291, 303, 304, 304, 302, 283, 171, and 0 m² respectively. Calculate the displacement of the ship in sea water of 1·025 tonne/m³.

CSA	SM	Product for Volume
5	1	5
118	4	472
233	2	466
291	4	1164
303	2	606
304	4	1216
304	2	608
302	4	1208
283	2	566
171	4	684
0	1	0
		$6995 = \Sigma_\nabla$

$$\text{Common interval} = \frac{180}{10} = 18 \text{ m}$$

$$\text{Volume of displacement} = \frac{h}{3}\Sigma_{\nabla}$$

$$= \frac{18}{3} \times 6995$$

$$= 41\ 970 \text{ m}^3$$

$$\text{Displacement} = \text{vol of displacement} \times \text{density}$$
$$= 41\ 970 \times 1{\cdot}025$$
$$= 43\ 019 \text{ tonne}$$

Example. The TPC values for a ship at 1·2 m intervals of draught commencing at the keel, are 8·2, 16·5, 18·7, 19·4, 20·0, 20·5 and 21·1 respectively. Calculate the displacement at 7·2 m draught.

Waterplane	TPC	SM	Product for Displacement
0	8·2	1	8·2
1·2	16·5	4	66·0
2·4	18·7	2	37·4
3·6	19·4	4	77·6
4·8	20·0	2	40·0
6·0	20·5	4	82·0
7·2	21·1	1	21·1
			$332{\cdot}3 = \Sigma_{\Delta}$

$$\text{Common interval} = 1{\cdot}2 \text{ m or } 120 \text{ cm}$$

$$\text{Displacement} = \frac{h}{3}\Sigma_{\Delta}$$

$$= \frac{120}{3} \times 332{\cdot}3$$

$$= 13\ 292 \text{ tonne}$$

Note: The common interval must be expressed in *centimetres* since the ordinates are tonne per *centimetre* immersion.

USE OF INTERMEDIATE ORDINATES

At the ends of the ship, where the curvature of a waterplane is considerable, it is necessary to reduce the spacing of the ordinates to ensure an accurate result. Intermediate ordinates are introduced to reduce the spacing to half or quarter of the normal spacing. While it is possible to calculate the area of such a waterplane by dividing it into separate sections, this method is not considered advisable. The following method may be used.

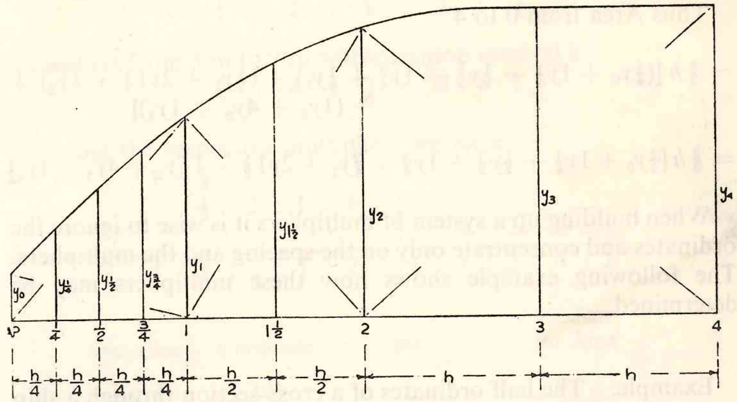


Fig. 25

If the length of the ship is divided initially into 10 equal parts, then:

$$\text{Common interval} = h = \frac{L}{10}$$

It is proposed to introduce intermediate ordinates at a spacing of $\frac{h}{4}$ from section 0 to section 1 and at a spacing of $\frac{h}{2}$ from section 1 to section 2. The ½ ordinates at sections AP, ¼, ½, ¾, etc. will be denoted by yo, $y\frac{1}{4}$, $y\frac{1}{2}$, $y\frac{3}{4}$, etc. respectively.

$$\text{Area from 0 to 1} = \frac{2}{3}\frac{h}{4}\left(1y_0 + 4y_{\frac{1}{4}} + 2y_{\frac{1}{2}} + 4y_{\frac{3}{4}} + 1y_1\right)$$

$$= \tfrac{2}{3}h\left(\tfrac{1}{4}y_0 + 1y_{\frac{1}{4}} + \tfrac{1}{2}y_{\frac{1}{2}} + 1y_{\frac{3}{4}} + \tfrac{1}{4}y_1\right)$$

$$\text{Area from 1 to 2} = \frac{2}{3}\frac{h}{2}\left(1y_1 + 4y_{1\frac{1}{2}} + 1y_2\right)$$

$$= \tfrac{2}{3}h\left(\tfrac{1}{2}y_1 + 2y_{1\frac{1}{2}} + \tfrac{1}{2}y_2\right)$$

$$\text{Area from 2 to 4} = \tfrac{2}{3}h\left(1y_2 + 4y_3 + 1y_4\right)$$

Thus Area from 0 to 4

$$= \tfrac{2}{3}h\left[\left(\tfrac{1}{4}y_0 + 1y_{\frac{1}{4}} + \tfrac{1}{2}y_{\frac{1}{2}} + 1y_{\frac{3}{4}} + \tfrac{1}{4}y_1\right) + \left(\tfrac{1}{2}y_1 + 2y_{1\frac{1}{2}} + \tfrac{1}{2}y_2\right) + \left(1y_2 + 4y_3 + 1y_4\right)\right]$$

$$= \tfrac{2}{3}h\left[\tfrac{1}{4}y_0 + 1y_{\frac{1}{4}} + \tfrac{1}{2}y_{\frac{1}{2}} + 1y_{\frac{3}{4}} + \tfrac{3}{4}y_1 + 2y_{1\frac{1}{2}} + 1\tfrac{1}{2}y_2 + 4y_3 + 1y_4\right]$$

When building up a system of multipliers it is wise to ignore the ordinates and concentrate only on the spacing and the multipliers. The following example shows how these multipliers may be determined

Example. The half ordinates of a cross-section through a ship are as follows:

WL	keel	0·25	0·50	0·75	1·0	1·5	2·0	2·5	3·0	4·0	5·0	6·0	7·0 m
½ ord	2·9	5·0	5·7	6·2	6·6	6·9	7·2	7·4	7·6	7·8	8·1	8·4	8·7 m

Calculate the area of the cross-section to the 7 m waterline.

Let the common interval $= h = 1$ m

Then interval from keel to 1m waterline $= \dfrac{h}{4}$

interval from 1 m to 3 m waterline $= \dfrac{h}{2}$

interval from 3 m to 7 m waterline $= h$

Multipliers from keel to 1 m with common interval $\dfrac{h}{4}$

$= 1 : 4 : 2 : 4 : 1$

or with common interval of h

$= \tfrac{1}{4} : 1 : \tfrac{1}{2} : 1 : \tfrac{1}{4}$

Multipliers from 1 m to 3 m with common interval $\dfrac{h}{2}$

$= 1 : 4 : 2 : 4 : 1$

or with common interval of h

$= \tfrac{1}{2} : 2 : 1 : 2 : \tfrac{1}{2}$

Multipliers from 3 m to 7 m with common interval h

$= 1 : 4 : 2 : 4 : 1$

Adding the respective multipliers we have:

$\tfrac{1}{4} : 1 : \tfrac{1}{2} : 1 : \tfrac{1}{4}$

$\qquad\qquad \tfrac{1}{2} : 2 : 1 : 2 : \tfrac{1}{2}$

$\qquad\qquad\qquad\qquad 1 : 4 : 2 : 4 : 1$

$\tfrac{1}{4} : 1 : \tfrac{1}{2} : 1 : \tfrac{3}{4} : 2 : 1 : 2 : 1\tfrac{1}{2} : 4 : 2 : 4 : 1$

Waterline	½ ordinate	SM	Product for Area
Keel	2·9	¼	0·73
0·25	5·0	1	5·00
0·50	5·7	½	2·85
0·75	6·2	1	6·20
1·0	6·6	¾	4·95
1·5	6·9	2	13·80
2·0	7·2	1	7·20
2·5	7·4	2	14·80
3·0	7·6	1½	11·40
4·0	7·8	4	31·20
5·0	8·1	2	16·20
6·0	8·4	4	33·60
7·0	8·7	1	8·70
			156·63

Area of cross-section $= \tfrac{2}{3} \times 1{\cdot}0 \times 156{\cdot}63$

$= 104{\cdot}42\ \text{m}^2$

f APPLICATION OF SIMPSON'S RULE TO FIRST AND SECOND MOMENTS OF AREA

It is often found necessary to determine the centroid of a curved plane such as a waterplane and the second moment of area of a waterplane.

Consider a plane ABCD (Fig. 26).

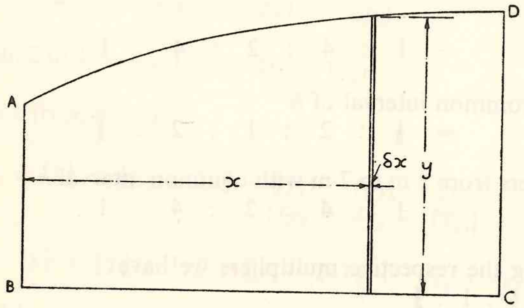


Fig. 26

Divide the plane into thin strips of length δx

Let one such strip, distance x from AB, have an ordinate y.

$$\text{Area of strip} = y \times \delta x$$

$$\therefore \text{Total area of plane} = (y_1 + y_2 + y_3 + \ldots)\delta x = \Sigma\, y\, \delta x$$

But the area of the plane may be found by putting the ordinates y through Simpson's Rule.

First moment of area of strip about AB $= x \times y\,\delta x = x\,y\,\delta x$

$\therefore$ First moment of area of plane about AB $=$

$$(x_1y_1 + x_2y_2 + x_3y_3 + \ldots)\delta x = \Sigma\, xy\, \delta x$$

Now it was mentioned earlier that Simpson's Rule may be used to find the area under any continuous curve, no matter what the ordinates represent. Such a curve may be drawn on a base equal to BC, with ordinates of xy, and the area under this curve may be found by putting the values of xy through Simpson's Rule

Second moment of area of strip about AB

$$= I_{NA} + A\bar{x}^2 = \tfrac{1}{12}y\,(\delta x)^3 + x^2y\,\delta x$$

This may be reduced to $(x^2y\,\delta x)$ since δx is very small

$\therefore$ Second moment of area of plane about AB

$$= (x_1^2y_1 + x_2^2y_2 + x_3^2y_3 + \ldots)\delta x = \Sigma\, x^2y\,\delta x$$

This may be found by putting the values of x^2y through Simpson's Rule.

First moment of area of strip about BC

$$= \tfrac{1}{2}y \times y\,\delta x = \tfrac{1}{2}y^2\,\delta x$$

First moment of area of plane about BC

$$= (y_1^2 + y_2^2 + y_3^2 + \ldots)\tfrac{1}{2}\,\delta x = \Sigma\,\tfrac{1}{2}\,y^2\,\delta x$$

This may be found by putting $\frac{1}{2}\,y^2$ through Simpson's Rule.

Second moment of area of strip about BC

$$= \tfrac{1}{12}y^3\,\delta x + (\tfrac{1}{2}y^2)y\,\delta x = \tfrac{1}{3}\,y^3\,\delta x$$

Second moment of area of plane about BC

$$= (y_1^3 + y_2^3 + y_3^3 + \ldots)\tfrac{1}{3}\,\delta x = \Sigma\,\tfrac{1}{3}\,y^3\,\delta x$$

This may be found by putting $\frac{1}{3}\,y^3$ through Simpson's Rule.

It is usually necessary to calculate area and centroid when determining the second moment of area of a waterplane about a transverse axis. Since the centroid is near amidships it is preferable to take moments about amidships. The following calculation shows the method used to determine area, centroid and second moment of area about the centroid for a waterplane having half ordinates of $y_0, y_1, y_2, \ldots y_{10}$ spaced h m apart commencing from aft.

The positive sign indicates an ordinate aft of midships.

The negative sign indicates an ordinate forward of midships.

Section	½ ord	SM	Product for area	Lever	Product for 1st moment	Lever	Product for 2nd moment
AP	y_0	1	$1y_0$	$+5h$	$+5y_0h$	$+5h$	$+25y_0h^2$
1	y_1	4	$4y_1$	$+4h$	$+16y_1h$	$+4h$	$+64y_1h^2$
2	y_2	2	$2y_2$	$+3h$	$+6y_2h$	$+3h$	$+18y_2h^2$
3	y_3	4	$4y_3$	$+2h$	$+8y_3h$	$+2h$	$+16y_3h^2$
4	y_4	2	$2y_4$	$+1h$	$+2y_4h$	$+1h$	$+2y_4h^2$
5	y_5	4	$4y_5$	$0h$	$\Sigma_{MA} \times h$	$0h$	—
6	y_6	2	$2y_6$	$-1h$	$-2y_6h$	$-1h$	$+2y_6h^2$
7	y_7	4	$4y_7$	$-2h$	$-8y_7h$	$-2h$	$+16y_7h^2$
8	y_8	2	$2y_8$	$-3h$	$-6y_8h$	$-3h$	$+18y_8h^2$
9	y_9	4	$4y_9$	$-4h$	$-16y_9h$	$-4h$	$+64y_9h^2$
FP	y_{10}	1	$1y_{10}$	$-5h$	$-5y_{10}h$	$-5h$	$+25y_{10}h^2$
			Σ_A		$\Sigma_{MF} \times h$		$\Sigma_I \times h^2$

Area of waterplane $A = \frac{2}{3} h \Sigma_A$

First moment of area of waterplane about amidships

$$= \tfrac{2}{3} h (\Sigma_{MA} + \Sigma_{MF}) h$$

(added algebraically)

Centroid from midships $x = \dfrac{\frac{2}{3} h^2 (\Sigma_{MA} + \Sigma_{MF})}{\frac{2}{3} h \Sigma_A}$

$$= h \frac{(\Sigma_{MA} + \Sigma_{MF})}{\Sigma_A} \text{ aft of midships}$$

(NOTE: If Σ_{MF} is greater than Σ_{MA}, the centroid will be forward of midships).

Second moment of area of waterplane about amidships

$$I_m = 2 h \Sigma_I \times h^2$$
$$= \tfrac{2}{3} h^3 \Sigma_I$$

Second moment of area of waterplane about the centroid

$$I_F = I_m - Ax^2$$

It should be noted that h remains in the table as a constant and left to the end of the calculation. It may be omitted from the table as will be seen from the worked examples.

Example. The half ordinates of a waterplane 180 m long are as follows:

section	AP	½	1	2	3	4	5	6	7	8	9	9½	FP
½ ord	0	5·0	8·0	10·5	12·5	13·5	13·5	12·5	11·0	7·5	3·0	1·0	0 m

Calculate:

(a) area of waterplane

(b) distance of centroid from midships

(c) second moment of area of waterplane about a transverse axis through the centroid.

Section	½ ordinate	SM	Product for Area	Lever	Product for 1st moment	Lever	Product for 2nd moment
AP	0	½	—	+5	—	+5	—
½	5·0	2	10·0	+4½	+45·0	+4½	+202·5
1	8·0	1½	12·0	+4	+48·0	+4	+192·0
2	10·5	4	42·0	+3	+126·0	+3	+378·0
3	12·5	2	25·0	+2	+50·0	+2	+100·0
4	13·5	4	54·0	+1	+54·0	+1	+ 54·0
					+323·0		
5	13·5	2	27·0	0		0	—
6	12·5	4	50·0	—1	—50·0	—1	+ 50·0
7	11·0	2	22·0	—2	—44·0	—2	+ 88·0
8	7·5	4	30·0	—3	—90·0	—3	+270·0
9	3·0	1½	4·5	—4	—18·0	—4	+ 72·0
9½	1·0	2	2·0	—4½	—9·0	—4½	+ 40·5
FP	0	½	—	—5	—	—5	—
			278·5		—211·0		+1447·0

$$\text{Common interval} = \frac{180}{10}$$
$$= 18 \text{ m}$$

(a) Area of waterplane $= \frac{2}{3} \times 18 \times 278{\cdot}5$

$= 3342{\cdot}0 \text{ m}^2$

(b) Centroid from midships $= \dfrac{18\ (323 - 211)}{278{\cdot}5}$

$= 7{\cdot}238$ m aft

(c) Second moment of area of waterplane about midships

$= \frac{2}{3} \times 18^3 \times 1447$

$= 5{\cdot}626 \times 10^6 \text{ m}^4$

Second moment of area of waterplane about centroid

$= 5{\cdot}626 \times 10^6 - 3342 \times 7{\cdot}238^2$

$= 5{\cdot}626 \times 10^6 - 0{\cdot}175 \times 10^6$

$= 5{\cdot}451 \times 10^6\ \text{m}^4$

To determine the second moment of area of the waterplane about the centreline of the ship, the half ordinates must be *cubed* and then put through Simpson's Rule.

½ ordinate	(½ ordinate)³	SM	Product for 2nd moment
y_0	y_0^3	1	$1y_0^3$
y_1	y_1^3	4	$4y_1^3$
y_2	y_2^3	2	$2y_2^3$
y_3	y_3^3	4	$4y_3^3$
y_4	y_4^3	2	$2y_4^3$
y_5	y_5^3	4	$4y_5^3$
y_6	y_6^3	2	$2y_6^3$
y_7	y_7^3	4	$4y_7^3$
y_8	y_8^3	2	$2y_8^3$
y_9	y_9^3	4	$4y_9^3$
y_{10}	y_{10}^3	1	$1y_{10}^3$
			ΣI_{CL}

Second moment of area of waterplane about the centreline.

$= \frac{2}{3} \times h \times \Sigma I_{CL} \times \frac{1}{3}$

$= \frac{2}{9}\, h\, \Sigma I_{CL}$

Example. The half ordinates of a waterplane 180 m long are as follows:

Section	AP	½	1	2	3	4	5	6	7	8	9	9½	FP
½ ord	0	5·0	8·0	10·5	12·5	13·5	13·5	12·5	11·0	7·5	3·0	1·0	0 m

Calculate the second moment of area of the waterplane about the centreline.

Section	½ ordinate	(½ ordinate)³	SM	2nd moment
AP	0	—	½	—
½	5·0	125·0	2	250·0
1	8·0	512·0	1½	768·0
2	10·5	1157·6	4	4630·4
3	12·5	1953·1	2	3906·2
4	13·5	2460·4	4	9841·6
5	13·5	2460·4	2	4920·8
6	12·5	1953·1	4	7812·4
7	11·0	1331·0	2	2662·0
8	7·5	421·9	4	1687·6
9	3·0	27·0	1½	40·5
9½	1·0	1·0	2	2·0
FP	0	—	½	—
				36521·5

Common interval = 18 m

Second moment of area of waterplane about centreline

$= \frac{2}{9} \times 18 \times 36\ 521{\cdot}5$

$= 146\ 086\ \text{m}^4$

It should be noted that the second moment of area about a transverse axis is considerably greater than the second moment about the centreline.

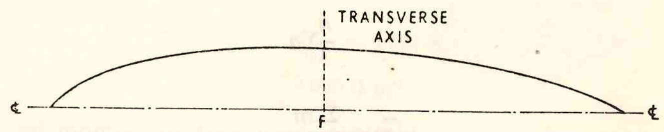


Fig. 27

Since each waterplane is symmetrical it is never necessary to calculate the first moment about the centreline. There are many occasions, however, on which the first moment of area of a tank surface must be calculated as shown by the following example.

Example. A double bottom tank extends from the centreline to the ship side. The widths of the tank surface, at regular intervals of h, are y_1, y_2, y_3, y_4 and y_5.

Calculate the second moment of area of the tank surface about a longitudinal axis through its centroid.

It is necessary in this calculation to determine the area, centroid from the centreline and the second moment of area.

Width	SM	Product for Area	(Width)²	SM	Product for 1st moment	(Width)³	SM	Product for 2nd moment
y_1	1	$1y_1$	y_1^2	1	$1y_1^2$	y_1^3	1	$1y_1^3$
y_2	4	$4y_2$	y_2^2	4	$4y_2^2$	y_2^3	4	$4y_2^3$
y_3	2	$2y_3$	y_3^2	2	$2y_3^2$	y_3^3	2	$2y_3^3$
y_4	4	$4y_4$	y_4^2	4	$4y_4^2$	y_4^3	4	$4y_4^3$
y_5	1	$1y_5$	y_5^2	1	$1y_5^2$	y_5^3	1	$1y_5^3$
		Σa			Σm			Σi

Area $a = \frac{h}{3}\Sigma a$

First moment of area about centreline

$$= \frac{h}{3} \times \Sigma m \times \tfrac{1}{2}$$

$$= \frac{h}{6} \Sigma m$$

$$\text{Centroid from centreline} = \frac{\frac{h}{6} \Sigma m}{\frac{h}{3} \Sigma a}$$

$$\bar{y} = \frac{\Sigma m}{2 \Sigma a}$$

Second moment of area about centreline

$$i_{CL} = \frac{h}{9} \Sigma i$$

Second moment of area about centroid

$$= i_{CL} - a\bar{y}^2$$

Example. A double bottom tank 21 m long has a watertight centre girder. The widths of the tank top measured from the centreline to the ship's side are 10·0, 9·5, 9·0, 8·0, 6·5, 4·0 and 1·0 m respectively. Calculate the second moment of area of the tank surface about a longitudinal axis through its centroid, for one side of the ship only.

Width	SM	Product for Area	(Width)^2	SM	Product for 1st moment	(Width)^3	SM	Product for 2nd moment
10·0	1	10·0	100·00	1	100·00	1000·0	1	1000·0
9·5	4	38·0	90·25	4	361·00	857·4	4	3429·6
9·0	2	18·0	81·00	2	162·00	729·0	2	1458·0
8·0	4	32·0	64·00	4	256·00	512·0	4	2048·0
6·5	2	13·0	42·25	2	84·50	274·6	2	549·2
4·0	4	16·0	16·00	4	64·00	64·0	4	256·0
1·0	1	1·0	1·00	1	1·00	1·0	1	1·0
		128·0			1028·50			8741·8

$$\text{Common interval} = \frac{21}{6} = 3·5 \text{ m}$$

$$\text{Area of tank surface} = \frac{3·5}{3} \times 128$$

$$= 149·33 \text{ m}^2$$

$$\text{Centroid from centreline} = \frac{1028·5}{2 \times 128}$$

$$= 4·018 \text{ m}$$

Second moment of area about centreline

$$= \frac{3·5}{9} \times 8741·8$$

$$= 3400·0 \text{ m}^4$$

Second moment of area about centroid

$$= 3400·0 - 149·33 \times 4·018^2$$
$$= 3400·0 - 2410·8$$
$$= 989·2 \text{ m}^4$$

A further application of first and second moments of area is the calculation of the load exerted by a liquid on a bulkhead and the position of the centre of pressure.

Let the widths of a bulkhead at intervals of h, commencing from the top, be y_0, y_1, y_2, ... y_6 (Fig. 28)

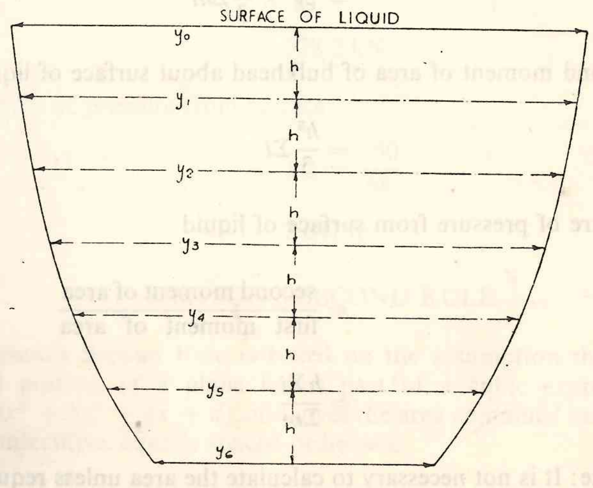


Fig. 28

Assume the bulkhead to be flooded to the top edge with liquid of density ρ on one side only

Width	SM	Product for Area	Lever	Product for 1st moment	Lever	Product for 2nd moment
y_0	1	$1y_0$	0	—	0	—
y_1	4	$4y_1$	1	$4y_1$	1	$4y_1$
y_2	2	$2y_2$	2	$4y_2$	2	$8y_2$
y_3	4	$4y_3$	3	$12y_3$	3	$36y_3$
y_4	2	$2y_4$	4	$8y_4$	4	$32y_4$
y_5	4	$4y_5$	5	$20y_5$	5	$100y_5$
y_6	1	$1y_6$	6	$6y_6$	6	$36y_6$
		Σa		Σm		Σi

$$\text{Area of bulkhead} = \frac{h}{3}\Sigma a$$

First moment of area of bulkhead about surface of liquid

$$= \frac{h^2}{3}\Sigma m$$

It was shown previously that:

Load on bulkhead = ρg × first moment of area

$$= \rho g \times \frac{h^2}{3}\Sigma m$$

Second moment of area of bulkhead about surface of liquid

$$= \frac{h^3}{3}\Sigma i$$

Centre of pressure from surface of liquid

$$= \frac{\text{second moment of area}}{\text{first moment of area}}$$

$$= \frac{h\,\Sigma i}{\Sigma m}$$

(Note: It is not necessary to calculate the area unless requested to do so).

Example. A fore peak bulkhead is 4·8 m deep and 5·5 m wide at the deck. At regular intervals of 1·2 m below the deck, the horizontal widths are 5·0, 4·0, 2·5 and 0·5 m respectively. The bulkhead is flooded to the top edge with sea water on one side only. Calculate:

(a) area of bulkhead

(b) load on bulkhead

(c) position of centre of pressure

Depth	Width	SM	Product for Area	Lever	Product for 1st moment	Lever	Product for 2nd moment
4·8	5·5	1	5·5	0	—	0	—
3·6	5·0	4	20·0	1	20·0	1	20·0
2·4	4·0	2	8·0	2	16·0	2	32·0
1·2	2·5	4	10·0	3	30·0	3	90·0
0	0·5	1	0·5	4	2·0	4	8·0
			44·0		68·0		150·0

Common interval = 1·2 m

(a) $$\text{Area of bulkhead} = \frac{1{\cdot}2}{3} \times 44$$

$$= 17{\cdot}6 \text{ m}^2$$

(b) $$\text{Load on bulkhead} = 1{\cdot}025 \times 9{\cdot}81 \times \frac{1{\cdot}2^2}{3} \times 68{\cdot}0$$

$$= 328{\cdot}2 \text{ kN}$$

(c) Centre of pressure from surface

$$= 1{\cdot}2 \times \frac{150}{68}$$

$$= 2{\cdot}647 \text{ m}$$

SIMPSON'S SECOND RULE

Simpson's Second Rule is based on the assumption that the curved portion of a plane forms part of a cubic expression ($y = ax^3 + bx^2 + cx + d$), and gives the area contained between *four* consecutive, equally spaced-ordinates.

$$\text{Area ABCD} = \tfrac{3}{8}\,h\,(1y_1 + 3y_2 + 3y_3 + 1y_4)$$

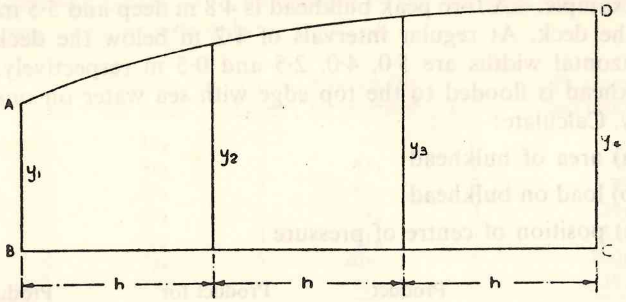


Fig. 29

This rule may be applied to successive areas in the same way as the first rule.

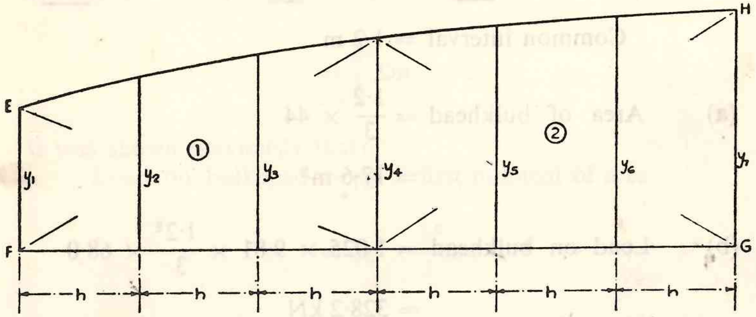


Fig. 30

$$\text{Area } 1 = \tfrac{3}{8}h(1y_1+3y_2+3y_3+1y_4)$$
$$\text{Area } 2 = \tfrac{3}{8}h(1y_4+3y_5+3y_6+1y_7)$$
$$\text{Area EFGH} = \tfrac{3}{8}h[(1y_1+3y_2+3y_3+1y_4)+(1y_4+3y_5+3y_6+1y_7)]$$
$$= \tfrac{3}{8}h[1y_1+3y_2+3y_3+2y_4+3y_5+3y_6+1y_7]$$

It may be shown that Simpson's Second Rule may be used to determine the area of a plane having a number of equal intervals divisible by *three*, i.e. 4, 7, 10, 13, 16 ordinates.

Example A waterplane 135 m long has equally-spaced half ordinates of 1·2, 4·4, 6·7, 7·8, 8·0, 8·0, 7·7, 6·1, 3·8 and 0 m respectively. Calculate the area of the waterplane.

½ ordinate	SM	Product for Area
1·2	1	1·2
4·4	3	13·2
6·7	3	20·1
7·8	2	15·6
8·0	3	24·0
8·0	3	24·0
7·7	2	15·4
6·1	3	18·3
3·8	3	11·4
0	1	0
		143·2

$$\text{Common interval} = \frac{135}{9} = 15 \text{ m}$$

$$\text{Area of waterplane} = \tfrac{3}{8} \times 15 \times 143{\cdot}2 \times 2$$

$$= 1611{\cdot}0 \text{ m}^2$$

TEST EXAMPLES 3

1. A ship 180 m long has ½ widths of waterplane of 1, 7·5, 12, 13·5, 14, 14, 14, 13·5, 12, 7 and 0 m respectively. Calculate:
   (a) waterplane area
   (b) TPC
   (c) waterplane area coefficient.

2. The waterplane areas of a ship at 1·5 m intervals of draught, commencing at the keel, are 865, 1735, 1965, 2040, 2100, 2145 and 2215 m² respectively. Calculate the displacement at 9 m draught.

3. A ship 140 m long and 18 m beam floats at a draught of 9 m. The immersed cross-sectional areas at equal intervals are 5, 60, 116, 145, 152, 153, 153, 151, 142, 85 and 0 m² respectively. Calculate:
   (a) displacement
   (b) block coefficient
   (c) midship section area coefficient
   (d) prismatic coefficient.

4. The ½ ordinates of a waterplane 120 m long are as follows:

Section	AP	½	1	1½	2	3	4	5	6	7	8	8½	9	9½	FP	
½ ord	1·2	3·5	5·3	6·8	8·0	8·3	8·5	8·5	8·5	8·4	8·2	7·9	6·2	3·5	0	m

Calculate:
   (a) waterplane area
   (b) distance of centroid from midships.

*f*5. The TPC values of a ship at 1·5 m intervals of draught, commencing at the keel, are 4·0, 6·1, 7·8, 9·1, 10·3, 11·4 and 12·0 respectively. Calculate at a draught of 9 m:
   (a) displacement
   (b) KB

*f*6. The ½ breadths of the load waterplane of a ship 150 m long, commencing from aft, are 0·3, 3·8, 6·0, 7·7, 8·3, 9·0, 8·4, 7·8, 6·9, 4·7 and 0 m respectively. Calculate:
   (a) area of waterplane
   (b) distance of centroid from midships
   (c) second moment of area about a transverse axis through the centroid

*f*7. The displacement of a ship at draughts of 0, 1, 2, 3 and 4 m are 0, 189, 430, 692 and 977 tonne. Calculate the distance of the centre of buoyancy above the keel when floating at a draught of 4 m, given:

VCB below waterline

$$= \frac{\text{area between displacement curve and draught axis}}{\text{displacement}}$$

*f*8. The widths of a deep tank bulkhead at equal intervals of 1·2 m commencing at the top, are 8·0, 7·5, 6·5, 5·7, 4·7, 3·8 and 3·0 m. Calculate the load on the bulkhead and the position of the centre of pressure, if the bulkhead is flooded to the top edge with sea water on one side only.

*f*9. A forward deep tank 12 m long extends from a longitudinal bulkhead to the ship's side. The widths of the tank surface measured from the longitudinal bulkhead at regular intervals are 10, 9, 7, 4 and 1 m. Calculate the second moment of area of the tank surface about a longitudinal axis passing through its centroid.

*f*10. A ship 160 m long has ½ ordinates of waterplane of 1·6, 5·7, 8·8, 10·2, 10·5, 10·5, 10·5, 10·0, 8·0, 5·0 and 0 m respectively. Calculate the second moment of area of the waterplane about the centreline.

*f*11. The immersed cross-sectional areas of a ship 120 m long, commencing from aft, are 2, 40, 79, 100, 103, 104, 104, 103, 97, 58 and 0 m². Calculate:
   (a) displacement
   (b) longitudinal position of the centre of buoyancy.

12. The ½ ordinates of the waterplane of a ship 180 m long are 1·6, 6·0, 9·2, 10·5, 11, 11, 10·2, 8·3, 5·1 and 0 m respectively. Calculate the area of the waterplane.

CHAPTER 4

CENTRE OF GRAVITY

The centre of gravity of an object is the point at which the whole weight of the object may be regarded as acting. If the object is suspended from this point, then it will remain balanced and will not tilt.

The distance of the centre of gravity from any axis is the total moment of *force* about that axis divided by the total *force*. If a body is composed of a number of different types of material, the force may be represented by the *weights* of the individual parts.

$$\text{Centre of gravity from axis} = \frac{\text{moment of weight about axis}}{\text{total weight}}$$

At any point on the earth's surface, the value of g remains constant. Hence the weight may be represented by *mass*, and:

$$\text{Centre of gravity from axis} = \frac{\text{moment of mass about axis}}{\text{total mass}}$$

If the body is of the same material throughout, then the weight depends upon the *volume* and moments of *volume* may therefore be used.

$$\text{Centre of gravity from axis} = \frac{\text{moment of volume about axis}}{\text{total volume}}$$

The centre of gravity of a uniform lamina is midway through the thickness. Since both the thickness and the density are constant, moments of *area* may be used. This system may also be applied to determine the centre of gravity, or, more correctly, *centroid* of an area.

$$\text{Centroid from axis} = \frac{\text{moment of area about axis}}{\text{total area}}$$

The position of the centre of gravity of a ship may be found by taking moments of the individual masses. The actual calculation of the centre of gravity of a ship is a very lengthy process, and since many of the masses must be estimated, is not considered to be sufficiently accurate for stability calculations. Such a calculation is usually carried out for a passenger ship in the initial design stages, but the results are confirmed by an alternative method when the ship is completed. Once the position of the centre of gravity of an empty ship is known, however, the centre of gravity of the ship in any loaded condition may be found.

It is usual to measure the *vertical* position of the centre of gravity (VCG) of the ship above the keel and this distance is denoted by *KG*. The height of the centre of gravity of an item on the ship above the keel is denoted by *Kg*. The *longitudinal* position of the centre of gravity (LCG) is usually given as a distance forward or aft of midships. If the ship is upright, the *transverse* centre of gravity lies on the centreline of the ship and no calculation is necessary.

Example. A ship of 8500 tonne displacement is composed of masses of 2000, 3000, 1000, 2000, and 500 tonne at positions 2, 5, 8, 10 and 14 m above the keel. Determine the height of the centre of gravity of the ship above the keel.

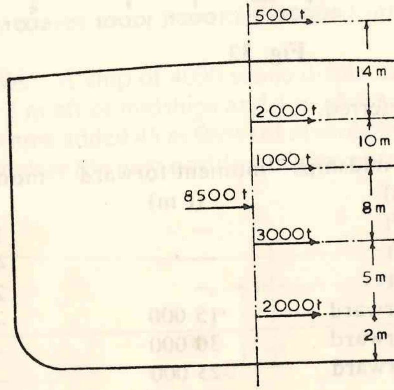


Fig. 31

This example is preferably answered in table form.

mass (tonne)	Kg (m)	Vertical moment (t m)
2000	2	4 000
3000	5	15 000
1000	8	8 000
2000	10	20 000
500	14	7 000
8500		54 000

$$KG = \frac{\text{total moment}}{\text{total displacement}}$$

$$= \frac{54\,000}{8500}$$

$$= 6{\cdot}353 \text{ m}$$

Example. A ship of 6000 tonne displacement is composed of masses of 300, 1200 and 2000 tonne at distances 60, 35 and 11 m aft of midships, and masses of 1000, 1000 and 500 tonne at distances 15, 30 and 50 m forward of midships. Calculate the distance of the centre of gravity of the ship from midships.

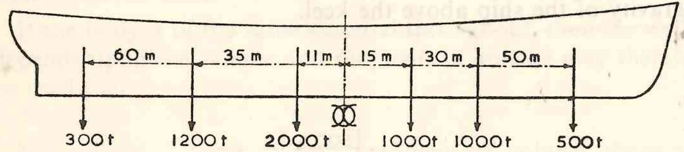


Fig. 32

A table is again preferred.

Mass (tonne)	Lcg from midships (m)	moment forward (t m)	moment aft (t m)
300	60 aft	—	18 000
1200	35 aft	—	42 000
2000	11 aft	—	22 000
1000	15 forward	15 000	—
1000	30 forward	30 000	—
500	50 forward	25 000	—
6000		70 000	82 000

The moment aft is greater than the moment forward and therefore the centre of gravity must lie aft of midships.

$$\text{Excess moment aft} = 82\,000 - 70\,000$$
$$= 12\,000 \text{ tonne m}$$

Centre of gravity aft of midships

$$= \frac{\text{excess moment}}{\text{total displacement}}$$

$$= \frac{12\,000}{6000}$$

$$= 2{\cdot}00 \text{ m}$$

SHIFT IN CENTRE OF GRAVITY DUE TO ADDITION OF MASS

When a mass is added to a ship, the centre of gravity of the ship moves towards the added mass. The distance moved by the ship's centre of gravity depends upon the magnitude of the added mass, the distance of the mass from the ship's centre of gravity and the displacement of the ship. If a mass is placed on the port side of the ship in the forecastle, the centre of gravity moves forward, upwards and to port. The actual distance and direction of this movement is seldom required but the separate components are most important, i.e. the longitudinal, vertical and transverse distances moved. When an item on a ship is removed, the centre of gravity moves away from the original position of that item.

Example. A ship of 4000 tonne displacement has its centre of gravity 1·5 m aft of midships and 4 m above the keel. 200 tonne of cargo are now added 45 m forward of midships and 12 m above the keel. Calculate the new position of the centre of gravity.

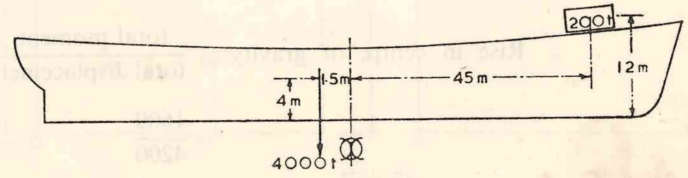


Fig. 33

Taking moments about midships:

$$\text{Moment aft of midships} = 4000 \times 1{\cdot}5$$
$$= 6000 \text{ t m}$$
$$\text{Moment forward of midships} = 200 \times 45$$
$$= 9000 \text{ t m}$$
$$\text{Excess moment forward} = 9000 - 6000$$
$$= 3000 \text{ t m}$$
$$= 3000 \text{ t m}$$

$$\text{Centre of gravity from midships} = \frac{\text{excess moment}}{\text{total displacement}}$$
$$= \frac{3000}{4000 + 200}$$
$$= 0{\cdot}714 \text{ m forward}$$

Taking moments about the keel:

$$\text{Centre of gravity from keel} = \frac{4000 \times 4 + 200 \times 12}{4000 + 200}$$
$$= \frac{16\,000 + 2400}{4200}$$
$$KG = 4{\cdot}381 \text{ m}$$

Thus the centre of gravity rises 0·381 m.

The same answer may be obtained by taking moments about the original centre of gravity, thus:

$$\text{Moment of ship about centre of gravity} = 4000 \times 0$$
$$\text{Moment of added mass about centre of gravity} = 200(12 - 4)$$
$$= 1600 \text{ t m}$$

$$\text{Rise in centre of gravity} = \frac{\text{total moment}}{\text{total displacement}}$$
$$= \frac{1600}{4200}$$
$$= 0{\cdot}381 \text{ m}$$

If the actual distance moved by the centre of gravity is required, it may be found from the longitudinal and vertical movements.

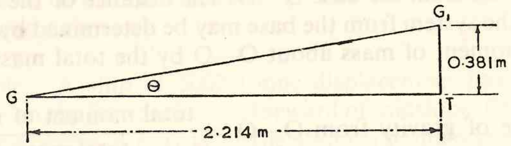


Fig. 34

Longitudinal shift in the centre of

$$\text{gravity } GT = 1{\cdot}5 + 0{\cdot}714$$
$$= 2{\cdot}214 \text{ m}$$
$$GG_1 = \sqrt{GT^2 + TG_1^2}$$
$$= \sqrt{2{\cdot}214^2 + 0{\cdot}381^2}$$
$$= 2{\cdot}247 \text{ m}$$

The angle θ which the centre of gravity moves relative to the horizontal may be found from Fig. 34.

$$\tan\theta = \frac{0{\cdot}381}{2{\cdot}214}$$
$$= 0{\cdot}712$$
$$\text{from which} \quad \theta = 9^\circ 45'$$

SHIFT IN CENTRE OF GRAVITY DUE TO MOVEMENT OF MASS

When a mass which is already on board a ship is moved in any direction, there is a corresponding movement in the centre of gravity of the ship in the same direction.

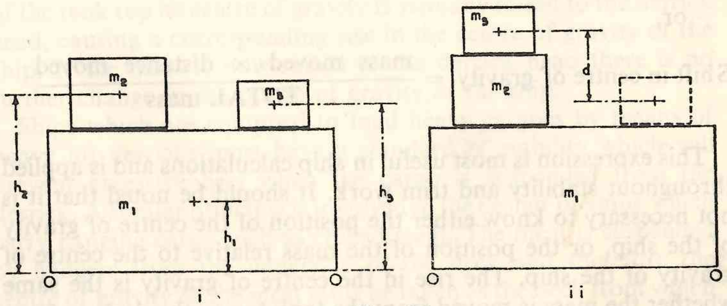


Fig. 35

Consider a system composed of masses of m_1, m_2 and m_3 as shown in Fig. 35 (i), the centre of gravity of each being h_1, h_2 and h_3 respectively from the base O—O. The distance of the centre of gravity of the system from the base may be determined by dividing the total moment of mass about O—O by the total mass.

$$\text{Centre of gravity from O—O} = \frac{\text{total moment of mass}}{\text{total mass}}$$

$$= \frac{m_1h_1 + m_2h_2 + m_3h_3}{m_1 + m_2 + m_3}$$

$$= y$$

If m_3 is now raised through a distance d to the position shown in Fig. 35 (ii), the centre of gravity of the system is also raised. New centre of gravity from O—O

$$= \frac{m_1h_1 + m_2h_2 + m_3(h_3 + d)}{m_1 + m_2 + m_3}$$

$$= \frac{m_1h_1 + m_2h_2 + m_3h_3}{m_1 + m_2 + m_3} + \frac{m_3d}{m_1 + m_2 + m_3}$$

$$= y + \frac{m_3d}{m_1 + m_2 + m_3}$$

Thus it may be seen that:

$$\text{Shift in centre of gravity} = \frac{m_3d}{m_1 + m_2 + m_3}$$

or,

$$\text{Shift in centre of gravity} = \frac{\text{mass moved} \times \text{distance moved}}{\text{TOTAL mass}}$$

This expression is most useful in ship calculations and is applied throughout stability and trim work. It should be noted that it is not necessary to know either the position of the centre of gravity of the ship, or the position of the mass relative to the centre of gravity of the ship. The rise in the centre of gravity is the same whether the mass is moved from the tank top to the deck or from the deck to the mast head as long as the distance moved is the same. The centre of gravity of the ship moves in the same direction as the centre of gravity of the mass. Thus if a mass is moved forward and down, the centre of gravity of the ship also moves forward and down.

Example. A ship of 5000 tonne displacement has a mass of 200 tonne on the fore deck 55 m forward of midships. Calculate the shift in the centre of gravity of the ship if the mass is moved to a position 8 m forward of midships.

$$\text{Shift in centre of gravity} = \frac{\text{mass moved} \times \text{distance moved}}{\text{displacement}}$$

$$= \frac{200 \times (55 - 8)}{5000}$$

$$= 1{\cdot}88 \text{ m}$$

EFFECT OF A SUSPENDED MASS

When a mass hangs freely from a point on a ship, its centre of gravity lies directly below that point. If the vessel now heels, the mass moves in the direction of the heel until it again lies vertically below the point of suspension, and no matter in which direction the vessel heels, the centre of gravity of the mass is always below this point. Thus it may be seen that the position of the centre of gravity of a hanging mass, relative to the ship, is at the point of suspension.

This principle proves to be very important when loading a ship by means of the ship's derricks. If, for example, a mass lying on the tank top is being discharged, then as soon as the mass is clear of the tank top its centre of gravity is virtually raised to the derrick head, causing a corresponding rise in the centre of gravity of the ship. If the mass is now raised to the derrick head there is no further change in the centre of gravity of the ship.

Ships which are equipped to load heavy cargoes by means of heavy lift derricks must have a standard of stability which will prevent excessive heel when the cargo is suspended from the derrick. A similar principle is involved in the design of ships which carry hanging cargo such as chilled meat. The meat is suspended by hangers from the underside of the deck and therefore the centre of gravity of the meat must be taken as the deck from which it hangs.

Example. A ship of 10 000 tonne displacement has a mass of 60 tonne lying on the deck. A derrick, whose head is 7·5 m above the centre of gravity of the mass, is used to place the mass on the tank top 10·5 m below the deck. Calculate the shift in the vessel's centre of gravity when the mass is:

(a) just clear of the deck
(b) at the derrick head
(c) in its final position.

(a) When the mass is just clear of the deck its centre of gravity is raised to the derrick head.

$$\text{Shift in centre of gravity} = \frac{\text{mass moved} \times \text{distance moved}}{\text{displacement}}$$

$$= \frac{60 \times 7{\cdot}5}{10\ 000}$$

$$= 0{\cdot}045 \text{ m up}$$

(b) When the mass is at the derrick head there is no further movement of the centre of gravity of the ship.

$$\text{Shift in centre of gravity} = 0{\cdot}045 \text{ m up}$$

$$\text{(c) Shift in centre of gravity} = \frac{60 \times 10{\cdot}5}{10\ 000}$$

$$= 0{\cdot}063 \text{ m down}$$

TEST EXAMPLES 4

1. A ship of 4000 tonne displacement has its centre of gravity 6 m above the keel. Find the new displacement and position of the centre of gravity when masses of 1000, 200, 5000 and 3000 tonne are added at positions 0·8, 1·0, 5·0 and 9·5 m above the keel.

2. The centre of gravity of a ship of 5000 tonne displacement is 6 m above the keel and 1·5 m forward of midships. Calculate the new position of the centre of gravity if 500 tonne of cargo are placed in the 'tween decks 10 m above the keel and 36 m aft of midships.

3. A ship has 300 tonne of cargo in the hold, 24 m forward of midships. The displacement of the vessel is 6000 tonne and its centre of gravity is 1·2 m forward of midships.

Find the new position of the centre of gravity if this cargo is moved to an after hold, 40 m from midships.

4. An oil tanker of 17 000 tonne displacement has its centre of gravity 1 m aft of midships and has 250 tonne of oil fuel in its forward deep tank 75 m from midships.

This fuel is transferred to the after oil fuel bunker whose centre is 50 m from midships.

200 tonne of fuel from the after bunker is now burned.

Calculate the new position of the centre of gravity:

(a) after the oil has been transferred
(b) after the oil has been used.

5. A ship of 3000 tonne displacement has 500 tonne of cargo on board. This cargo is lowered 3 m and an additional 500 tonne of cargo is taken on board 3 m vertically above the original position of the centre of gravity. Determine the alteration in position of the centre of gravity.

6. A ship of 10 000 tonne displacement has its centre of gravity 3 m above the keel. Masses of 2000, 300 and 50 tonne are removed from positions 1·5, 4·5 and 6m above the keel. Find the new displacement and position of the centre of gravity.

7. A vessel of 8000 tonne displacement has 75 tonne of cargo on the deck. It is lifted by a derrick whose head is 10·5 m above the centre of gravity of the cargo, and placed in the lower hold 9 m below the deck and 14 m forward of its original position. Calculate the shift in the vessel's centre of gravity from its original position when the cargo is:

(a) just clear of the deck
(b) at the derrick head
(c) in its final position.

CHAPTER 5

STABILITY OF SHIPS

Statical stability is a measure of the tendency of a ship to return to the upright if inclined by an external force.

In theory it is possible to balance a pencil on its point on a flat surface. The pencil will be balanced if its centre of gravity is vertically above its point. In practice this is found to be impossible to achieve. It is, however, possible to balance the pencil on its flat end, since, if the pencil is very slightly inclined, the centre of gravity may still lie within the limits of the base and the pencil will tend to return to the upright. Fig. 36 is exaggerated to show this.

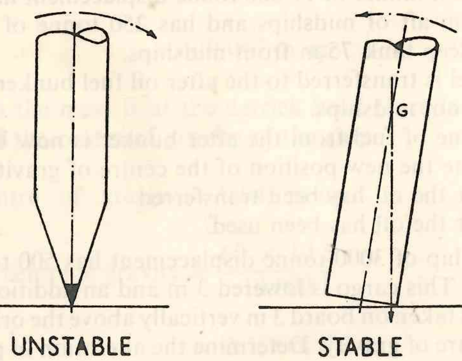


Fig. 36

The only times a ship may be assumed to be stationary and upright are before launching and when in dry dock. Thus it is essential to consider practical conditions and to assume that a ship is always moving. If the vessel is stated to be upright it should be regarded as rolling slightly about the upright position.

In the upright position (Fig. 37), the weight of the ship acts vertically down through the centre of gravity G, while the upthrust acts through the centre of buoyancy B. Since the weight is equal to the upthrust, and the centre of gravity and the centre of buoyancy are in the same vertical line, the ship is in equilibrium.

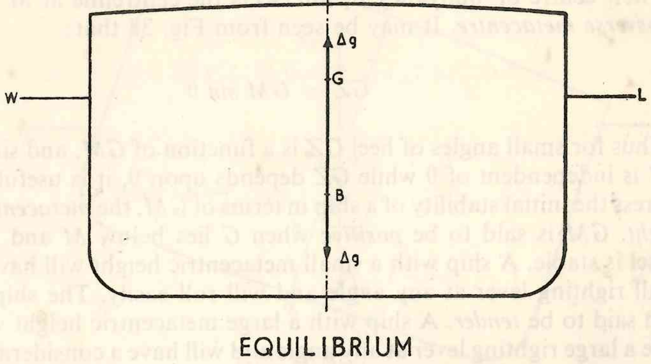


Fig. 37

When the ship is inclined by an external force to an angle θ, the centre of gravity remains in the same position but the centre of buoyancy moves from B to B_1 (Fig. 38).

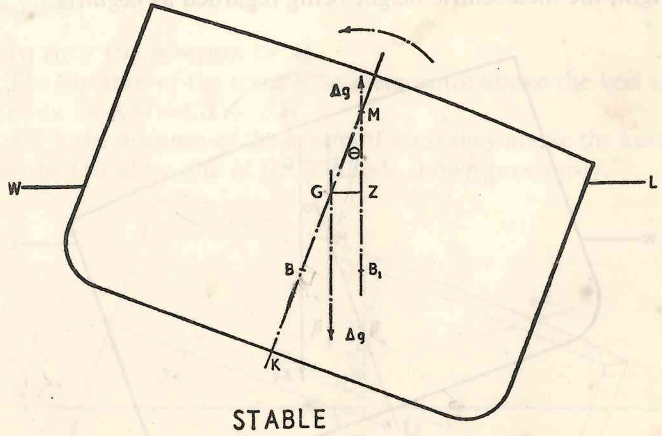


Fig. 38

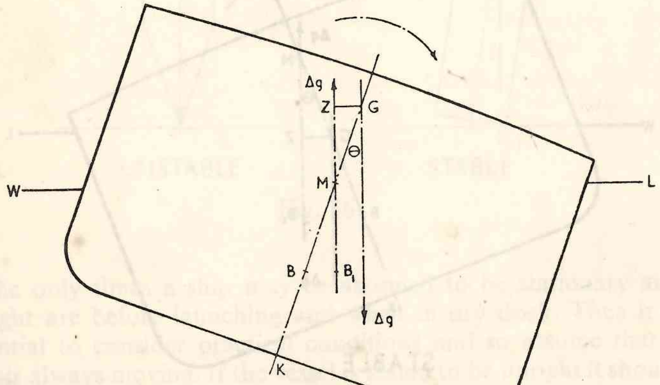
The buoyancy, therefore, acts up through B_1 while the weight still acts down through G, creating a moment of $\triangle g \times GZ$ which tends to return the ship to the upright. $\triangle g \times GZ$ is known as the *righting moment* and GZ the *righting lever*. Since this moment tends to right the ship the vessel is said to be *stable*.

For small angles of heel, up to about 10°, the vertical through the new centre of buoyancy B_1 intersects the centreline at M the *transverse metacentre*. It may be seen from Fig. 38 that:

$$GZ = GM \sin \theta$$

Thus for small angles of heel GZ is a function of GM, and since GM is independent of θ while GZ depends upon θ, it is useful to express the initial stability of a ship in terms of GM, the *metacentric height*. GM is said to be *positive* when G lies below M and the vessel is stable. A ship with a small metacentric height will have a small righting lever at any angle and will roll easily. The ship is then said to be *tender*. A ship with a large metacentric height will have a large righting lever at any angle and will have a considerable resistance to rolling. The ship is then said to be *stiff*. A stiff ship will be very uncomfortable, having a very small rolling period and in extreme cases may result in structural damage.

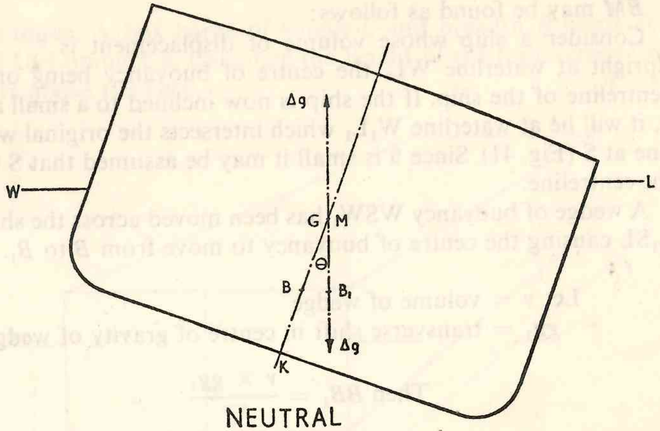
If the centre of gravity lies above the transverse metacentre (Fig. 39), the moment acts in the opposite direction, increasing the angle of heel. The vessel is then *unstable* and will not return to the upright, the metacentric height being regarded as *negative*.



UNSTABLE

Fig. 39

When the centre of gravity and transverse metacentric coincide (Fig. 40), there is no moment acting on the ship which will therefore remain inclined to angle θ. The vessel is then said to be in *neutral equilibrium*.



NEUTRAL

EQUILIBRIUM

Fig. 40

Since any reduction in the height of G will make the ship stable, and any rise in G will make the ship unstable, this condition is regarded as the point at which a ship *becomes* either stable or unstable.

TO FIND THE POSITION OF M

The distance of the transverse metacentre above the keel (KM) is given by $KM = KB + BM$.

KB is the distance of the centre of buoyancy above the keel and may be found by one of the methods shown previously.

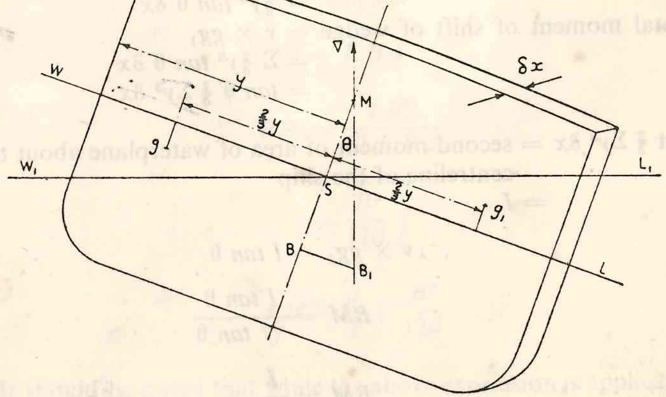


Fig. 41

BM may be found as follows:

Consider a ship whose volume of displacement is ∇, lying upright at waterline WL, the centre of buoyancy being on the centreline of the ship. If the ship is now inclined to a small angle θ, it will lie at waterline W_1L_1 which intersects the original waterline at S (Fig. 41). Since θ is small it may be assumed that S is on the centreline.

A wedge of buoyancy WSW_1 has been moved across the ship to L_1SL causing the centre of buoyancy to move from B to B_1.

Let v = volume of wedge

gg_1 = transverse shift in centre of gravity of wedge

$$\text{Then } BB_1 = \frac{v \times gg_1}{\nabla}$$

$$\text{But } BB_1 = BM \tan\theta$$

$$\therefore BM \tan\theta = \frac{v \times gg_1}{\nabla}$$

$$BM = \frac{v \times gg_1}{\nabla \tan\theta}$$

To determine the value of $v \times gg_1$, divide the ship into thin transverse strips of length δx, and let the half width of waterplane in way of one such strip be y.

$$\text{Volume of strip of wedge} = \tfrac{1}{2}y \times y \tan\theta\,\delta x$$

$$\text{Moment of shift of strip of wedge} = \tfrac{4}{3}y \times \tfrac{1}{2}y^2 \tan\theta\,\delta x$$

$$= \tfrac{2}{3}y^3 \tan\theta\,\delta x$$

$$\text{Total moment of shift of wedge} = v \times gg_1$$

$$= \Sigma\, \tfrac{2}{3}y^3 \tan\theta\,\delta x$$

$$= \tan\theta\, \tfrac{2}{3}\,\Sigma y^3\,\delta x$$

But $\frac{2}{3}\,\Sigma y^3\,\delta x$ = second moment of area of waterplane about the centreline of the ship $= I$

$$\therefore v \times gg_1 = I \tan\theta$$

$$BM = \frac{I \tan\theta}{\nabla \tan\theta}$$

$$BM = \frac{I}{\nabla}$$

Example. A box barge of length L and breadth B floats at a level keel draught d. Calculate the height of the transverse metacentre above the keel.

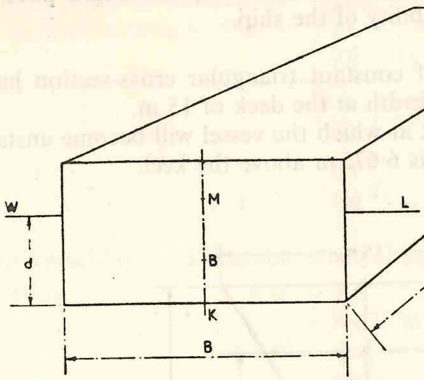


Fig. 42

$$KM = KB + BM$$

$$KB = \frac{d}{2}$$

$$BM = \frac{I}{\nabla}$$

$$I = \tfrac{1}{12}LB^3$$

$$\nabla = L.B.d$$

$$BM = \frac{LB^3}{12.L.B.d}$$

$$= \frac{B^2}{12d}$$

$$\therefore KM = \frac{d}{2} + \frac{B^2}{12d}$$

It should be noted that while the above expression is applicable only to a box barge, similar expressions may be derived for vessels of constant triangular or circular cross sections. The waterplane in each case is in the form of a *rectangle*, the second moment of which is $\frac{1}{12}$ × length × breadth³. As long as the length of a vessel having constant cross-section exceeds the breadth, the length does not affect the transverse stability of the ship.

Example. A vessel of constant triangular cross-section has a depth of 12 m and a breadth at the deck of 15 m.

Calculate the draught at which the vessel will become unstable if the centre of gravity is 6·675 m above the keel.

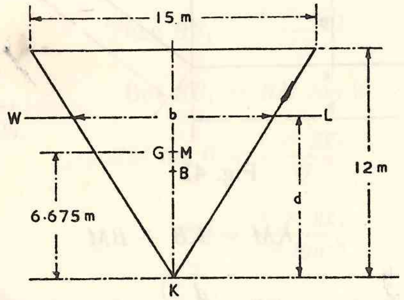


Fig. 43

Let d = draught

b = breadth at waterline

By similar triangles $\quad \frac{b}{d} = \frac{B}{D}$

$$\therefore b = \frac{15}{12} d$$

$$= \frac{5}{4} d$$

$$KB = \tfrac{2}{3} d$$

$$\nabla = \tfrac{1}{2} L\, b\, d$$

$$I = \tfrac{1}{12} L\, b^3$$

(Note that b is the breadth at the waterline).

$$BM = \frac{I}{\nabla}$$

$$= \tfrac{1}{12} L\, b^3 \div \tfrac{1}{2} L\, b\, d$$

$$= \frac{b^2}{6d}$$

$$= \frac{1}{6d}\left(\frac{5}{4} d\right)^2$$

$$= \frac{25}{96} d$$

The vessel becomes unstable when G and M coincide.

Thus $\quad KM = KG$

$$= 6{\cdot}675 \text{ m}$$

$$6{\cdot}675 = \tfrac{2}{3} d + \tfrac{25}{96} d$$

$$= \frac{89}{96} d$$

$$d = 6{\cdot}675 \times \frac{96}{89}$$

Draught d = 7·2 m

METACENTRIC DIAGRAM

Since both KB and BM depend upon draught, their values for any ship may be calculated for a number of different draughts, and

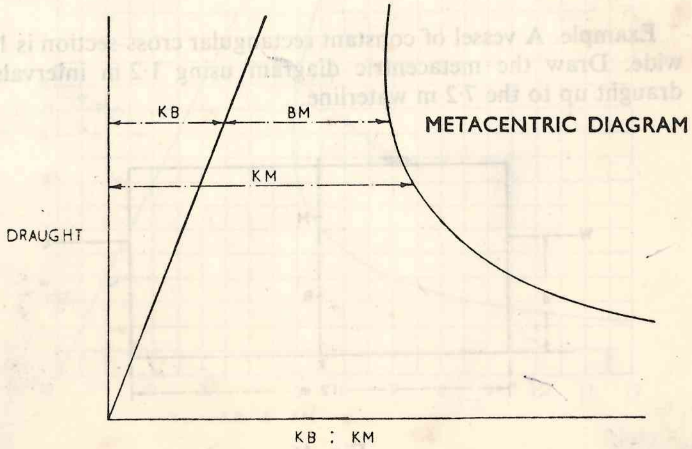
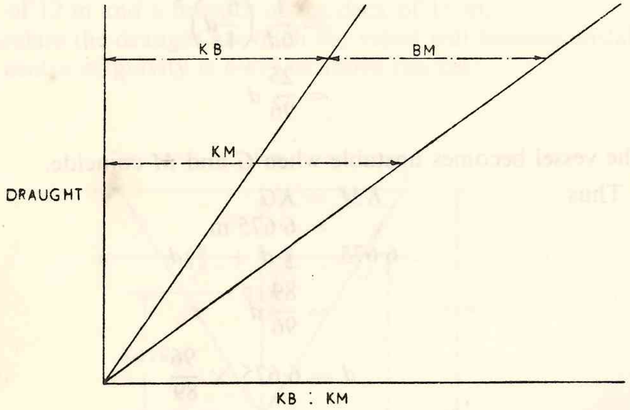


Fig. 44

plotted to form the *metacentric diagram* for the ship. The height of the transverse metacentre above the keel may then be found at any intermediate draught.

The metacentric diagram for a box barge is similar to that for a ship (Fig. 44), while the diagram for a vessel of constant triangular cross-section is formed by two straight lines starting from the origin (Fig. 45).



METACENTRIC DIAGRAM FOR VESSEL OF CONSTANT TRIANGULAR CROSS-SECTION

Fig 45

Example. A vessel of constant rectangular cross-section is 12 m wide. Draw the metacentric diagram using 1·2 m intervals of draught up to the 7·2 m waterline.

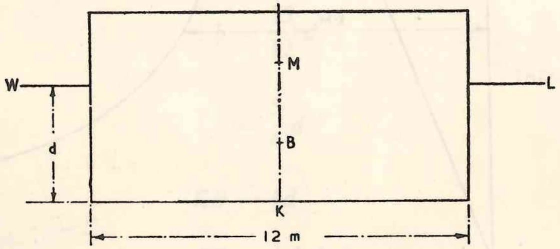


Fig. 46

It is useful in an example of this type to derive an expression for KB and KM in terms of the only variable-draught-and substitute the different draught values in tabular form.

$$KB = \frac{d}{2}$$

$$BM = \frac{B^2}{12d}$$

$$= \frac{12^2}{12d}$$

$$= \frac{12}{d}$$

$$KM = KB + BM$$

d	KB	BM	KM
0	0	∞	∞
1·2	0·6	10·00	10·60
2·4	1·2	5·00	6·20
3·6	1·8	3·33	5·13
4·8	2·4	2·50	4·90
6·0	3·0	2·00	5·00
7·2	3·6	1·67	5·27

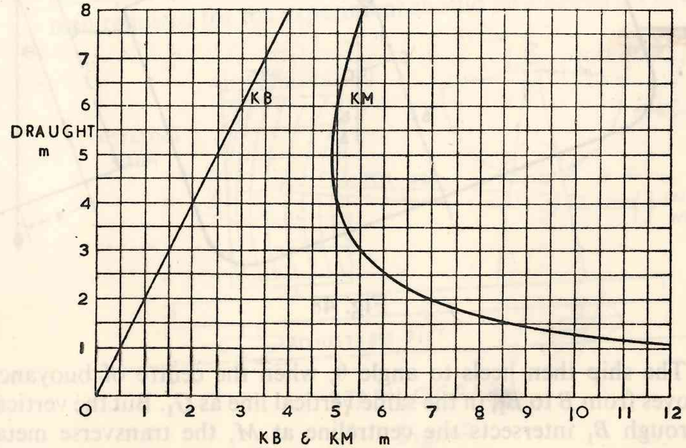


Fig. 47

INCLINING EXPERIMENT

This is a simple experiment which is carried out on the completed ship to determine the metacentric height, and hence the height of the centre of gravity of the ship. If the height of the centre of gravity of the empty ship is known, it is possible to calculate its position for any given condition of loading. It is therefore necessary to carry out the inclining experiment on the empty ship (or as near to empty as possible).

The experiment is commenced with the ship upright.

A small mass m is moved across the ship through a distance d. This causes the centre of gravity to move from its original position G on the centreline to G_1. (Fig. 48).

If $\triangle$ = displacement of ship

Then $$GG_1 = \frac{m \times d}{\triangle}$$

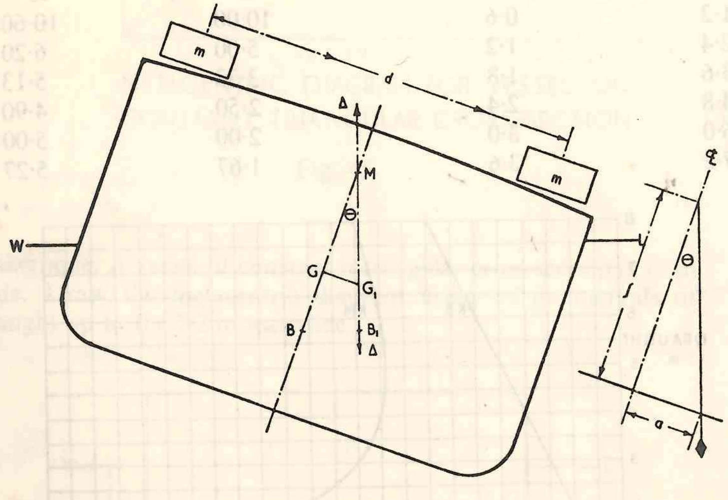


Fig. 48

The ship then heels to angle θ, when the centre of buoyancy moves from B to B_1, in the same vertical line as G_1. But the vertical through B_1 intersects the centreline at M, the transverse metacentre.

CONDUCT OF EXPERIMENT

The experiment must be carried out very carefully to ensure accurate results. At least two pendulums are used, one forward and one aft. They are made as long as possible and are suspended from some convenient point, e.g. the underside of the hatch. A stool is arranged in way of each pendulum on which the deflections are recorded. The pendulum bobs are immersed in water or light oil to dampen the swing.

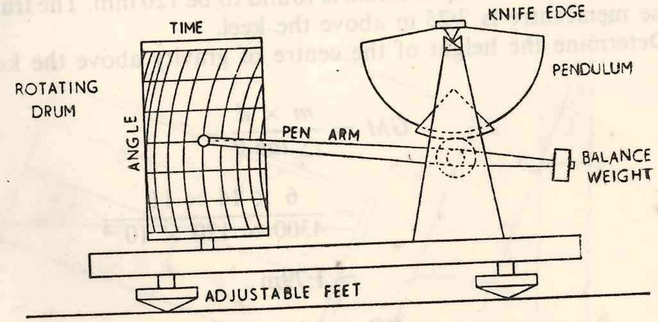
Four masses A, B, C and D are placed on the deck, two on each side of the ship near midships, their centres being as far as possible from the centreline.

The mooring ropes are slackened and the ship-to-shore gangway removed. The draughts and density of water are read as accurately as possible.

The inclining masses are then moved, one at a time, across the ship until all four are on one side, then all four on the other side and finally two on each side. The deflections of the pendulums are recorded for each movement of mass. An average of these deflections is used to determine the metacentric height. Thus if there are eight movements of mass, and the recorded deflections of pendulum are a_1, a_2, a_3, a_4............................a_8, then

$$\text{average deflection} = \frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8}{8}$$

The ship should be in a sheltered position, e.g. graving dock, and the experiment should be carried out in calm weather. Only those men required for the experiment should be allowed on board. Any



STABILOGRAPH

Fig. 49

$$GG_1 = GM \tan\theta$$

$$\therefore \quad GM \tan\theta = \frac{m \times d}{\triangle}$$

$$GM = \frac{m \times d}{\triangle \tan\theta}$$

To determine the angle of heel it is necessary to suspend a pendulum from, say, the underside of a hatch. The deflection a of the pendulum may be measured when the mass is moved across the deck.

Thus if $\quad l$ = length of pendulum

$$\tan\theta = \frac{a}{l}$$

$$\text{and} \quad GM = \frac{m \times d \times l}{\triangle \times a}$$

The height of the transverse metacentre above the keel may be found from the metacentric diagram and hence the height of the centre of gravity of the ship may be determined.

$$KG = KM - GM$$

Example. A mass of 6 tonne is moved transversely through a distance of 14 m on a ship of 4300 tonne displacement, when the deflection of an 11 m pendulum is found to be 120 mm. The transverse metacentre is 7·25 m above the keel.

Determine the height of the centre of gravity above the keel.

$$GM = \frac{m \times d}{\triangle \tan\theta}$$

$$= \frac{6 \times 14 \times 11}{4300 \times 120 \times 10^{-3}}$$

$$= 1{\cdot}79\text{m}$$

$$KG = KM - GM$$

$$= 7{\cdot}25 - 1{\cdot}79$$

$$= 5{\cdot}46 \text{ m}$$

movement of liquid affects the results and therefore all tanks should be empty or pressed up tight. The magnitude and position of any mass which is not included in the lightweight of the ship should be noted and it is therefore necessary to sound all tanks and inspect the whole ship. Corrections are made to the centre of gravity for any such masses.

An instrument for recording inclination is in use by many shipyards. It consists of a heavy metal pendulum balanced on knife edges, geared to a pen arm which records the angle of heel on a rotating drum. The advantages of using this instrument, known as a *Stabilograph*, are that a permanent record is obtained and the movement of the ship may be seen as the experiment is in progress. If, for instance, the mooring ropes are restricting the heel, the irregular movement will be seen on the drum.

FREE SURFACE EFFECT

When a tank on board a ship is not completely full of liquid, and the vessel heels, the liquid moves across the tank in the same direction as the heel. The centre of gravity of the ship moves away from the centreline, reducing the righting lever and increasing the angle of heel (Fig. 50).

The movement of the centre of gravity from G to G_1 has been caused by the transfer of a wedge of liquid across the tank. Thus if

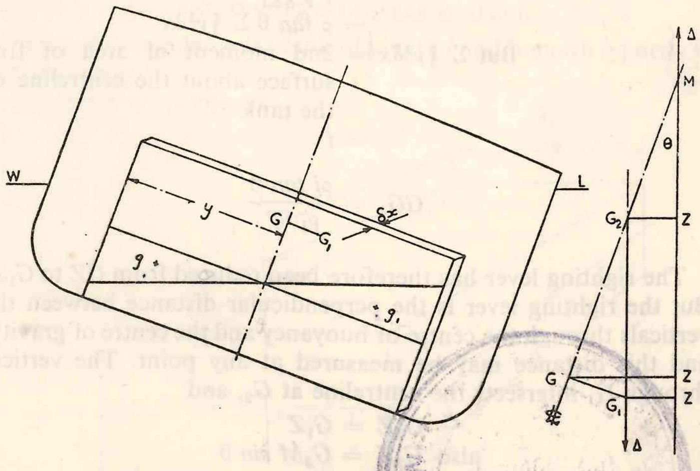


Fig. 50

m is the mass of the wedge and gg_1 the distance moved by its centre, then

$$GG_1 = \frac{m \times gg_1}{\triangle}$$

$$\begin{aligned} \text{But } m &= v \times \rho \\ \text{where } v &= \text{volume of wedge} \\ \rho &= \text{density of liquid} \\ \text{and } \triangle &= \nabla \times \rho_1 \\ \text{where } \nabla &= \text{volume of displacement} \\ \rho_1 &= \text{density of water} \end{aligned}$$

$$\therefore \quad GG_1 = \frac{v \times \rho \times gg_1}{\nabla \times \rho_1}$$

Divide the tank into thin, transverse strips of length δx and let one such strip have a half width of free surface of y

$$\begin{aligned} \text{Volume of strip of wedge} &= \tfrac{1}{2} y \times y \tan\theta\,\delta x \\ &= \tfrac{1}{2} y^2 \tan\theta\,\delta x \\ \text{Mass of strip of wedge} &= \rho \times \tfrac{1}{2} y^2 \tan\theta\,\delta x \end{aligned}$$

Moment of transfer of strip of wedge

$$\begin{aligned} &= \tfrac{4}{3} y \times \rho \times \tfrac{1}{2} y^2 \tan\theta\,\delta x \\ &= \rho \times \tfrac{2}{3} y^3 \tan\theta\,\delta x \end{aligned}$$

Total moment of transfer of wedge

$$\begin{aligned} &= v\,\rho\, gg_1 \\ &= \rho \tan\theta\, \Sigma\, \tfrac{2}{3} y^3 \delta x \end{aligned}$$

But $\Sigma\, \frac{2}{3} y^3 \delta x$ = 2nd moment of area of free surface about the centreline of the tank $= i$

$$\therefore GG_1 = \frac{\rho i \tan\theta}{\rho_1 \nabla}$$

The righting lever has therefore been reduced from GZ to G_1Z. But the righting lever is the perpendicular distance between the verticals through the centre of buoyancy and the centre of gravity, and this distance may be measured at any point. The vertical through G_1 intersects the centreline at G_2, and

$$G_2Z = G_1Z$$

$$\text{also } G_2Z = G_2M \sin\theta$$

but G_1Z does not equal $G_1M \sin\theta$

Since the initial stability of a ship is usually measured in terms of metacentric height, it is useful to assume that the effect of a free surface of liquid is to raise the centre of gravity from G to G_2, thus reducing the metacentric height of the vessel.

GG_2 is termed the *virtual reduction in metacentric height due to free surface* or, more commonly, *the free surface effect.*

$$\text{Now } GG_1 = GG_2 \tan\theta$$

$$\therefore GG_2 = \frac{\rho\, i \tan\theta}{\rho_1\, \nabla \tan\theta}$$

Free surface effect $\quad GG_2 = \dfrac{\rho\, i}{\rho_1 \nabla} \quad$ or $\quad = \dfrac{\rho\, i}{\Delta}$

Example. A ship of 5000 tonne displacement has a rectangular tank 6m long and 10 m wide. Calculate the virtual reduction in metacentric height if this tank is partly full of oil (rd 0·8).

$$\begin{aligned} \rho &= 1000 \times 0{\cdot}8 \ \text{kg/m}^3 \\ i &= \tfrac{1}{12}\, 6 \times 10^3 \ \text{m}^4 \\ \rho_1 &= 1025 \ \text{kg/m}^3 \\ \nabla &= \frac{5000}{1{\cdot}025} \ \text{m}^3 \end{aligned}$$

$$\begin{aligned} \therefore GG_2 &= \frac{1000 \times 0{\cdot}8 \times 6 \times 10^3 \times 1{\cdot}025}{1025 \times 5000 \times 12} \\ &= 0{\cdot}08 \text{ m} \end{aligned}$$

THE EFFECT OF TANK DIVISIONS ON FREE SURFACE

Consider a rectangular tank of length l and breadth b partly full of sea water.

(a) WITH NO DIVISIONS

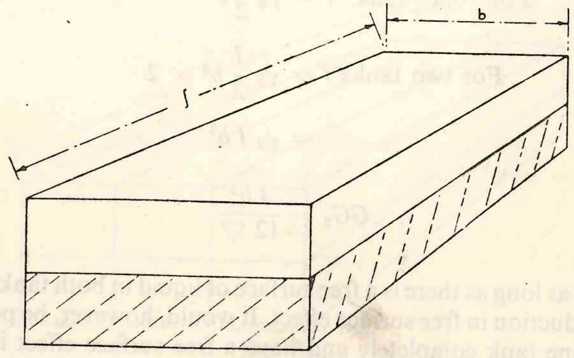


Fig. 51

$$GG_2 = \frac{\rho\, i}{\rho_1 \nabla}$$

$$= \frac{i}{\nabla} \text{ since } \rho = \rho_1$$

$$i = \tfrac{1}{12}\, l\, b^3$$

$$\therefore GG_2 = \frac{l\, b^3}{12\, \nabla}$$

(b) WITH A MID-LENGTH, TRANSVERSE DIVISION

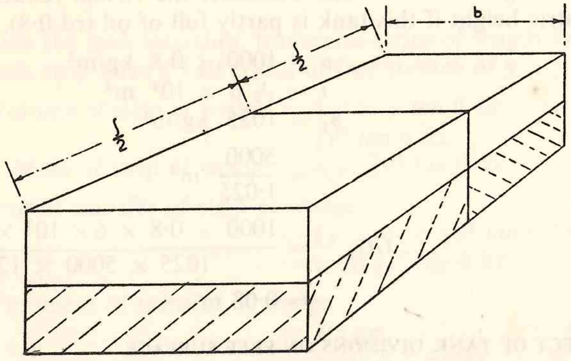


Fig. 52

$$\text{For one tank } i = \tfrac{1}{12}\, \frac{l}{2}\, b^3$$

$$\text{For two tanks } i = \tfrac{1}{12}\, \frac{l}{2}\, b^3 \times 2$$

$$= \tfrac{1}{12}\, l\, b^3$$

$$\therefore GG_2 = \frac{l\, b^3}{12\, \nabla}$$

Thus as long as there is a free surface of liquid in both tanks there is no reduction in free surface effect. It would, however, be possible to fill one tank completely and have a free surface effect in only one tank.

(c) WITH A LONGITUDINAL, CENTRELINE DIVISION

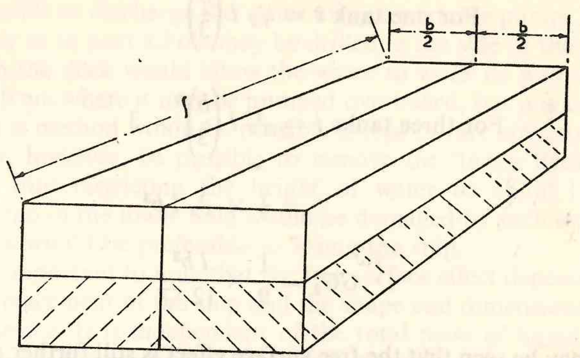


Fig. 53

$$\text{For one tank } i = \tfrac{1}{12}\, l \left(\frac{b}{2}\right)^3$$

$$\text{For two tanks } i = \tfrac{1}{12}\, l \left(\frac{b}{2}\right)^3 \times 2$$

$$= \tfrac{1}{4} \times \tfrac{1}{12}\, l\, b^3$$

$$\therefore GG_2 = \frac{1}{4} \times \frac{l\, b^3}{12\, \nabla}$$

Thus the free surface effect is reduced to one quarter of the original by introducing a longitudinal division.

(d) WITH TWO LONGITUDINAL DIVISIONS FORMING THREE EQUAL TANKS

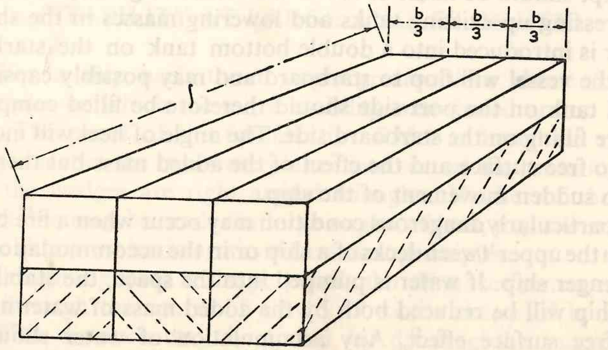


Fig. 54

$$\text{For one tank } i = \tfrac{1}{12}\, l \left(\frac{b}{3}\right)^3$$

$$\text{For three tanks } i = \tfrac{1}{12}\, l \left(\frac{b}{3}\right)^3 \times 3$$

$$= \tfrac{1}{9} \times \tfrac{1}{12}\, l\, b^3$$

$$\therefore GG_2 = \frac{1}{9} \times \frac{l\, b^3}{12\, \nabla}$$

It may be seen that the free surface effect is still further reduced by the introduction of longitudinal divisions.

If a tank is sub-divided by n longitudinal divisions forming *equal* tanks, then

$$GG_2 = \frac{1}{(n+1)^2}\, \frac{l\, b^3}{12\, \nabla}$$

PRACTICAL CONSIDERATIONS

The effect of a free surface of liquid may be most dangerous in a vessel with a small metacentric height and may even cause the vessel to become unstable. In such a ship, tanks which are required to carry liquid should be pressed up tight. If the ship is initially unstable and heeling to port, then any attempt to introduce water ballast will reduce the stability. Before ballasting, therefore, an attempt should be made to lower the centre of gravity of the ship by pressing up existing tanks and lowering masses in the ship. If water is introduced into a double bottom tank on the starboard side the vessel will flop to starboard and may possibly capsize. A small tank on the port side should therefore be filled completely before filling on the starboard side. The angle of heel will increase due to free surface and the effect of the added mass but there will be no sudden movement of the ship.

A particularly dangerous condition may occur when a fire breaks out in the upper 'tween decks of a ship or in the accommodation of a passenger ship. If water is pumped into the space, the stability of the ship will be reduced both by the added mass of water and by the free surface effect. Any accumulation of water should be avoided. Circumstances will dictate the method used to remove the

water, and will vary with the ship, cargo and position of fire. It may be possible to discharge the water using a portable pump. In calm weather or in port a hole may be drilled in the side of the ship. A hole in the deck would allow the water to work its way into the bilges from where it may be pumped overboard, but it is doubtful if such a method would be possible except in rare circumstances. It may, however, be possible to remove the 'tween deck hatch covers thus restricting the height of water to about 150 mm. The cargo in the lower hold would be damaged by such a method but this would be preferable to losing the ship.

It is important to note that the free surface effect depends upon the displacement of the ship and the shape and dimensions of the *free surface*. It is independent of the total mass of liquid in the tank and of the position of the tank in the ship.

The ship with the greatest free surface effect is, of course, the oil tanker, since space must be left in the tanks for expansion of oil. Originally tankers were built with centreline bulkhead and expansion trunks. Twin longitudinal bulkheads were then introduced without expansion trunks and were found to be successful, since the loss in metacentric height due to free surface was designed for. It is not possible to design dry cargo vessels in the same way, since the position of the centre of gravity of the ship varies considerably with the nature and disposition of the cargo. Thus while the free surface effect in a tanker is greater than in a dry cargo ship, it is of more importance in the latter.

The effect of a suspended mass on the stability of a ship may be treated in the same way as a free surface. It may be shown, as stated in Chapter 4, that the centre of gravity of the mass may be taken as acting at the point of suspension.

*f*STABILITY AT LARGE ANGLES OF HEEL

When a ship heels to an angle greater than about 10°, the principles on which the initial stability were based are no longer true. The proof of the formula for BM was based on the assumption that the two waterplanes intersect at the centreline and that the wedges are right angled triangles. Neither of these assumptions may be made for large angles of heel, and the stability of the ship must be determined from first principles.

The righting lever is the perpendicular distance from a vertical axis through the centre of gravity G to the centre of buoyancy B_1. This distance may be found by dividing the moment of buoyancy about this axis by the buoyancy. In practice recourse is made to an instrument known as an integrator which may be used to determine the area of any plane and the moment of the plane about a given axis. The method used is as follows.

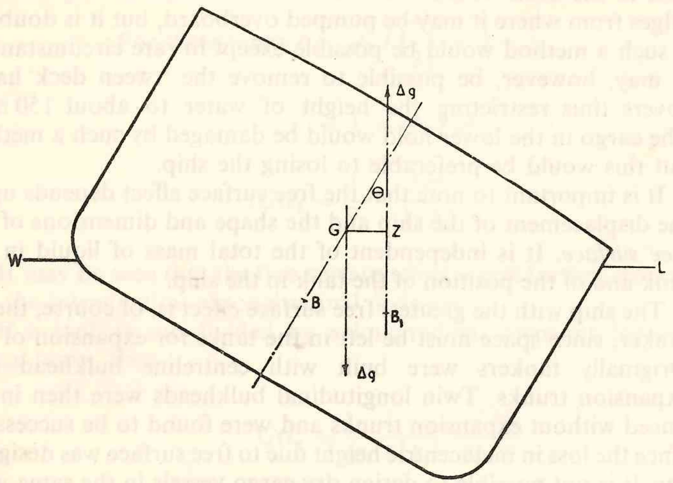
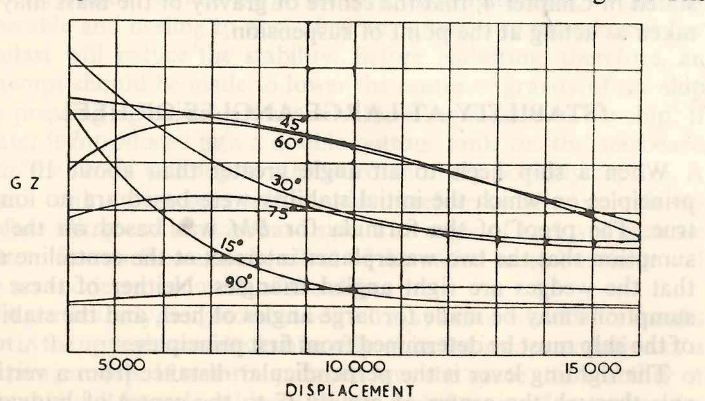


Fig. 55

The position of the centre of gravity G must be assumed at some convenient position above the keel, since the actual position is not known. Sections through the ship are drawn at intervals along the ship's length. These sections are inclined to an angle of, say 15°. The integrator is set with its axis in the vertical through G. The



CROSS CURVES OF STABILITY
Fig. 56

outline of each section is traced by the integrator up to a given waterline and the displacement and righting lever obtained. This is repeated for different waterlines and for angles of 30°, 45°, 60°, 75° and 90°. The GZ values at each angle are plotted on a base of displacement to form the *cross curves of stability* for the ship.

The displacement, height of centre of gravity and metacentric height of a vessel may be calculated for any loaded condition. At this displacement the righting levers may be obtained at the respective angles for the assumed position of the centre of gravity. These values must be amended to suit the actual height of the centre of gravity.

Let G = assumed position of centre of gravity
G_1 = actual position of centre of gravity

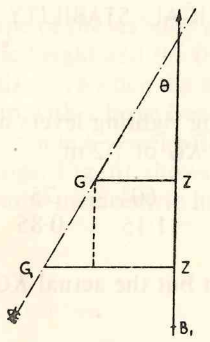


Fig. 57

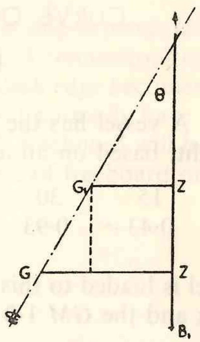


Fig. 58

If G_1 lies *below* G (Fig. 57), then the ship is *more* stable and

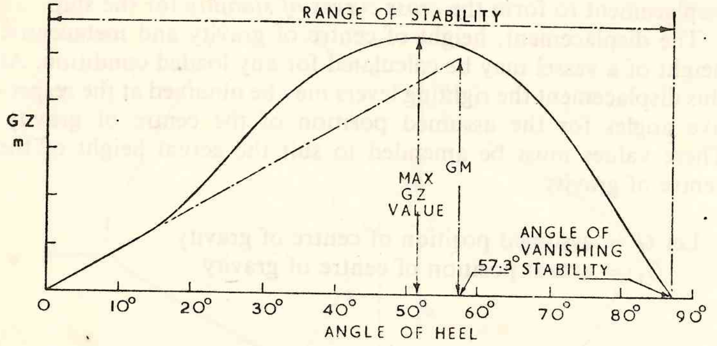
$$G_1Z = GZ + GG_1 \sin \theta$$

If G_1 lies *above* G (Fig. 58), then the ship is *less* stable and

$$G_1Z = GZ - GG_1 \sin \theta$$

The amended righting levers are plotted on a base of angle of heel to form the *Curve of Statical Stability* for the ship in this condition of loading. The initial slope of the curve lies along a line drawn from the origin to GM plotted vertically at one radian (57·3°).

The area under this curve to any given angle, multiplied by the gravitational weight of the ship, is the work done in heeling the ship to that angle and is known as the *Dynamical Stability*.



CURVE OF STATICAL STABILITY

Fig. 59

Example. A vessel has the following righting levers at a particular draught, based on an assumed KG of 7·2 m

θ	0°	15°	30°	45°	60°	75°	90°
GZ	0	0·43	0·93	1·21	1·15	0·85	0·42 m

The vessel is loaded to this draught but the actual KG is found to be 7·8 m and the GM 1·0 m.

Draw the amended statical stability curve.

$$GG_1 = 0{\cdot}6 \text{ m}$$
$$G_1Z = GZ - GG_1 \sin\theta$$

(i.e. the vessel is *less* stable than suggested by the original values).

Angle θ	sin θ	GG_1 *sin* θ	GZ	G_1Z
0	0	—	0	0
15°	0·259	0·15	0·43	0·28
30°	0·500	0·30	0·93	0·63
45°	0·707	0·42	1·21	0·79
60°	0·866	0·52	1·15	0·63
75°	0·966	0·58	0·85	0·27
90°	1·000	0·60	0·42	−0·18

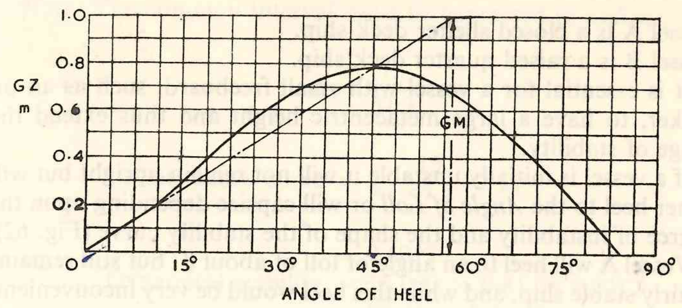


Fig. 60

The shape of the stability curve of a ship depends largely on the metacentric height and the freeboard. A tremendous change takes place in this curve when the weather deck edge becomes immersed. Thus a ship with a large freeboard will normally have large range of stability while a vessel with a small freeboard will have a much smaller range. Fig. 61 shows the effect of freeboard on two ships with the same metacentric height.

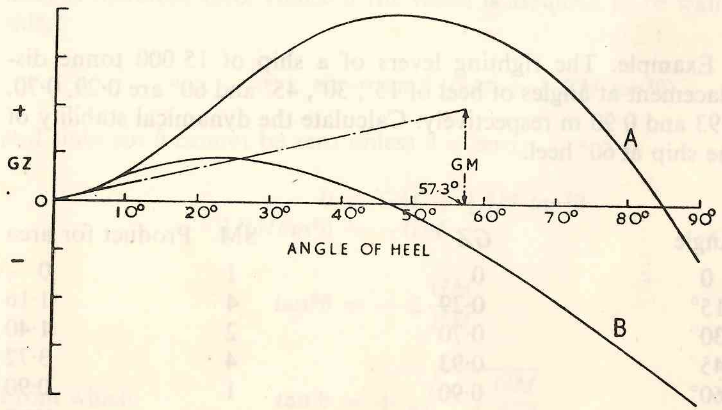


Fig. 61