1

EARTH

1.1 SHAPE OF THE EARTH :

The Earth is not a true sphere. Its shape is that of an oblate spheroid, the equatorial diameter being more than the polar diameter. The equatorial diameter is 7926.7 statute miles while the polar diameter is 7899.5 statute miles. In kilometers the equatorial radius is 6378.16 km and the polar radius is 6356.77 km. The difference of about 27 miles between these diameters as compared to the average diameter of 7913 miles is so small that the Earth may be considered a true sphere for most purposes.

Axis The axis of the Earth is the diameter about which it rotates.

Poles The geographic poles of the Earth are the two points where the axis meets the Earth's surface.

The Earth rotates about its axis once each day. This rotation carries each point on the Earth's surface towards East. West is the direction 180° from East, North is the direction 90° to the left of East, and South the direction 90° to the right of East. The two poles of the Earth are designated North Pole and South Pole, accordingly.

A Great Circle is a circle on the surface of a sphere, the plane of which passes through the centre of the sphere.

There is only one great circle through any two points on the sphere's surface, except if the points are at the two ends of a diameter when an infinite number of great circles are possible.

A Small Circle is a circle on the surface of a sphere, the plane of which does not pass through the centre of the sphere.

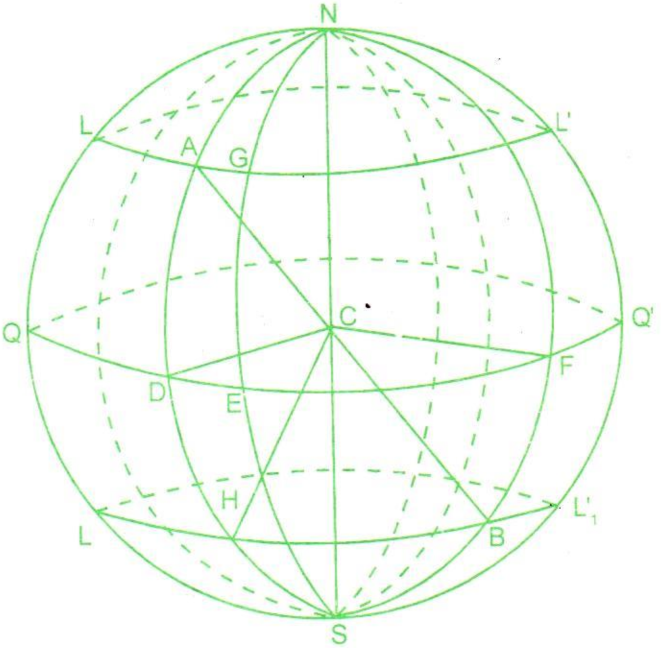
Equator The Equator is a great circle on the surface of the Earth, the plane of which is perpendicular to the Earth's axis. The Equator divides the Earth into the north and the south hemispheres. Latitudes are measured North or South from the Equator.

Parallels of Latitude Parallels of Latitude are small circles on the Earth's surface, the planes of which are parallel to the plane of the Equator. All parallels therefore run East-West.

Meridians Meridians are semi-great circles on the Earth, joining the two poles. The other half of the same great circle forms yet another meridian.

All meridians intersect the Equator and parallels of latitude at 90°. Since the meridians join the poles, all meridians run North-South.

Prime Meridian is the meridian which passes through Greenwich. The other meridians are named East or West from the Prime meridian.



(FIG.1.1)

In Fig. 1.1. QDQ' is a great circle as its plane passes through C, the centre of the sphere.
LGL' is a small circle
N & S are the North Pole and South Pole respectively.
NCS the Earth's axis
QQ' the Equator
LL' are parallels of latitudes
NDS, NES and NFS are meridians
NGS the Prime meridian (through Greenwich)
Latitude of A = arc AD or angle ACD (The lat. is North)
Longitude of A = arc ED or angle GNA (the long. is West)
Latitude of B = arc FB or angle FCB (the lat. is South)
Longitude of B = arc EF or angle ENF (the long. is East)

d'lat from A to B = arc AH or angle ACH (the d'lat is South)
d'long from A to B = arc DF or angle ANB (the d'long is East)

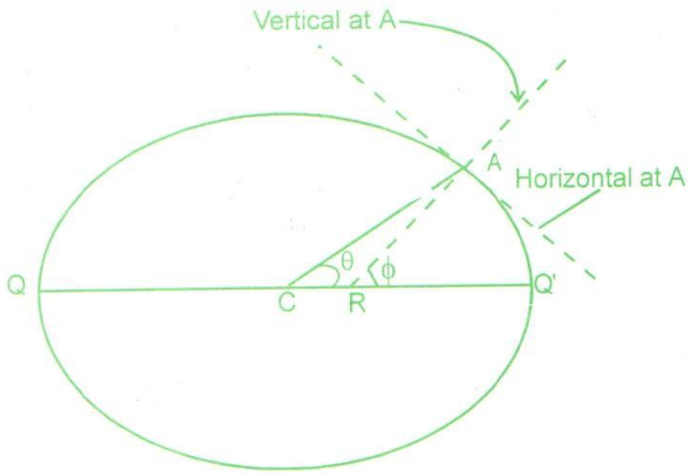
Geocentric Latitude of a place

is the arc of a meridian or the angle at the centre of the Earth con tained between the Equator, and the parallel of latitude through that place. Latitudes are measured from 0° to 90°, and named North or South according to the place being North or South of the Equator.

Geographic Latitude of a place

is the angle between the plane of the Equator and the vertical at that place. In navigation, the term latitude implies, the latitude as observed, that is the geographic latitude.

The Geographic latitude differs from the Geocentric latitude as the Earth is not a true sphere. The difference between them is nil at the Equator and at the poles. They differ by a maximum of about 11.6' at 45°N and 45°S. The geocentric latitude is approximately equal to :- Geographic latitude - (11.6 x sin 2 geographic latitude).



(FIG.1.2)

QQ' the plane of the Equator
- Geocentric latitude of A
- Geographic latitude of A

Longitude of a place

is the arc of the Equator or the angle at the poles contained between the Prime meridian and the meridian through that place. Longitudes are measured from 0° to 180°, and named East or West according to the place being East or West of the Prime meridian.

Any position on the Earth is established, if its latitude and longitude are defined.

Difference in Latitudes (d'lat)

The d'lat between two places is the arc of a meridian or angle at the centre of Earth contained between the parallels of latitude through the two places.

D'lat is named North or South according to the direction from the first place to the second e.g. d'lat from 30°N to 20°N is 10°S and d'lat from 10°S to 15°N is 25°N.

Difference in longitude (d'long)

The d'long between two places is the shorter arc of the Equator or the smaller angle at the poles contained between the meridians through the two places. D'long is named East or West according to the direction from the first place to the second place. The following examples will make it more clear.

	d'long from	070°E	to	110°E	=	40°E
	d'long from	090°W	to	040°W	=	50°E
	d'long from	020°E	to	030°W	=	50°W
*	d'long from	160°W	to	170°E	=	30°W
*	d'long from	155°E	to	070°W	=	135°E

* The shorter arc crosses the 180th Meridian and therefore the d'long is named in the direction of the 180th meridian from the first place.

Mean Latitude

The Mean latitude between two latitudes is the arithmetic mean between them.

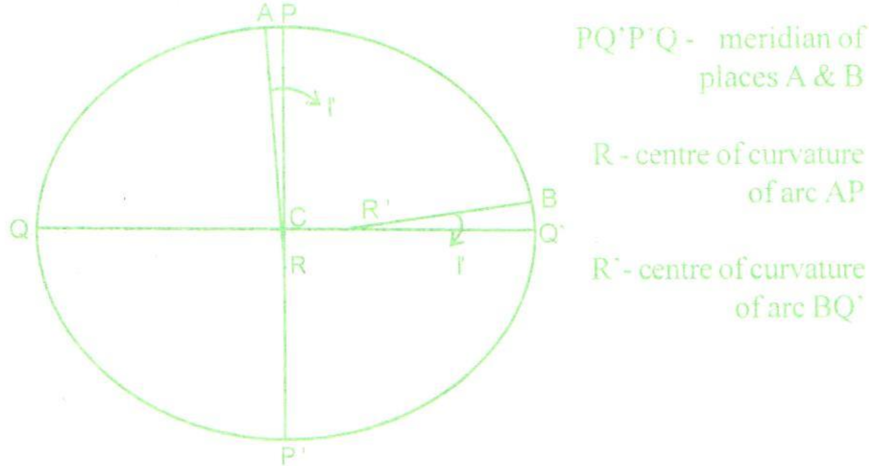
1.2 DISTANCE & DIRECTIONS

1.2.1 Distance

Various units are used for measuring distances on the Earth.

Nautical mile

The nautical mile at any place is the length of the arc of a meridian subtending an angle of 1' at the centre of curvature of that place. It may also be defined as the length of a meridian between two Geographic latitudes which differ by 1', that is 1' of d'lat.



(FIG.1.3)

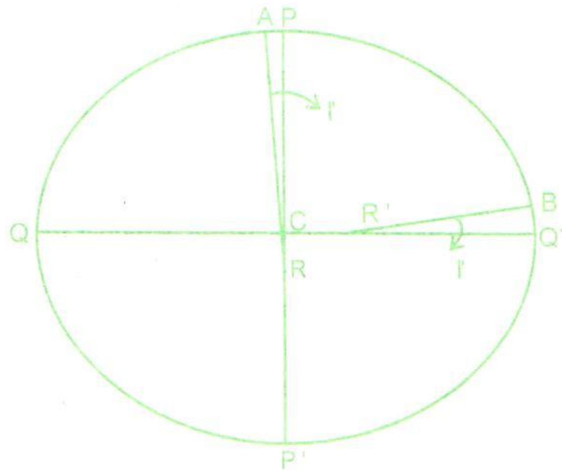
Since RA is greater than R'B, AP the nautical mile near the pole is also greater than BQ', the nautical mile near the Equator. The length of the nautical mile varies with the latitude, due to the varying curvature of the Earth's surface.

At the poles where the curvature is least, the nautical mile measures 1861.7m; (6107.8ft.) while at the Equator, where the curvature is largest, the nautical mile measures 1842.9m; (6046.4ft.). This is so because the Earth being flattened at the poles and bulged at the Equator, the centre of curvature of the polar region will be further away from the Earth's surface than the centre of curvature of the equatorial region. The arc subtended by the same angle of 1' would therefore be larger at the Poles and smaller at the Equator. The small variation in the length of the nautical mile has no significance in practical navigation as the distance in nautical miles between two places on the same meridian is the d'lat between them in minutes; and the two units vary together.

For certain purposes, a standard unit is necessary. Therefore a mean length of 1852.3m (6080ft.) is adopted as the standard nautical mile. **The length of the Nautical Mile in latitude ϕ is obtained as $1852.3 - 9.4 \cos 2\phi$.**

Knot is a unit of speed equal to one nautical mile per hour.

Geographical mile is the length of the arc of the Equator subtending an angle of 1' at the centre of the Earth. It is constant in length, equal to 1855.3m (6087.2ft.).



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Statute mile or land mile is an arbitrary measure of length equal to 5280ft.

Kilometer is the approximate length of 1 / 10,000 part of a meridian between the Equator and the pole. (90° x 60 = 5400' x 1.8523 = 10,002.43 km)

1.2.2 Directions

Directions are measured as angles in degrees and minutes with reference to the Geographic North, which is indicated by all meridians. The angle is measured clock-wise from North in 360° notation. In the quadrantal system, the angles are measured from North to East or West and from South to East or West. Thus 160° in the 360° notation would be S20°E in the quadrantal system.

True course is the angle at the ship between True North and the ship's head, that is, the angle between the true meridian and the ship's fore and aft line.

True Bearing The true bearing of an object is the angle at the observer between True North indicated by the meridian and the line joining the observer and the object.

Magnetic meridians & variation Magnetic meridians are lines joining the magnetic poles of the Earth. Since these poles are not in the same position as the geographic poles, there is an angle between the magnetic and the geographic meridians. The angle between them is known as the variation. Variation is different at different places. It is termed East, if the Magnetic North lies to the East or right of the True North and West if the Magnetic North lies to the West or left of the True North. The value of the variation at a place is not constant. It changes because the position of the magnetic poles of the Earth is constantly changing. This change is called the secular change in variation. The variation and the amount of yearly change in it are indicated on the compass roses on the charts. The value of the variation at any place may also be obtained from the variation chart of the World.

A magnetic compass undisturbed by any other magnetic field will point towards the Magnetic North. In a ship made of steel, the magnetism of the ship's structure also creates a further magnetic field at the compass position. This deviates the compass from the direction of Magnetic North.

Deviation is the angle between the magnetic meridian and the North-South line of the compass card. Deviation is termed Easterly if the compass North lies to the East or right of the Magnetic North and Westerly if the compass North lies to the West or left of Magnetic North. The deviation of a compass varies as the ship's head changes.

It should be noted that for the same ship's head, the deviation remains the same for all bearings, as deviation depends on the ship's head and not on the bearings.

Compass error

The compass error is the algebraic sum of the deviation and the variation. Deviation, variation and the error are to be applied as follows to courses and bearings.

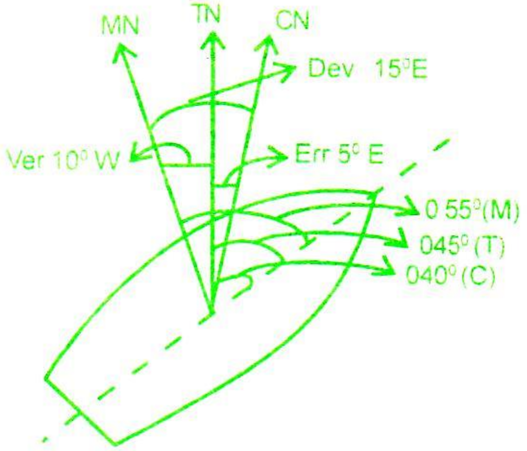
True	(-)E (+)W Variation	=	Magnetic	(-)E (+)W Deviation	=	Compass
Compass	(+)E (-)W Deviation	=	Magnetic	(+)E (-)W Variation	=	True
True	(-)E (+)W Error	=	Compass			
Compass	(+)E (-)W Error	=	True			

The above rule can be understood better by drawing appropriate figures in each case.

Example

Find the true course for a compass course of 040°, Deviation 15°E, Variation 10°W.

Dev.	15°E	Comp.Co.	040°(C)
Var.	10°W	Dev.	15°(E)
Error	5°E	Mag.Co.	055°(M)
Comp. Co.	040°(C)	Var.	10°(W)
True Co.	045°(T)	True Co.	045°(T)



(FIG.1.4)

As an exercise in the application of the above, the following table should be completed.

True Course	Variation	Magnetic Course	Deviation	Compass Course	Error
097°	5°E	-	3°W	-	-
-	13°W	164°	6°W	-	-
273°	-	280°	-	282°	-
-	7°W	-	6°E	168°	-
343°	-	338°	5°W	-	-

EXERCISE 1

1. Find the d'lat and d'long between the following positions :
   a) From 30°10.0'N 019°25.2'W to 37°15.7'N 020°04.2'W
   b) From 08°12.6'N 015°03.8'E to 02°08.0'S 017°18.6'W
   c) From 11°11.6'N 178°32.0'E to 15°14.0'S 176°00.2'W
   d) From 08°14.2'S 160°40.0'W to 03°53.8'S 130°27.2'E

2. Find the mean latitude between the following latitudes :
   a) 10°12.0'N and 46°36.0'N
   b) 12°04.0'N and 23°08.0'S

3. Given initial position 12°49.5'S 176°48.7'E, d'lat 30°12.0'N, d'long 12°36.5'E. Find the final position.

4. Given initial position 15°30.6'N 008°20.8'W,d'lat 02°56.8'N,d'long 32°11.6'E. Find the final position.

5. If the vessel's arrival position was 29°10.0'S, 003°28.3'E and she had made good a d'lat of 62°16.3'S and d'long of 29°52'E, what was the initial position ?

6. Given Compass error 3°E, Variation 7°E, find the Deviation.

7. Given Compass error 6°W, Deviation 2°E, find the Variation.

THEORY QUESTIONS

1. Define : (a) Nautical mile
   (b) Geographical mile
   (c) Statute mile,

Explain clearly why the length of the nautical mile varies.

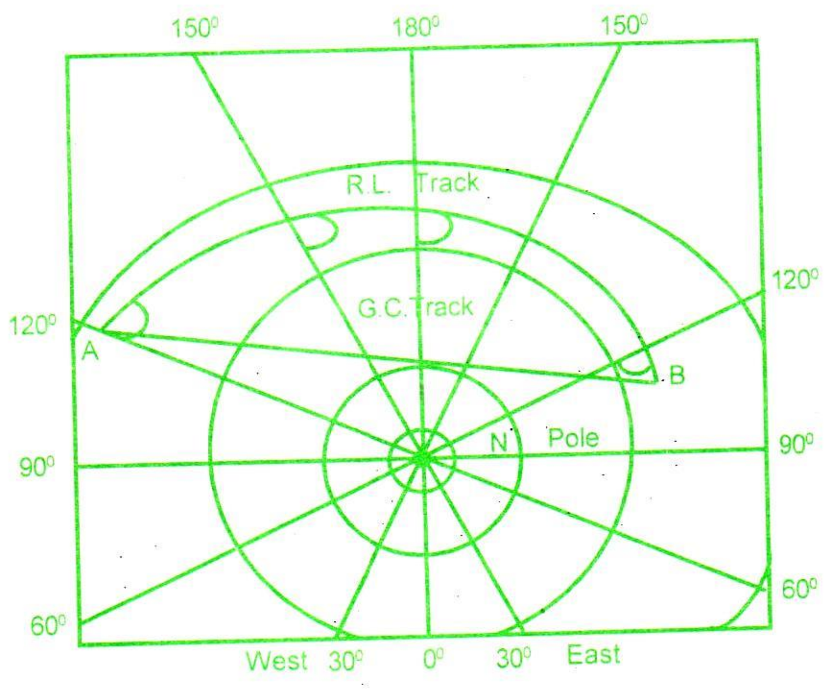
2. Define Variation and Deviation. Is the Variation at a place constant ? Why ?

3. Define : (a) Equator
   (b) D'long
   (c) Latitude.

4. Show by drawing a suitable figure, the difference between "Geocentric latitude" and "Geographic latitude".

2
PARALLEL & PLANE SAILING

Sailing between two positions on the Earth's surface involves calculating the course and distance between them. The shortest distance between any two points on the Earth is the shorter arc of the great circle through those points. It can be seen from Figure 2.1, that the great circle track crosses the various meridians at differing angles. Thus a ship following a great circle track would have to continually alter her course, throughout the passage. Therefore in navigating from one place to another, the usual method is to sail along a rhumb line track.

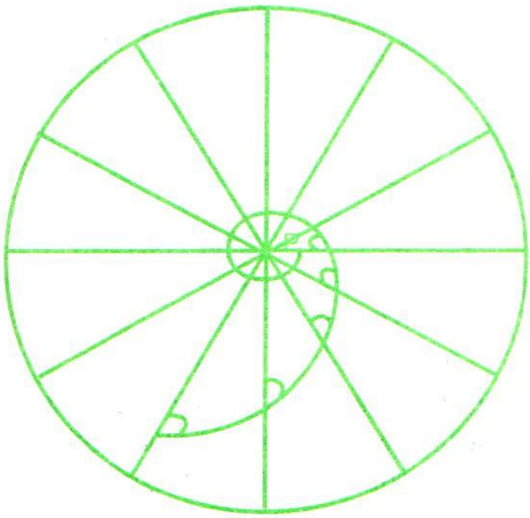


(FIG.2.1)

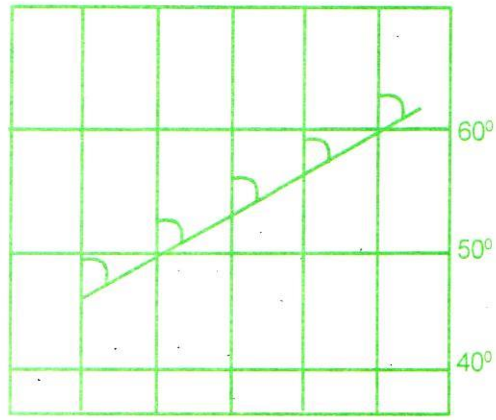
Rhumb line

A Rhumb line or Loxodrome is a line on the Earth's surface, crossing all meridians at the same angle. It can thus be seen that the rhumb line is the most convenient track to follow as the course of the ship remains constant for the entire passage.

The Equator, all parallels of latitude and meridians are particular cases of rhumb lines, as the course along the first two is always 090° or 270° and the course along any meridian is always 000° or 180°. On the surface of the Earth, all other rhumb lines will be curves spiralling towards the pole of the hemisphere. This is so because on the Earth the meridians converge towards the poles. (Fig.2.2a)



(FIG.2.2a)



(FIG.2.2b)

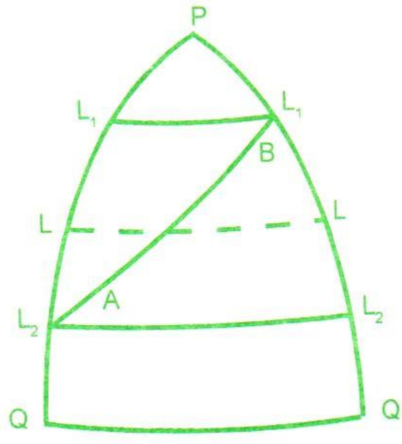
On a Mercator chart however, a rhumb line appears as a straight line, as the meridians on a Mercator Chart are represented as straight lines, parallel to each other. (Fig.2.2b)

Departure

The departure between two places is the east-west distance between them in nautical miles.

When the two places are on the same latitude the departure is the distance between them along their parallel of latitude. This fact is used in parallel sailing problems.

When the two places (A and B in Fig.2.3) are in different latitudes the departure between them will be smaller than the distance L_2L_2 and greater than the distance L_1L_1. When the latitudes of the two places are fairly close to each other, the departure between them may, for practical purposes, be considered equal to the east-west distance between the two meridians measured along the mean latitude LL (Fig.2.3). This concept is used without appreciable loss of accuracy in mean latitude sailing problems.



(FIG.2.3)

When the latitudes of the two places are widely separated, the above assumption would be incorrect. The true departure between the two places then, will be the east-west distance between the meridians, measured along the "middle latitude" between them.

Middle Latitude

The middle latitude between two places is the latitude in which the true departure lies, when sailing between them. It may also be defined as the latitude whose secant is the d'long in minutes divided by the departure in nautical miles between the two places. (Relationship proved later).

To convert mean latitude to middle latitude, some nautical tables provide a table of difference between the mean and middle latitude, as a function of the mean latitude and the d'lat between them.

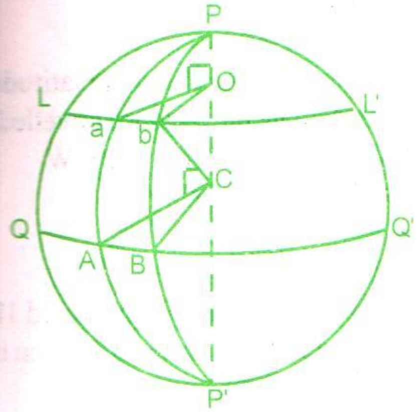
"Middle latitude sailing" is based on this concept. The use of middle latitude sailing for the purposes of practical navigation is now generally discouraged.

2.1 PARALLEL SAILING

When the starting and destination positions are on the same latitude, the ship could sail along a rhumb line, due East or West. Her track would therefore lie along the parallel of latitude of the two places. Sailing in this manner is therefore called parallel sailing. Since the distance travelled is due East or West, it is equal to the departure between the two positions.

A very important relationship exists between departure and d'long in such cases.

Proof of the parallel sailing formula



P & P' represent the poles of the Earth

PP' the Earth's axis

PAP' and PBP' are two meridians

C centre of the Earth

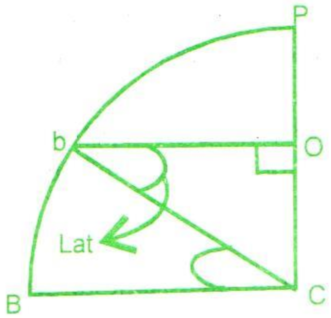
O centre of the circle of latitude LL'

QQ' the Equator.

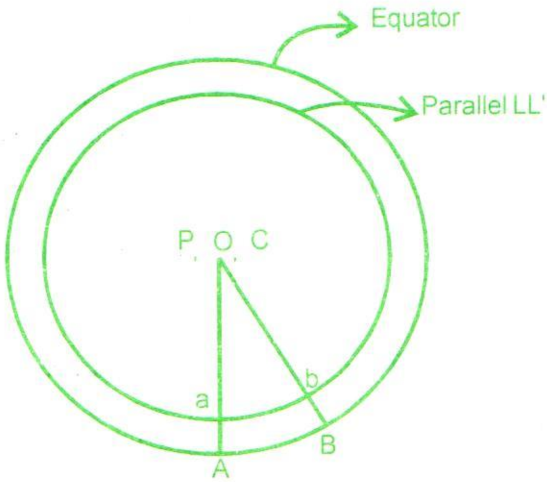
CA = CB = Cb as they are radii of Earth

a & b two places on the latitude LL'

(FIG.2.4)



(FIG.2.4a)



(FIG.2.4b)

Arc ab is the departure between the two places and arc AB on the Equator is their d'long.

Arc ab / Arc AB = dep. / d'lomg = radius Ob / radius CB , as arcs are proportional to radius in concentric circles, as shown in fig. 2.4 (b)
Since Cb = CB (both radii of the Earth), we have,
dep. / d'long = radius Ob / radius Cb
Since the triangle ObC is a plane triangle, right angled at O.
Ob / Cb = sin OCb = sin(90-lat) = cos lat.
dep. / d'long = cos lat.

Example 1

A vessel in lat. 47°S long. 054°W steers a course of 270°(T) for a distance of 412 miles. Find the position arrived.

dep. / d'long	=	cos lat or d'long = dep.x sec lat = 412. sec 47°
	=	604.1'W = 10°04.1'W

Long arrived = 54°W + 10°04.1'W = 064°04.1'W
Position arrived = 47°S; 064°04.1'W

Example 2

A vessel in latitude 37°12'N, proceeds along the same latitude from longitude 013°04'E to 005°37'W, calculate the distance travelled.
d'long made good = 13°04' + 05°37' = 18°41'W = 1121'W
dep. = d'long cos lat = 1121. cos 37°12' = 892.9 M
Distance travelled = 892.9 miles.

Example 3

Two vessels on the Equator, were 60 miles apart. Both steered 180°(T) until they reached latitude 30°S. Find the distance between them on latitude 30°S.
We know that dep. / d'long = cos lat.
Since the vessels are in 0° latitude, dep. / d'long = cos 0° = 1

Therefore departure (the distance between them) is equal to the d'long between them. Thus d'long = 60'.

As both ships have steered 180°(T), i.e. along their respective meridians, the d'long between them remains the same on reaching latitude 30°S.

Since the two vessels are on the same latitude, the departure i.e. the east-west distance between them equals d'long. cos lat. = 60.cos 30° = 51.96 miles.

EXERCISE II

1. Find the d'long for 200 miles of departure in latitude 60°N

2. Two ships on the Equator are 60 miles apart. Both steer 180°(T) at equal speeds. How many miles would each have to proceed till they are 40 miles apart ?
   Hint The number of minutes of d'lat = the distance in miles steamed South.

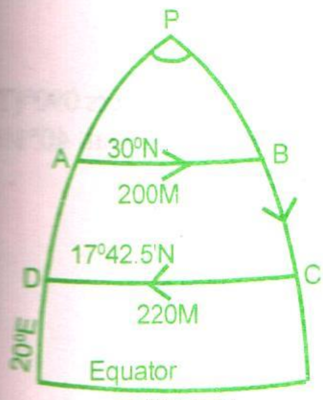
3. Two vessels in the same latitude and 300 miles apart, steer 000°(T) at the same speed. On reaching latitude 40°N, their d'long is found to be 5°30'. What distance did they cover ?

4. Two aircrafts in lat. 60°N, long. 090°W depart at the same time, one flying East and the other West at 500 knots. In what longitude will they meet, if there is a 30 knot Easterly wind ?

5. In what latitude will the number of miles of departure equal half the number of minutes of d'long ?

HARDER PROBLEMS

1. A ship in position 30°N 020°E, steers a course 090°(T) at 10 knots for 20 hours. She then alters course 90° to starboard and covers a certain distance. Thereafter the course is altered a further 90° to starboard. She sails on this course for 22 hours and arrives in longitude 020°E. Find the distance covered by her whilst heading South.



(FIG.2.5)

In lat. 30°N dep. made good eastwards = 20 x 10 = 200 miles
d'long made good = dep.x sec lat = 200 sec 30°= 230.9' From the figure, it is clear that the d'long for CD = d'long for AB = 230.9'.
At CD, cos lat = dep. / d'long = 220 / 230.9
Lat of CD = 17°42.5'N
The distance covered South (BC in figure) is equal to d'lat in minutes.
Distance the V/L covered while heading South
= 30°N - 17°42.5'N
= 12°17.5' = 737.5 M

2. Find the difference in speed at which two places, one in lat.22°S and the other in lat.43°N are carried round by the Earth's rotation.

A place on the Equator is carried round by the Earth's rotation at
360° x 60' / 24 = 900'/hour
d'long at the Equator = 900'/hour

Since dep. = d'long. cos lat.; a place in Lat 22°S will be carried round at 900.cos 22° = 834.46 M/hour

Similarly a place in latitude 43°N will be carried round at 900. cos 43° = 658.21 M/hour.

Difference in speeds between them = 834.46 - 658.21 = 176.25 miles/hour.

3. Ship A in lat 42°S, steers due West at 20 knots. Ship B in lat 30°S, also steers due West. They commenced from the same longitude. If after 24 hours, they remained due North and South of each other, calculate B's speed.

Distance covered by A, in 24 hours on a course
270°(T) = 24 x 20 = 480 M = departure

The d'long made by A = dep.x sec lat = 480. sec 42° = 645.9'
The d'long made by B = d'long made by A = 645.9'
departure made by B = d'long. cos lat. = 645.9.
cos 30°

= 559.4 M
Speed of B = 559.4 / 24 = 23.31 knots.

4. A vessel in position, lat. 40°10'N, long. 25°10'E, steers 090°(T) at 15 kts. After 8 hours, her position was found to be lat. 40°10'N, long. 28°E. Find the set and drift of current.

departure = 15 Kts. x 8 hours =		120 M
d'long for 120 M of dep.	=	dep. x sec lat.
	=	120. sec 40°10'
	=	157.03'
	=	2°37.03'
Long left 25°16'E; d'long	=	2°37.30'E
DR long arrived	=	27°47.03'E
Obs long	=	28°00.00'E
d'long due to current	=	0°12.97'E
dep. for d'long of 12.97	=	12.97 cos 40°10' = 9.91 M.

Since DR and observed positions are on the same latitude, the set is East and the drift 9.91 M.

5. A Ship 'X' on the Equator is steering a course of 270° (T) at 20 kts, while ship 'Y' on a certain south parallel of latitude is steering a course of 090°(T) at 15 kts. When Ship X makes a d'long of 80', ship Y makes a d'long of 75'. Calculate the latitude of ship Y.

d'long of ship X = 80' on the Equator = dist covered.

Time taken = 80 / 20 = 4 hours

In 4 hours, dist. covered by Y = 4 x 15 = 60 M = departure she makes. d'long made by Y in the same period = 75'

cos lat = dep. / d'long = 60 / 75 = 0.8
Latitude of Y = 36°52.2'S.

6. A ship on the Equator, steers 270°(T) at 18 kts. Another ship in a south latitude steers 090°(T) at 15 kts. While the first ship makes a d'long of 1°40', the second ship makes a d'long of 2°. Find the latitude of the second ship.

To make a d'long of 1°40', the first ship will take 100 / 18 hours.

The distance covered by the second ship in the same interval
100 / 18 x 15 = 83.33 miles = her dep.
cos lat of second ship = dep. / d'long = 83.33 / 120 = 0.6944
Latitude of second ship = 46°01'S.

7. Two ships X and Y depart from the same meridian and steer 090°(T). X is on the Equator and Y in a north latitude. X proceeds at 1¼ times the speed of Y. Find Y's latitude, if she remains true North of X throughout.

   Since Y remains North of X throughout, both X and Y make the same d'long in equal periods.
   Let the speed of Y = a kts.
   then speed of X = 5/4 a kts.
   Distance covered by Y in one hour = a = departure
   Distance covered by X in one hour 5/4 a = d'long of Y

   dep. / d'long = cos lat = a / 5a / 4 = 4 / 5 = 0.8

   Latitude of Y = 36°52'N

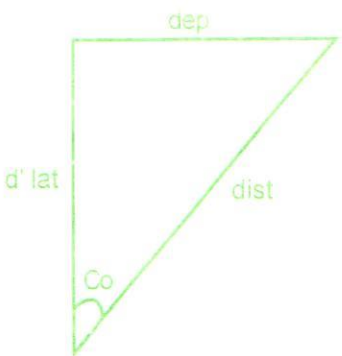
THEORY QUESTIONS

With the help of a figure, establish the relationship between departure, d'long and latitude.

2.2 PLANE SAILING

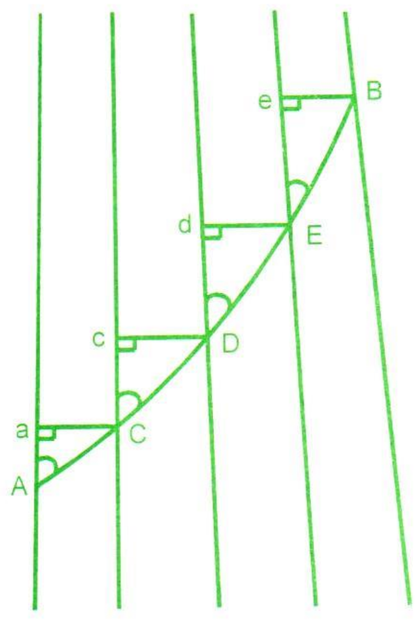
Plane sailing is sailing along a rhumb line from one position to another, which are not situated on the same latitude.

When the vessel sails along any rhumb line, except a meridian or a parallel of latitude; as an artifice, the d'lat, departure and distance may be considered as the three sides of a plane right angled triangle. The angle opposite the side, which represents the departure would then represent the course.



(FIG.2.6)

AB represents the rhumb line track from A to B (Fig. 2.7). The rhumb line AB is divided into a large number of very small equal parts, AC, CD etc. Ca, Dc etc. are arcs of parallels of latitude through C, D etc. respectively. Since the sections are very small, the triangles AaC, CcD etc. may be considered to be right angled plane triangles. It should be understood that the Earth's surface is not being considered as a plane surface. It is the very small areas covered by each triangle which are being considered as flat surfaces.



(FIG.2.7)

The course angles at A,C,D etc. are all equal because AB is a rhumb line.
In sailing from A to B

Sections Aa, Cc, Dd etc. are sections of d'lat.

Sections aC, cD, dE etc. are sections of dep.

and Sections AC, CD, DE etc. are sections of distance.

aC = AC sin course, cD = CD sin course, dE = DE sin course etc. Adding, aC + cD + dE etc. = (AC + CD + DE etc.) sin co.

Thus dep. = distance. sin course. Similarly, it can be shown that d'lat = distance cosine course. From the above formulae it can be seen that :
dep. / d'lat = tan course AND Distance = d'lat.sec course

In sailing between two positions, it must be understood that the departure is made good in every latitude through which the ship sails. Thus the departure to be used in the above formulae is the **true** departure between the places and not the departure at the latitude left or at the latitude arrived.

Therefore if the **true** departure is used, the above relationships hold good for **all distances and courses.** If the departure used is that at the latitude left or at the latitude reached, inaccuracies will result. The inaccuracies will be least when

(a) the distances are small,
(b) sailing near the Equator and
(c) sailing nearly North or South.

However any result obtained by calculations involving the use of d'lat, dist. or course (but not dep.) will always be accurate, for all distances and courses. It can be seen that the plane sailing formulae connect **dep., d'lat, dist** and **course** only. It does **not** involve **d'long**. Thus, knowing only the d'lat and d'long between two places, the course or distance between them cannot be found by the above formulae.

In practical navigation problems, the course is initially found by Mercator sailing or Middle latitude sailing formulae (explained later) and thereafter the distance obtained by using the Plane sailing formula, Distance = d'lat. sec co.

However academic problems based on the Plane sailing formulae are necessary to understand the principles involved. Using the plane sailing formulae, the following exercises should be worked out.

EXERCISE II (A)

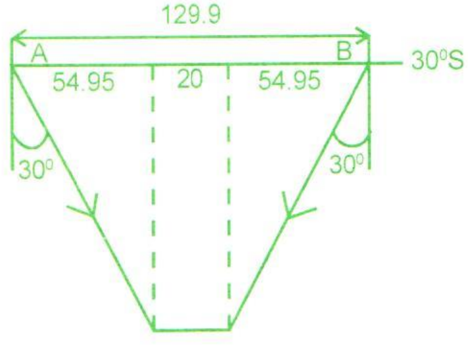
1. A vessel sails on a course 240° for 350 M. Find the d'lat and dep. she makes.

2. Find the course and distance, made good by a ship if she made a departure of 260 M. East and a d'lat of 165' North.

3. Find the course in the SE quadrant on which the d'lat will be 1/6th of the departure.

HARDER PROBLEMS

1. Two ships A and B doing equal speeds are both in lat 30°S, B being to the East of A. The d'long between the two ships is 2°30'.'A' steers 150°(T), while 'B' steers 210°(T). Find the latitude reached when they are 20 miles apart.

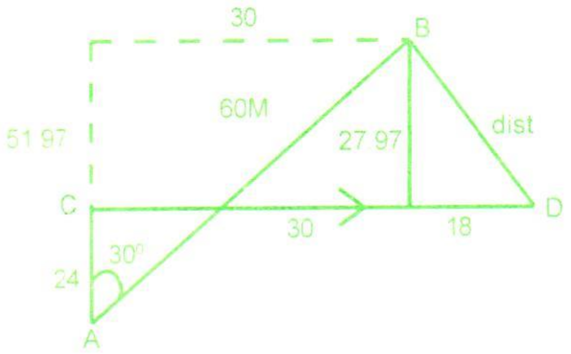


(FIG.2.8)

Departure between the ships in latitude 30°S
= d'long. cos lat. = 150. cos 30° = 129.9 M
d'lat = dep.x cot co. = 54.95. cot 30° = 95.18 = 1°35.18'S
Lat.left = 30°00'S; Lat reached = 31°35.18'S.

2. Two ships start from the same point in the Northern hemisphere. While the first ship steered 030°(T) at 10 kts., the second steered 000°(T) at 12 kts. for 2 hours and then altered course 090°(T). Calculate the distance between the two ships, 6 hours after starting.

d'lat made by first ship
= dist. cos co = 60. cos 30° = 51.97'
dep. made by first ship = 60. sin 30 = 30 M
d'lat made by 2nd ship = 24'
dep. made by 2nd ship = 4 x 12 = 48 Miles
diff of d'lat between two ships = 51.97' - 24' = 27.97'
diff of dep. between two ships = 48 - 30 = 18 miles



(FIG.2.9)

tan co. = dep. / d'lat = 18.0 / 27.97 = 0.6435
Co = 32°46.3'
Dist. = d'lat. sec co = 27.97. sec 32°46.3' = 33.25 M

3. From a position in lat 24°17' N, long 17°12'W, a course was set to a position 24°54'N, 17°12'W. After steaming for 34 miles, it was discovered that the compass error had been applied the wrong way and the ship had reached the position 24°49'N, 17°24.6'W. Find the actual error of the compass.

Hint - True course to be made good = 000°(T). Find the actual co. made good. The difference between the two gives double the error of the compass as the error was applied the wrong way. **Ans.** 9°52.5'W

THEORY QUESTIONS

1. Discuss the limitations involved in the use of Plane sailing formulae.

Before proceeding to Mercator sailing, it is necessary to understand the principles on which the Mercator charts are constructed. It is therefore necessary to introduce the topic of 'Charts' at this stage. We shall return to sailings after this topic is covered.

3
CHARTS

Maps and Charts are representations of portions of the Earth's surface, to a suitable scale, on a flat surface.

Charts differ from maps in that charts show a large amount of information for navigational usage.

A surface is said to be "developable" if it can be placed flat without being stretched or torn i.e. distorted. The curved surface of a sphere like that of the Earth is 'non-developable' since it cannot be placed flat without distortion. Therefore distortion is inescapable in any map or chart representing the Earth's surface.

There are various projections used in map making. A projection is an arrangement of lines representing meridians and parallels of latitude. A map projection is therefore a representation of the meridians and parallels of latitude, on a plane surface. It does not imply a projection in the geometric sense. The graticule representing meridians and parallels may be constructed on a mathematical basis, in no way connected with the geometric projection.

In choosing a particular projection, for constructing a chart, we first decide as to what kind of distortion is least objectionable and as to what particular properties are to be fulfilled by the chart.

To a navigator, it is important that his chart should represent the shape of the land correctly in any particular vicinity (i.e. the chart should be orthomorphic). As the most common form of sailing is along rhumb line tracks, it would be advantageous if rhumb lines can be laid off as straight lines on the chart. It should also be fairly easy to measure distances.

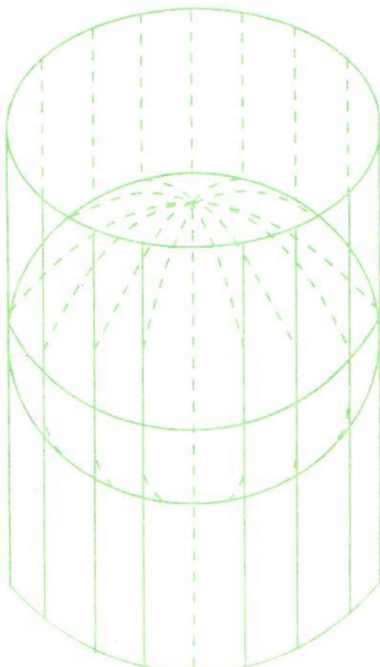
A projection is said to be orthomorphic, if in the immediate neighbourhood of any point represented, the scale along the meridian, along any radial line and along the parallel of latitude are all equal.

Such a projection will exhibit correctness of shape over small areas. The scale of the graticule may vary from one latitude to another, so that the shape of an entire land mass may differ considerably from its shape on the Earth. What is important to note is that the correctness of shape is always maintained over small areas. For example, on a Cylindrical Orthomorphic projection of the world, the shape of the area around Bombay in India is just as correctly shown as the shape of the area around Cape Farewell in Greenland, but Greenland as a whole appears more than four times the size of India, though India is in fact one and a half the size of Greenland.

3.1 MERCATOR CHART

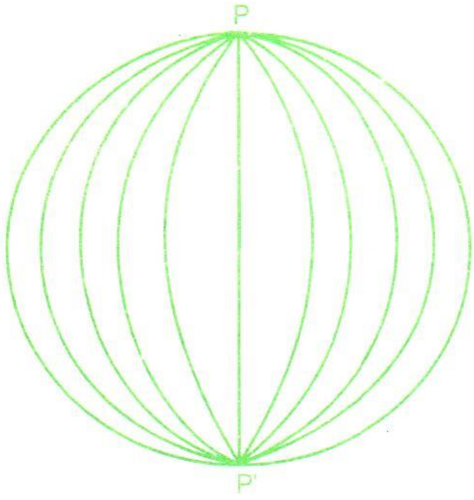
Most navigational charts are constructed on the Mercator projection, as they fulfill the important needs of the navigator, as stated earlier. This projection was initially used by Gerard Kremer, the latin form of whose name is Mercator. Among cartographers, the Mercator projection is said to be a "Cylindrical Orthomorphic Projection". It is derived mathematically and is not a perspective projection in the geometric sense. Apart from being orthomorphic, the projection is also stated to be cylindrical as it fulfills the conditions for a cylindrical projection. In a cylindrical projection the meridians are represented by parallel straight lines at right angles to the Equator. They divide the Equator into 360 equal parts.

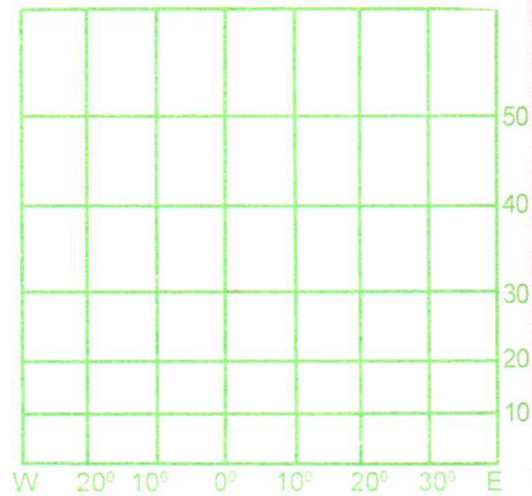
On a Mercator chart the Equator and parallels of latitude appear as horizontal parallel straight lines at selected distances from the Equator and from each other. The spacing between the parallels is selected on a mathematical principle designed to best satisfy the conditions the chart is intended to fulfill.



(FIG.3.1)

On the Earth's surface the meridians converge towards the poles. The distance between them is therefore maximum at the Equator and reduces as the latitude increases. On a Mercator chart however the meridians are represented by equidistant parallel straight lines. It therefore follows that the east-west distortion on the chart increases as the latitude increases.





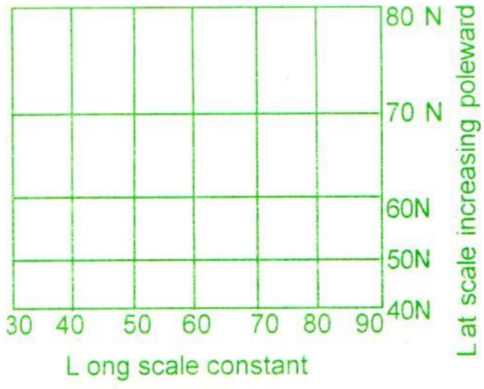
(FIG.3.2) (FIG.3.3)

To maintain the orthomorphic property over the entire chart, it is therefore necessary to deliberately introduce an equal north-south distortion, which like the east west distortion should increase poleward. It can thus be seen that the distances between successive parallels of latitude on Mercator Chart will increase towards the pole.

On the Earth's surface the east-west distance between two meridians reduces as the cosine of the latitude, because the departure on any latitude is equal to the d'long multiplied by the cosine of that latitude. On a Mercator Chart however if the distance between the meridians is represented by x cm, at the Equator, it will be represented by the same x cm at all other latitudes also. Thus distortion on the chart at any latitude ϕ is equal to $x / x \cos\phi = \sec\phi$. Since the east-west distortion is proportional to the secant of latitude, the latitude scale should also vary as the secant of latitude to maintain the orthomorphic property. Since secant 0° is 1, it implies that at the Equator the latitude scale = longitude scale.In other latitudes,

Lat.Scale = Long. Scale x sec lat. The longitude scale on a Mercator chart is constant throug out the chart. Due to this, the distances and areas on a Mercator chart are exaggerated proportional to secant of latitude

The nautical mile has been defined earlier as the length of a meridian between two geographic latitudes which differ by 1'; that is 1' of d'lat. On a Mercator chart, the latitude scale is therefore used for measuring distances. Since the lat. scale increases with latitude, the length of a nautical mile on the chart also increases poleward.



(FIG.3.4)

3.1.1. Meridional Parts

On a Mercator Chart, since the distance between successive parallels of latitude increases towards the poles, the length of a meridian between those parallels will also increase towards the pole. For example, the length of the meridian between latitudes 5° and 10° will be larger than its length between 0° and 5° latitudes.

The Meridional parts

The Meridional parts for any latitude is the length of a meridian between the Equator and that latitude, on a Mercator Chart, measured in units of longitude scale i.e. the number of times one minute of longitude can be laid along a meridian between the Equator and that latitude, on a Mercator Chart. The meridional parts for navigable latitudes are tabulated in the Nautical Tables, assuming the Earth to be spheroidal in shape.

Difference in Meridional parts (DMP)

DMP between two latitudes is the length of a meridian between those latitudes on a Mercator Chart expressed in units of longitude scale.

DMP between two latitudes may be obtained using the meridional part table as the difference or sum of the meridional parts of the two latitudes, similar to obtaining the d'lat.

The meridional parts table for the spheroidal Earth has been compiled using the expression, meridional parts for lat:

$$L = 7915.7 \log_{10} \tan (45 + L/2) - 23.4 \sin L + 0.01 \sin 3 L.$$

For the sphere however, the meridional parts could be obtained using only the first term of the expression. Thus, for the sphere

$$MP = 7915.7 \log_{10} \tan (45 + L/2)$$

The properties / features of a Mercator chart may be summarized under advantages and disadvantages of the chart.

Advantages

(1) Rhumb line courses are easily laid off as straight lines.
(2) Distances are easily measured as scale of distance = scale of latitude.
(3) Shapes of land masses in the neighbourhood of a point are correctly shown.
(4) Angles between rhumb lines are unaltered between the Earth and the chart.
(5) Directions remain correct though distortions of areas occur.
(6) Directions and position lines can be transferred correctly from one part of the chart to another as parallel lines. This facility which is often used by a navigator for obtaining running fixes is not available in most other projections.

Disadvantages

(1) Great circle courses cannot be laid off easily as they would appear curved.
(2) Polar regions cannot be represented due to extremely large distortions.
(3) The scale of distance which is the scale of latitude is a varying unit.
(4) Areas cannot be compared due to the varying distortion.

3.1.2 Natural Scale

The natural scale of a chart is the ratio that the distance between two points on the chart bears to the actual distance between them on the Earth.

For example a natural scale of 1 / 25,000 means, that one unit of length on the chart represents 25,000 units of length on the Earth. In other words 1 cm on the chart represents 25,000 cm on the Earth, or one foot on the chart represents 25,000 ft. on the Earth etc. The natural scale of a Mercator chart varies from latitude to latitude. Therefore any natural scale stated on the chart is valid for a particular latitude only.

Natural scale is normally expressed as the relationship that one minute of longitude on the chart bears to one minute of longitude on the Earth, in that latitude.

If one minute of longitude on a chart is represented by 5mm in latitude 60°, the natural scale in that latitude can be obtained as follows:

Natural Scale = Chart Distance / Earth Distance

The chart distance for 1' of long. = 5 mm

Since one minute of longitude on the Earth at the Equator is equal to 1 mile = 1852 m = 1852 x 1000 mm, the length of one minute of longitude in latitude 60° would be the departure in that latitude corresponding to a difference of longitude of 1' i.e. 1 x 1852 x 1000 x cos 60°.

Natural scale = 5 / 1 x 1852000 x ½ = 10 / 1852000 = 1 / 185200

3.1.3 Construction of Mercator Charts

From the foregoing, it will be realized that a Mercator chart of any area can be constructed quite accurately to a given natural scale in a particular latitude.

Example

Construct a Mercator chart of the area 28°N to 32°N, 15°W to 20°W to a natural scale of 1 / 1000,000 in latitude 30°N.

We must first calculate the longitude scale from the given natural scale. The length of one degree of longitude in latitude 30° = 60' x 1852 x 1000 x cos 30° = 96,229,920 mm

To a scale of 1 / 1,000,000 this length on the Earth would be represented by 96,229,920 / 1,000,000 = 96.23 mm (approx.) on the chart.

Draw in the limiting latitude of 28°N and on it, mark off the meridians 96.23 mm apart. Erect the meridians perpendicular to the limiting latitude and parallel to each other. We have to now calculate the latitude scale. To be very correct, the length of each minute of latitude should be calculated separately. Sufficient accuracy can be obtained particularly in fairly low latitudes if the length of each degree of latitude is calculated.

The natural scale we have chosen is, 1° of longitude = 96.23 mm.

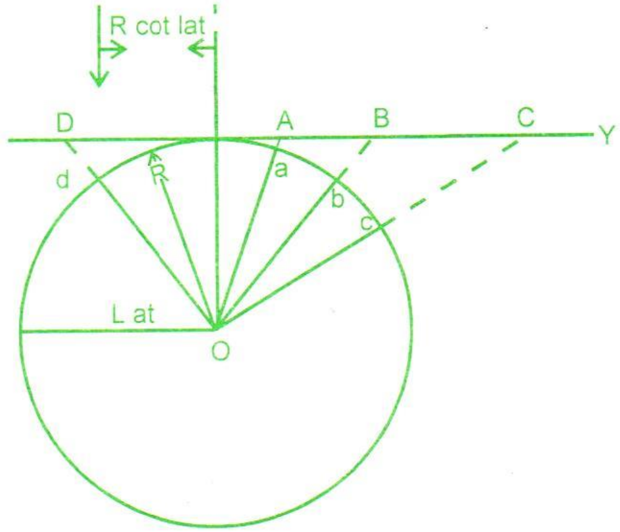
1 minute of longitude = (96.23 / 60) mm. From its definition, we know that DMP between two latitudes is the number of times 1' of d'long can be placed along a meridian between those latitudes on a Mercator chart. It therefore follows that the distance on the chart between latitude 28°N and latitude 29°N can be obtained, as the product of DMP between the two latitudes and length of 1' of long. to the scale we have already chosen.

MP for lat 28°	=	1740.2
MP for lat 29°	=	1808.1
DMP	=	67.9

Distance between latitudes 28° and 29° on the chart = 67.9 x 96.23 / 60 = 108.9 mm. Mark off this distance of 108.9 mm from the limiting latitude along any meridian. Draw in the 29° parallel of latitude, through the point marked off, parallel to the limiting latitude and perpendicular to the meridians. Repeat this process successively between 29° and 30°, 30° and 31° and so on till the other limiting latitude is reached. It should be noted that the formula for latitude scale (latitude scale = longitude scale x secant latitude) has not been used for this purpose as it holds good for any particular latitude only and not when dealing with distances between two latitudes.

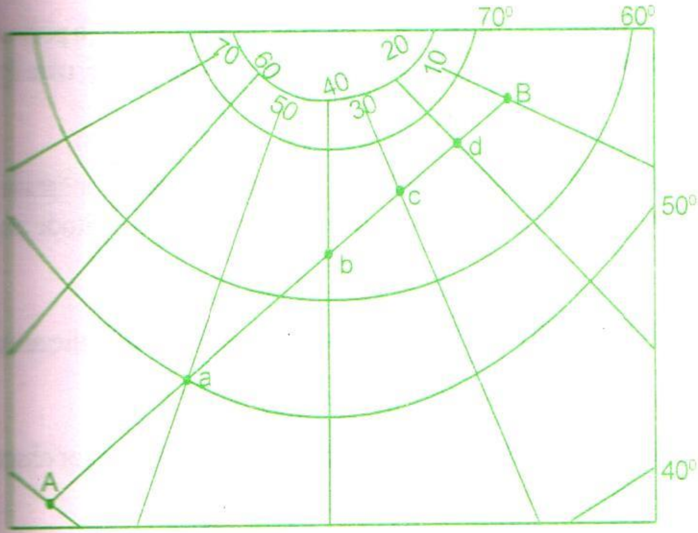
3.2 GNOMONIC CHART

If a navigator is to follow the shortest route between two positions, he must sail along a great circle. It would therefore be convenient to have charts on which great circles are represented by straight lines. The Gnomonic chart has this property. It is constructed on the gnomonic or tangential projection. In this projection, all points on the surface of a sphere are projected from the centre of the sphere to a plane which is tangential to the sphere. The tangent point chosen is usually around the centre of the area to be represented.

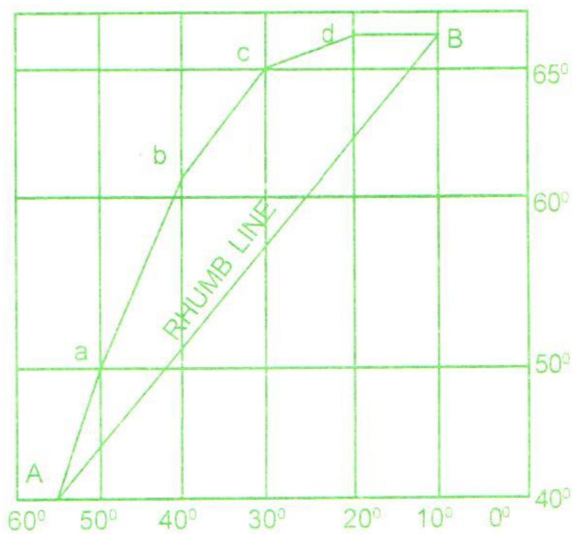


(FIG.3.5)

As with any projection used, distortions will be present on a gnomonic chart. It can be seen from figure 3.5. that distortion is nil at the tangent point and increases as the distance from the tangent point increases. If the tangent point is one of the poles, the chart would be a polar gnomonic chart. On a gnomonic chart, all great circles appear as straight lines. Therefore, meridians appear as straight lines converging towards the poles. Small circles and rhumb lines appear curved. Compass roses are not shown on gnomonic charts as they would be valid only for that particular location, since meridians are convergent.



(FIG.3.6)



(FIG.3.7)

Gnomonic charts are used to plot great circle courses between the departure and arrival positions, as straight lines (Fig. 3.6). Positions are then taken off at convenient intervals of longitude along the track. These positions are then transferred to a Mercator chart and rhumb line courses are laid off between the successive positions (Fig. 3.7). By sailing along the various rhumb line courses, the ship would make a track which would approximate very closely to the actual great circle course, without having to change course continuously as would have been necessary, if following a true great circle.

Advantages of a Gnomonic Chart

1. All areas of the world including polar regions can be reprsented on gnomonic charts.
2. Great circle courses are easily laid off as straight lines.

Disadvantages

1. Rhumb line courses and bearings cannot be laid off easily as they appear curved.

2. Bearings and positions lines cannot be transferred from one part of the chart to another as parallel lines, because the meridians are convergent.

3. Measurement of distances and courses is difficult.

3.3 PLAN CHARTS

Plan charts are representations of very small areas of the Earth's surface, such as an anchorage , a port or a harbour. Since the area represented is very small, it is considered flat and a plane drawing made, usually to a natural scale of 1: 50,000 or larger.

The plan usually shows a scale of latitude, which is also the distance scale and a separate scale of longitude. The areas represented being very small, on a plan chart, the scale of latitude and the scale of longitude are constant over the entire chart.

On a plan chart, it is usual to state the exact latitude and longitude of some reference point in the area covered by the chart.

Examples

1. Find the length of 1° of longitude, if 1° of latitude on a Mercator chart measures 12 cm in latitude 40°S.

   Lat. scale = long. scale x sec lat.
   Long. scale = lat. scale x cos lat.
   1° of longitude = 12 cm x cos 40° = 9.192 cm

2. In measuring a distance on Mercator chart, in latitude 45°S, the longitude scale was used by mistake. If the measured distance was 47', find the actual distance.

   Let 1' of longitude, on the chart be z cm
   The distance on the chart = 47 z cm
   1' of latitude i.e. 1 mile in lat. 45°S will be = z sec 45°.

   Actual distance= 47 z cm / z cm. sec 45° = 47/sec 45°=33.235 M.

3. One degree of longitude on a Mercator chart measures 2.8 cm. Find the distance in miles, between 2 points in lat 50°N, 5.6 cm apart on the chart.

   1° of longitude = 2.8 cm
   1° of latitude in 50°N = 2.8 sec 50°
   1' of latitude i.e. 1 Mile in 50° N = 2.8 sec 50° / 60
   Distance between the points = 5.6 x 60 / 2.8 sec 50° = 77.135 M.

Exercise III

1. If 1° of longitude on a Mercator chart measures 3 cm, find the length of 1° of latitude and 1 Mile in latitude 35°N.

2. In what latitude will 60 miles on a Mercator chart equal 2° of longitude ?

3. Assuming the Earth to be sphere, calculate the DMP between latitude 30°S and 32°40'S, without using the meridional parts table.

4. If the longitude scale on a Mercator chart is 1° = 2.5 cm, find the chart distance between two positions 20 miles apart in latitude 60°N.

HARDER PROBLEMS

1. The distance between 2 points on a Mercator chart in latitude 32°30'N was 22 miles. How many minutes of longitude can be placed ?

   Let 1' of latitude i.e. 1 mile in latitude 32°30' = z cm
   Then distance on chart = 22 z cm
   1' of long. = lat. scale x cos lat. = z cm x cos 32°30'
   No of minutes of longitude that can be placed between them
   = 22 z cm / z cm x cos 32°30' = 26.085'

2. The longitudes on a Mercator chart are drawn to a scale of 1° = 6 cm. The distance on that chart between the parallel of latitude 24°S and another latitude to the North of it was 22.3 cm. Find the second latitude.

   1° of longitude = 6 cm
   1' of longitude = 6 / 60 = 0.1 cm
   Distance on the chart between the two parallels = 22.3 cm
   No.of minutes of long. i.e.DMP between them 22.3 / 0.1 = 223

MP of latitude 24°S	=	1474.5
DMP	=	223
MP of second latitude	=	1251.5
Second latitude	=	20°32.5'S.

3. Find the distance between parallels of latitude 1° apart, to construct the graticule for Mercator chart of the area 20°S to 25°S and 070°E to 075°E, to a scale of 1° of longitude is equal to 3 cm. Give the extreme dimensions of the chart.

   1° of longitude = 3 cm
   Difference in longitude between 070°E & 075°E = 5°

Width of chart = 5 x 3 = 15 cm
1° of longitude is 60' of longitude = 3 cm
1' of longitude = 3 / 60 = 0.05 cm

(a) MP of lat. 20°S = 1217.14
MP of lat. 21°S = 1280.81
DMP = 63.67

Distance between 20° and 21° on the chart = 63.67 x 0.05 = 3.1835 cm.

(b) MP of lat. 21°S = 1280.81
MP of lat. 22°S = 1344.92
DMP = 64.11

Distance between 21° and 22° on the chart = 64.11 x 0.05 = 3.2055 cm.

(c) MP of lat. 22°S = 1344.92
MP of lat. 23°S = 1409.49
DMP = 64.57

Distance between 22° and 23° on the chart = 64.57 x 0.05 = 3.2285 cm.

(d) MP of lat. 23°S = 1409.49
MP of lat. 24°S = 1474.54
DMP = 65.05

Distance between 23° and 24° on the chart = 65.05 x 0.05 = 3.2525 cm.

(e) MP of lat. 24°S = 1474.54
MP of lat. 25°S = 1540.11
DMP = 65.57

Distance between 24° and 25° on the chart = 65.57 x 0.05 = 3.2785 cm.
Dimensions of the chart 15 cm x 16.1485 cm.

4. Two latitudes are complementary, the latitude scale at one of them is double that at the other. Find the two latitudes.

Let one latitude be = x
Then the other latitude = (90 - x)
If the scale at lat. x is double the scale at lat. (90 - x)
sec x = 2 sec (90 - x)
sec x = 2 cosec x
sec x / cosec x = 2
tan x = 2

x = 63°26'
the other lat. = 26°34'

Theory Questions

1. Discuss the Mercator projection and the advantages and disadvantages of a Mercator chart.

2. Define Natural scale, Meridional parts, Difference of meridional parts.

3. Describe, how a Mercator chart covering the area 20°N to 25°N and 080°E to 085°E could be constructed.

4. How would a circle of radius 600 miles on the Earth's surface, with its centre in latitude 60°S appear on a Mercator chart ?

5. Discuss the use, advantages and limitations of a Gnomonic chart.

6. Describe briefly a Plan chart.

CALCULATIONS ON NATURAL SCALE

Ex. 1. Find the length between meridians 1° apart on a Mercator chart drawn to a Natural Scale of $\frac{1}{1000,000}$ in latitude 30^0 S.

$$\text{N.Scale} = \frac{\text{Chart distance}}{\text{Earth distance}}$$

$$\frac{1}{1000\,000} = \frac{\text{x mm}}{60 \times \cos 30 \times 1852 \times 1000\text{mm}}$$

$$x = 96.23\text{mm}$$

Ex.2 . On a mercator chart, 1° of longitude is represented by 5 cm. What is the natural scale of the chart in latitude 60° N.

$$\text{N.scale} = \frac{\text{chart distance}}{\text{Earth distance}} = \frac{5}{60 \times 1852 \times 100 \times \cos 60^0}$$

$$= \frac{1}{1111200}$$

Exercise

1. Two positions on a Mercator chart drawn to a natural scale of 1/500,000 are 25 cms. apart . Find the actual distance between them in nautical miles.

Ans : 67.49M.

2. A mercotar chart is drawn to a longitude scale of 1° = 5 cms. Find the distance on that chart between two positions 40M. apart in latitude 50° N. Also find the Natural Scale of the chart in latitude 50° N.

Ans : 5.186 cms Natural Scale $\frac{1}{1428532}$

3. A Mercator chart is drawn to a scale of 1:1,800,000 in latitude 39° N. Calculate the distance on that chart between 27°00 'N 52°00 'E and 29°00' N 55° 00 'E
Hint : the distance on the Earth varies directly as cos latitude

$$\text{Scale in the M latitude } 28^0{}_N = \frac{1}{1{,}800{,}000 \times \frac{\cos 28}{\cos 39}} = \frac{1}{2045055}$$

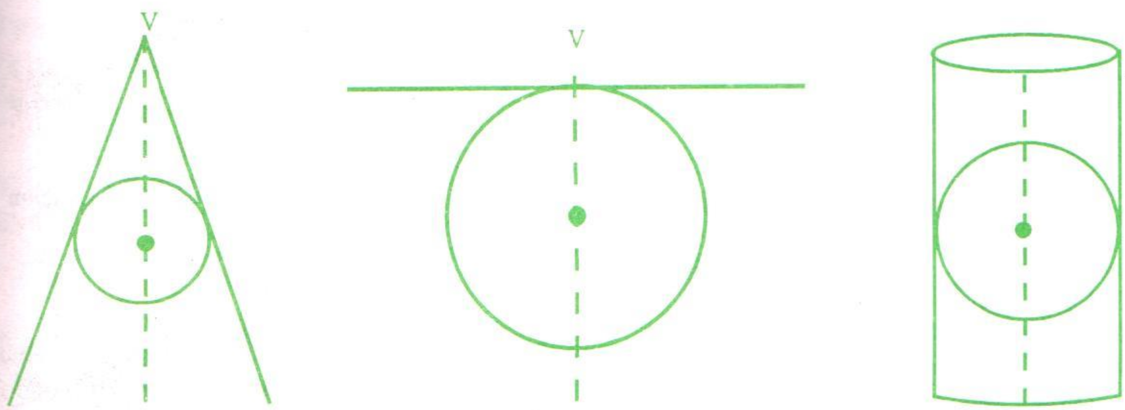
Now find actual distance on the Earth between the positions and then the distance chart.

Ans : 180.35 mm

3.4 OTHER MAP OR CHART PROJECTIONS :

Projections may be categorised into two classes. The first class is obtained from the cone, this includes all conical, zenithal and cylindrical projections. These projections may be constructed mathematically (as in the case of the Mercator chart) and also as true perspective projections. Example of these are the Mercator, Gnomonic and Lamberts projections. The second class of projections are the "Conventional" which are constructed only mathematically. Example of these include Mollweides, Bonnes, Sinusoidal and Equal Area projections.

Conical projections are obtained by rolling a plain in the form of a cone over a sphere representing the Earth. All points on the sphere are then projected on to the cone from a chosen center of projection lying on the diameter of the sphere, drawn from the vertex of the cone or on the diameter produced. (Fig. a)



Conical (a) Zenithal (b) Cylindrical (c)

If the angle at the vertex is increased, the vertex approaches the surface of the sphere, When the angle becomes 180°, the cone becomes a plane tangential to the sphere and the vertex becomes the tangent point. The projection is then said to be "zenithal". Since the true bearings of all places depicted, from the tangent point, will be correct, the zenithal projection is also called on "azitmuthal projection". An example of this is the Gnomonic projection.

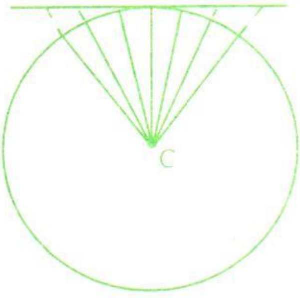
In the Gnomonic projection, the center of projection is the center of the sphere. It will be seen that this is a perspective projection and not a mathematical construction. (Fig b)

When the angle at the vertex is reduced to zero, the cone becomes a cylinder. If all points are projected on to the cylinder from the center of the sphere, the projection we obtain is a cylindrical projection. An example of this is the Mercator projection.

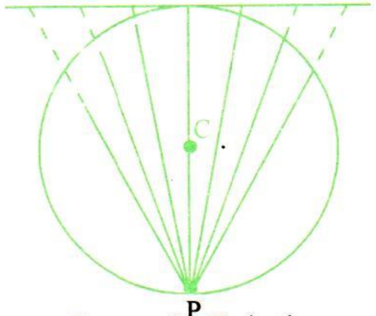
All zenithal, i.e. tangential projections are made by projecting the points on the sphere on to the tangent plane from a center of projection which lies on the diameter of the sphere drawn from the tangent point or this diameter produced. Some important projections result, depending on the position of the center of projection.

i) The Gnomonic projection is obtained if the center of projection is the center of the sphere. In this projection, less than half a hemisphere only can be projected. Distortion is nil at the tangent point, but increases considerably towards the outer points. The direction to any point depicted, from the tangent point is accurate.

ii) The Stereographic projection is obtained if the center of projection is the other extremity of the diameter. In this projection, more than a complete hemisphere can be projected, without very large distortions.

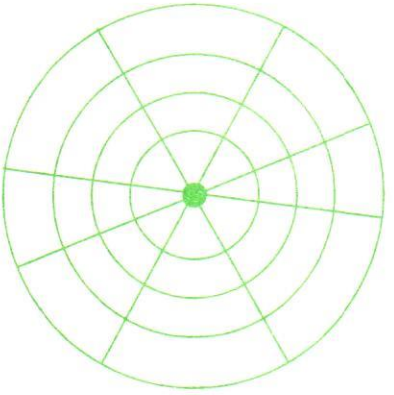


Gnomonic Projection

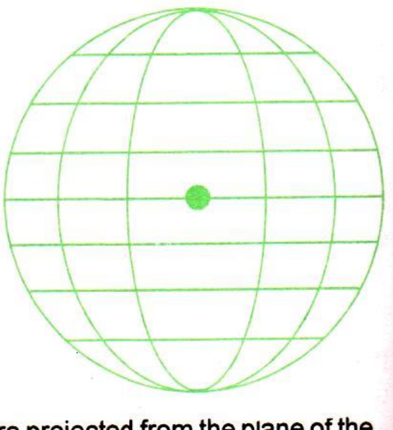


Stereogrphic Projection

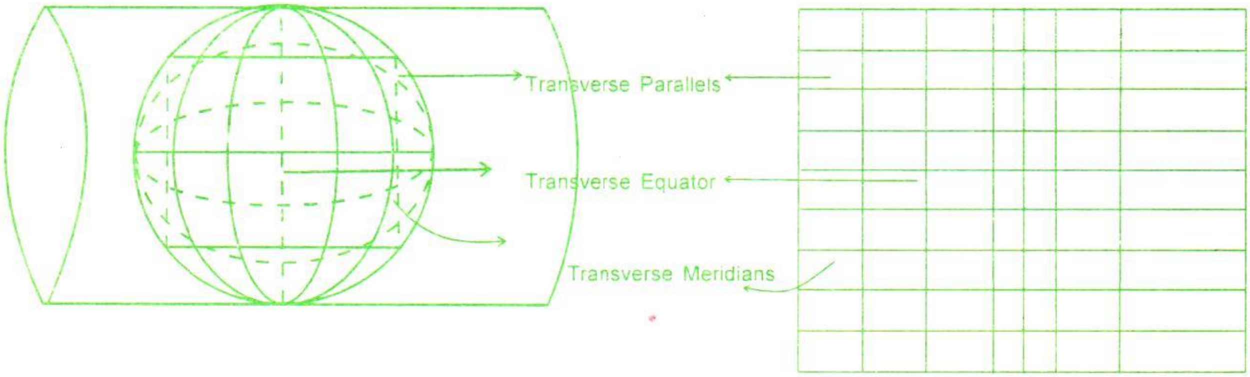
iii) The orthographic projection is obtained by moving the center of projection to an infinite distance from the center of the sphere, along the diameter produced. The graticule will then be the appearance of the meridians and parallels of the earth as seen from a far distance.



Where projected from the axis of the Earth produced



Where projected from the plane of the equator



In the transverse mercator projections, the geographic equator and transverse meridians will be straight lines, parallel to each other, equidistant from each other and lying in the E-W direction. The transverse parallels will be straight lines, lying N-S, parallel to each other and at increasing distances from the transverse equator, proportional to the secant of their angular distance from the transverse equator. The geographic meridians will therefore be curves, converging towards the pole. The geographic parallels of latitude will also be curves, concavity facing the pole of the geographic hemisphere.

THE UNIVERSAL TRANSVERSE MERCATOR SYSTEM

The projection used in this system is the transverse Mercator projection, therefore, all geographic parallel of latitude and all geographic meridians, except the geographic equator and the central meridian will appear curved.

UTM is a system of world co-ordinates (like latitude and longitude) covering the entire surface of the earth from 80° S to 84° N. The rectangular co-ordinates or measurement are in meters. The UTM lines are therefore at right angles to each other i.e. they are "orthogonal". UTM easting is the distance east, in meters from the chosen central meridian (transverse equator) of the area depicted. UTM northing is the distance north, in meters, from the geographic equator. Because the distortion increase away from the central meridian, UTM maps are made of zones, 6° wide in longitude. Sixty such maps of 6° wide zones in longitude are made to cover the entire world. UTM northing is devided into 8° high zones and are designated by letters C for the zone 80°S to 72° S, D for the zone 72° S to 64° S and so on to L for the zone 08° S to 00°. The equator is designated M. For areas in North latitude, the 8° high zones are designated from N for the zone 00° to 08° N to X for the zone 80° N to 84° N. the letters A, B and Y, Z are not used in this system as they are used for the universal polar system, in the polar regions.

Since UTM easting is the distance east in meters form the central meridians, places to the west of the central meridian would have had negative co-ordinates. To avoid this, a system of "false" easting is introduced by designating the central meridian of the area depicted as 500,000 instead of zero. Thus the easting co-ordinate of places, say 10m to the east of the central meridian would have an easting co-ordinate of 500,010. The easting co-ordinate of places 10m to the west of the central meridian would have an easting co-ordinate of 499,990. Thus the easting co-ordinates of all positions within the area (6° wide in longitude) depicted, on that map, will be positive. Similarly, 10,000,000 is added to the negative northing co-ordinates (distance in meters, south of the equator) in the south hemisphere. Since the original definition of the meter was "1/10,000,000th of the distance from the equator to the pole of the earth" all places in the south hemisphere will also have positive northing co-ordinates. It should be noted that the original definition of the meter is no longer exact. The northing co-ordinate of latitude 45° N, which would have been 5000,000m as per the original definition is actually 4,986,272.

Since the map scale increases to each side of the central meridian, in a transverse mercator projection, the scale is reduced to 0.9996 at the central meridian on UTM maps. Thus the scale will be accurate at two meridians, one on each side of the central meridian. This improves the average scale accuracy of the entire map.

In stating the UTM co-ordinates of a place, the easting co-ordinate is stated first and the northing co-ordinate thereafter.

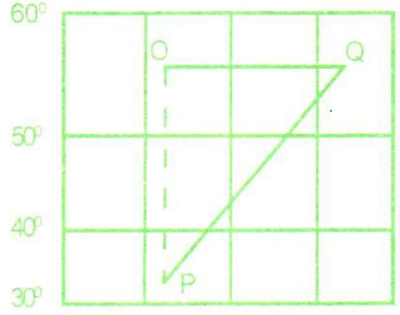
4

SAILING

(MERCATOR, MIDDLE / MEAN AND TRAVERSE)

4.1 MERCATOR SAILING

Mercator sailing method is used to find the rhumb line course and distance between two positions, the latitudes and longitudes of which are known.



(FIG.4.1)

The above figure represents a Mercator chart, with a rhumb line course laid from P to Q. PQ is the distance from P to Q and angle OPQ is the course. OQ is the d'long, and OP the d'lat between the two positions. It should however, be noted that the units of OQ, the d'long and OP, the d'lat are different. Thus d'long / d'lat cannot be said to be tangent course. If however, OP is also measured in terms of longitude scale, OQ / OP would be equal to tan course. We have already seen that OP measured in terms of longitude scale is the DMP between the two latitudes. It can therefore be seen that **d'long / DMP = tan course.** It can also be seen that since the distance is measured in terms of latitude scale, **Distance = d'lat x sec course.**

The secant of angles approaching 090° & 270° increases very rapidly. When a vessel is on a course which is nearly East or West, the secant of the course should therefore be obtained with great care when calculating the distance.

Example

Find the course and distance from 12°14'N 073°12'E to 23°37'S 010°19'E.

Lat.	12°14'N	Long.	073°12'E	MP	=	734.7
Lat.	23°37'S	Long.	010°19'E	MP	=	1449.5
d'lat	35°51'S	d'long	62°53'W	DMP	=	2184.2

d'long / DMP = tan co. = 3773 / 2184.2
Course = S59°56'W
Distance = d'lat x sec co. = 2151 x sec 59°56'= 4293.5 M.

Exercise IV

1. Find the course and distance from P in latitude 15°32'N, longitude 024°06' W, to Q in latitude 45°56'N, longitude 064°38 W.

2. Find the course and distance from 38°10'S, 178°00'E to 02°50'S, 081°10'W.

3. Find the course and distance between :
   A in 13°48'N, 166°55'E and B in 16°11'S, 157°48'W.

4. A vessel steered 046°(C), from 32°10'N, 178°50'E, and reached 33°34'N, 177° 52.5'W. Find the deviation of the compass, if the variation was 14°E.

HARDER PROBLEMS

1. A vessel sails on a course 144°(T) from latitude 15°40'N and makes a d'long of 47°50'. Find the distance covered and the latitude reached.

Lat. left	=	15°40'N, MP	=	945.6 (N)
DMP	=	d'long cot co. = 2870. cot 36°	=	3950.2 (S)
MP of latitude arrived			=	3004.6 (S)
Latitude arrived	=	44° 53.8'S		
Latitude left	=	15° 40.0'N		
d'lat	=	60° 33.8'S		
Distance = d'lat x sec co.	=	3633.8. sec 36°		
	=	4491.6 Miles		

2. A vessel in North latitude sailed from long 60°W, on a course of 036°(T) and made a departure of 160 M, and a DMP of 260. Find departure and arrival positions of the ship.

   d'long = DMP x tan co = 260. tan 36° = 188.90'E = 3°08.90'E
   d'lat = dep.x cot co = 160. cot 36° = 220.22'N = 3°40.22'N
   dep / d'long = 160 / 188.9 = cos mean lat.
   Mean lat = 32°06.8'N

Mean lat	32°06.8'N		32°06.8'N
½ d'lat	1°50.1'		- 01°50.1'
	33°56.9'N		30°16.7'N
Long. left	=	60°00.0'W	
d'long	=	3°08.9'E	
Long. arrived	=	56°51.1'W	
Position left	:	30°16.7'N, 60°00.0'W	
Position arrived	:	33°56.9'N, 56°51.1'W	

3. By sailing N44°W for 1600 miles, a vessel arrived in position 12°13'S 176°17'E. Find the vessel's departure position.
   (Hint - Obtain d'lat to find latitude left. Now obtain d'long as DMP x tan course)

   Ans. 31°23.9'S 163°45.5'W

4. A vessel left lat. 46°50'N and steered 253°(T), making a d'long of 15°31'. Find the latitude reached.
   (Hint - Obtain the DMP as d'long x cot course. Apply the DMP to the MP of the departure latitude to obtain MP of the arrival latitude).

   Ans. 43°28.7'N.

4.2 MIDDLE / MEAN LAT SAILING

In Plane sailing, the parameters used were departure, distance, course and d'lat. In Parallel sailing, departure, d'long and the latitude were used, the course being always East or West.

In sailing from one position to another, where d'lat and d'long are involved, Parallel sailing or Plane sailing cannot be used. To solve such problems, Middle latitude sailing may be used. Thus Middle latitude sailing can be used for

(i) finding the course and distance between two given positions,
(ii) to determine the arrival position, given the departure position, course and distance.

From what has been learnt earlier, it will be recalled that both the above types of problems are solved easily by the Mercator sailing method.

It will be seen that the Middle latitude method of solving such problems is more cumbersome. Thus for Practical Navigation, Middle latitude sailing problems are redundant. However, academic problems which can be solved only by the Middle latitude method may be encountered.

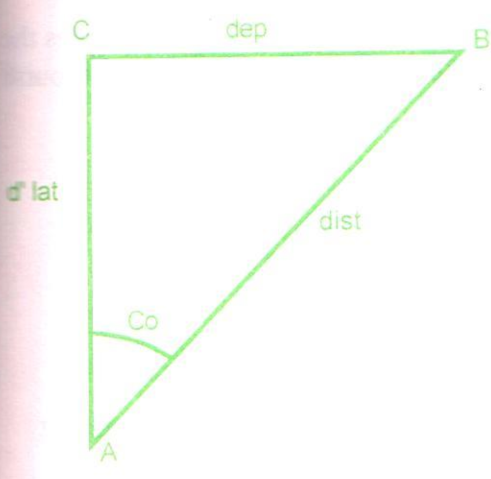
The Middle latitude was defined as the latitude in which the true departure lies, when sailing between two latitudes. Thus the Parallel sailing formula may be modified as dep / d'long = cos Middle lat., where the departure involved is the true departure.

The middle latitude may be obtained by applying a correction to the mean latitude. This correction is tabulated in some nautical tables. Having thus obtained the middle latitude the departure may be found by the expression :
dep = d'long x cosine middle latitude.
The course may then be obtained by the expression : **tan course = true departure / d'lat**
The distance can then be found by either of the expressions :
Distance = d'lat x sec course or Distance = true departure x cosec course.

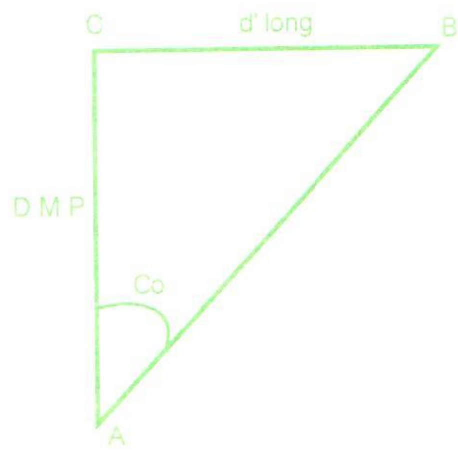
When on a course nearly East or West, it would be better to use the latter expression, as the secant of an angle changes rapidly for angles near 090° and 270°. Similarly for courses near N or S, it would be better to use the former expression, as the value of cosecant changes rapidly for angles near 000° and 180°.

Where the latitudes involved are not high and where the d'lat is small the mean latitude may be used instead of middle latitude without appreciable loss of accuracy. Due to the above and because the table of correction for converting, mean latitude to middle latitude is not available in all nautical tables, the problems on this topic have been solved using mean latitude instead of middle latitude.

A relationship which is not directly apparent may be seen from the Plane sailing and Mercator Sailing triangles which are similar.



(FIG.4.2)



(FIG.4.3)

It is evident from the two similar triangles, (FIG. 4.2 & 4.3)
dep / d'long = cos Middle lat.
d'lat / DMP also = cos Middle lat.

Examples

1. Find the course and distance, by Mean latitude sailing between A in 32°12'S, 178°14'E and B in 34°05'S 179°11'W.

A	32°12'S	;	178°14'E
B	34°05'S	;	179°11'W

d'lat 1°53'S ; d'long 2°35'E = 155'
Mean lat = 33°08.5'S
dep = d'long.cos Mean lat = 155'. cos 33°08.5 = 129.8 M
tan course = dep / d'lat = 129.8 / 113
Co = S48°57.3'E
Dist = d'lat . sec co = 113. sec 48°57.3 = 172.1 M

2. A vessel sails 030°(T), 240M and makes a d'long of 3°30'. Between what latitudes did she sail ?

dep = distance x sin co = 240 x sin 30° = 120M
dep / d'long = cos Mean Lat = 120 / 210
Mean Lat = 55°09'
dep / d'lat = tan co
d'lat = 120 x cot 30° = 207.8'
1/2 d'lat = 01°43.9
Lat. left and reached = 55°09' ± 1/2 d'lat
= 56°52.9' and 53°25.1'N or S

3. In sailing a certain course and distance, the d'lat is 1.5 times the departure and 0.8 times the d'long . Find the Middle lat. and course made good.

d'lat = 3 / 2 dep = 0.8 d'long
Let dep = z, then d'lat = 1.5z
dep / d'lat = tan co = z / 1.5z = 2 / 3 = 0.6666
Course 33°41.4'
Again, we know 3 / 2 dep = 0.8 d'long
dep = 2 / 3 x 0.8 d'long
dep / d'long = os Middle lat. = 2 x 0.8d'long / 3 x d'long
= 1.6 / 3
Middle lat. = 57°46'N or S
Course N 33°41.4'E or W
or
S 33°41.4'E or W

4. Two ships P and Q steer the same course. P is three times as fast as Q, but P makes only twice the d'long made by Q. If P is in latitude 17°S, find Q's latitude.

The departure made by the two ships will be proportional to the distances covered by them, as they are on the same course. Since the distance covered by P is 3 times the distance covered by Q, the departure made by P is also 3 times the departure made by Q.

When Q makes a departure t miles, P makes a departure of 3t miles
In making a dep. of 3t miles, the d'long made by
P = dep / cos lat. = 3t / cos 17° = 3.137t
During the same interval,the d'long made by Q is ½ that of P
3.137t / 2 = 1.5685t
For Q, dep / d'long = t / 1.5685t = cos lat.
Latitude of Q = 50°23.5'N or S

5. The middle latitude between two positions is 41°06'N. In covering a distance of 350M, between the positions, a vessel makes a d'long of 4°01'. What course in the NW quadrant did she make good ?

dep = d'long x cos mean lat = 241 x cos 41°06' = 181.6M
sin co = dep / dist = 181.6 / 350
Course N31°15.5'W

Exercise IV (A)

1. A vessel makes a d'lat of 01°44'N and a d'long of 5°34'W, in sailing a distance 255M. What course did she make good and between what latitudes did she sail ?

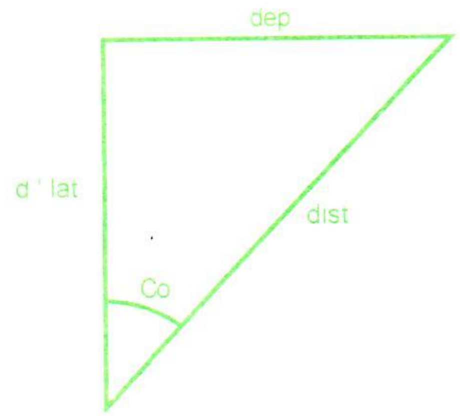
2. A ship in latitude 50°S, steers a course of 250°(T), making a d'long of 20'/hour. Calculate the ship's speed.

3. In covering a certain distance, the d'long in minutes equals the distance in miles and twice the d'lat in minutes. Find the middle lat.

4. A vessel sailed from latitude 45°N on a steady course making a DMP 1½ times the d'lat. Calculate the latitude reached.
(Hint - d'lat / DMP = cos middle lat.)

4.3 TRAVERSE SAILING

Traverse Table



(FIG.4.4)

In solving plane sailing problems. we use the above right angled triangle. The parameters dealt with are the COURSE, DISTANCE, D'LAT and DEPARTURE. Given any two of these parameters, the others can be obtained by solving the right angled triangle. The 'traverse table' is a ready made solution of plane right angled triangles for distances upto 600 miles, for each degree of course angle from 0° to 90°.

This table therefore enables solution of right angled plane triangles by inspection, without having to do any calculations.

While the traverse table is intended for the solution of sailing problems, it should be noted that, it can be used to solve right angled triangles for other purposes also.

In quadrantal notation, for course angles upto 45°, the angles and column headings, are at the top of the page and for course angles between 45° and 90°, angles and column headings are at the bottom of the page. When the course and distance are known, the d'lat and departure may be read off directly against the distance on the page for that course angle. When, however, the d'lat and departure are known and the course and distance are required, one has to search the tables, until the given d'lat and departure are found together. The distance can be read off against them, and the course angle read off from top or bottom of the page, as the case may be. Usually some interpolation may be necessary in the use of traverse tables.

When the d'lat is greater than the departure, the course angle will be less than 45°. Conversely when the departure is greater than the d'lat, the course angle will be more than 45°.

The following examples illustrate the use of the traverse tables:

Examples

1. Find the d'lat and departure made good after covering 75.5 miles on a course 158°(T).
   158°(T) = S22°E

Since the course angle is less than 45°, entering the traverse table with 22°, as course at the top of the page, the d'lat and departure read off against the distance of 75.5 miles gives d'lat = 70.0'S Departure = 28.3'E.

2. Obtain the d'lat and departure for a distance of 50.8 miles on a course 303°(T).
303°(T) = N57°W.

Since the course angle is over 45°, we enter the table with the course angle of 57°, and the other column headings at the bottom of the page. To avoid interpolation, and to obtain greater accuracy, we may read off d'lat and departure against a distance of 508' instead of 50.8', and then divide the values obtained by 10 to obtain the correct d'lat and departure. This is possible as the sides of right angled triangles having the same acute angles are proportional to each other.
d'lat = 27.7'N
departure = 42.6'W

3. A vessel made a d'lat of 343.6'S and a departure of 268.4'W. Find the course and distance made good by her.

Since the d'lat is greater than the departure, the course angle will be less than 45°.

Therefore the traverse table must be inspected with the d'lat and departure headings at the top of the page. By inspection we find that for a course angle of 38°, the d'lat and departure coincide. The distance read off against them is 436 miles.

Since the d'lat was South and the departure was West, the course made good is S38°W i.e. 218°(T) and the distance as already obtained is 436 miles.

4. If a vessel made a d'lat of 135.7'N, and a departure of 364.8E, find the course and distance made good.

Since the departure is more than the d'lat, we enter the table from below. When the departure and d'lat are about equal, the course angle will be around 45°. When the difference in their values is large, the course angle will be closer to 1° or 89°. In this case, since the difference is fairly large, we inspect the table with a course angle of say 75°, and find that for a departure of 364.8', the d'lat is about 97.7', which is too little. We therefore turn the pages towards 45°, and find that the values are agreeing around course angles of 70° or 69°.

For course 70°

departure	d'lat	distance
364.6	132.7	388
364.8	?	?
365.5	133.0	389

By interpolation, for departure 364.8 we get d'lat 132.8 and distance 388.2.

For course 69°

departure	d'lat	distance
364.1	139.8	390
364.8	?	?
365.0	140.1	391

By interpolation for departure 364.8', we get d'lat 140.0' and distance 390.8'.

d'lat	distance	course
132.8	388.2	70
135.7	?	?
140.0	390.8	69

By interpolation we get, Distance = 389.2 miles
Course = 69.6°

As will be observed, the interpolation involved is rather laborious. When the d'lat and departure are known, it would therefore be easier to obtain the course and distance by the expression,

dep / d'lat = tan co
& dist = d'lat. sec co.

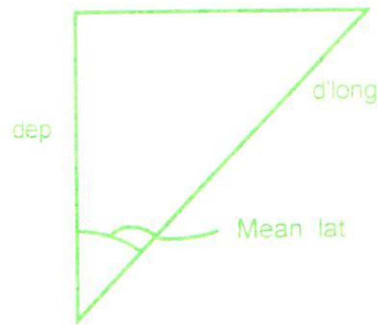
Exercise IV B

In the following table, find the missing values :

No.	course	distance	d'lat	departure
1	S50°E	310	-	-
2	-	-	140.5N	331.0W
3	132°	-	96.3S	-
4	-	153.5	108.6S	-
5	-	-	201.7N	348.0W

Use of the Traverse Tables for the relation dep / d'long = cos mean (or middle) latitude

In middle or mean latitude sailing, we use the expression dep / d'long = cos mean latitude. Since this is a trigonometrical relationship, it can be represented by a right angled triangle, in which the 'Mean Latitude' is the angle, departure is the 'adjacent side' and d'long the 'hypotenuse'.



(FIG.4.5)

As stated earlier, traverse tables are ready-made solutions of right angled triangles. We may therefore use the traverse tables for the above relationship between d'long, departure and mean latitude. Given any two, we can find the third, by entering the traverse table with the mean latitude as the course angle, d'long in the distance column and departure in the d'lat column.

In using the traverse table for this purpose, beginners are advised to exercise care in picking up the required values from the columns, as indicated above.

Examples :

1. Find the dep. for d'long of 5°30' in a mean lat. of 35°30'N.
   35°dep = 270.3
   36°dep = 267.0
   35°30' dep = **268.6 miles**

2. Find the d'long for a departure of 240.4 miles in the mean latitude of 49°20'.
   In Mean lat. 49° for dep. of 240.4', d'long = 366.4'
   In Mean lat. 50° for dep. of 240.4', d'long = 374.0'
   In Mean lat. 49°20' for dep. of 240.4', d'long = 368.9'
   = **6°08.9'**

3. In the following table, find the missing values :

No.	mean lat.	departure	d'long
1	26°	473.3	-
2	51°40'	300.4	-
3	-	274.2	418.0

1. d'long=526.5'=8°46.5'
2. d'long=484.4'=8°04.4'
3. Mean lat=49°

Problems involving use of traverse tables :

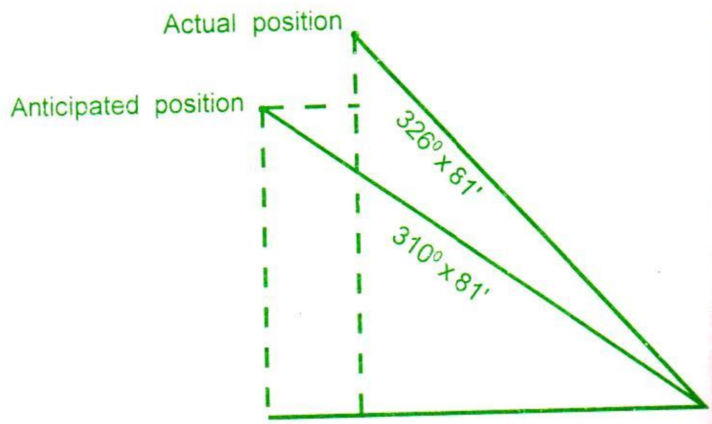
1. A ship proceeding at 18 knots was to steer 310°(T), for the next 4½ hours. After covering the distance, it was found that the compass error of 8°E had been applied the wrong way. Using traverse tables, find how far she is from the anticipated position.

Hint

	True course to steer	=	310°(T)
	Wrong error applied	=	8°(W)
	Compass course steered	=	318°(C)
	Actual error	=	8°(E)
	True course steered	=	326°(T)

The difference between the two d'lats and the two departures gives the d'lat and departure between the position expected to be reached and the position actually reached.

Ans. 22.5 miles.



(FIG.4.6)

2. Two dumb barges in latitude 30°S are in longitude 179°11'W and 179°39'E respectively. Both barges drift with a current setting 150°(T)

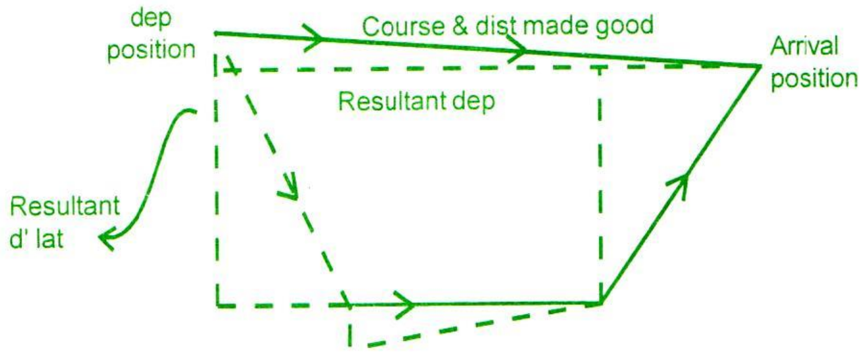
for 50 miles. Find their new positions and distance apart.

Ans. 30°43.4'S, 178°42'W and 30°43.4'S, 179°52'W.
Distance = 60.18 miles

4.4 DAYS WORK

When a vessel sails on several rhumb line courses for short distances, the irregular track that she follows is called a traverse. To find the direct course and distance between the departure and arrival positions, the several rhumb line distances that she sailed may be considered as the hypotenuse of plane sailing triangles.(FIG.4.7)

We can thus obtain the d'lat and departure for each leg of the traverse from the traverse tables. The algebraic sum of the various d'lats and that of the various departures would then give the resultant d'lat and departure that she made between the initial and destination positions.



(FIG.4.7)

The d'lat so obtained when applied to the departure latitude gives the latitude reached. We can then obtain the mean latitude and convert the resultant departure into d'long. The d'long when applied to the longitude of departure gives the longitude arrived at. The position so obtained is termed the **Dead Reckoning (DR)** position. The estimated set and drift of the current during the period under consideration can also be allowed as a leg of the traverse and allowance also made for any leeway in the courses. The final position so obtained would be referred to as the **Estimated position (E P).**

Examples

1. A vessel sailed from lat.27°12'N, long.178°42'E doing 15 kts by engines. She steered 067°(C), {Dev. 3°E}, for 10 hours. Course was then altered to 096°(C) {Dev. 1°E} and this course was maintained for 8 hours. Thereafter she steered, 230°(C), {Dev. 3°W} for another 6 hours. Find the position arrived, if she experienced a current setting 324°(T) at 2.5 knots throughout. Also find the course and distance she made good. Variation 7°W, throughout.

	1st course	2nd course	3rd course	current
Comp.co	067°(C)	096°(C)	230°(C)	
Dev.	3°E	1°(E)	3°W	
Mag.co	070°(M)	097°(M)	227°(M)	
Var.	7°W	7°W	7°W	
True co	063°(T)	90°(T)	220°(T)	324°(T)
Course	N63°E	East	S40°W	N36°W
Dist.	150M	120M	90M	60M

T. Co.	Distance	d'lat		departure	
		N	S	E	W
N63°E	150	68.1	-	133.7	-
East	120	-	-	120	-
S40°W	90	-	68.9	-	57.9
N36°W	60	48.5	-	-	35.3
		116.6	68.9	253.7	93.2

Resultant d'lat	=	116.6'N - 68.9S	=	47.7'N
Resultant dep.	=	253.7'E - 93.2W	=	160.5'E
Departure latitude			=	27°12.0'N
d'lat			=	47.7'N
Arrived lat.			=	27°59.7'N
Mean latitude			=	27°36.0'N

Using mean lat. 27°36' converting departure of 160.5' to d'long, using traverse table, d'long = 181.1'E = 3°01.1'E

Dep. long.	:	178°42.0'E
d'long	:	3°01.1'E
Arrived long.	:	181°43.1'E = 178°16.9'W
Arrived E.P.	:	27°59.7'N, 178°16.9'W

Using the resultant d'lat and dep, the course and distance made good can be found from the traverse table. The course could also be found by the Mercator sailing formula : tan co. = d'long/DMP . The course and distance may also be found by the plane sailing formulas tan co = dep/d'lat and Distance = d'lat x sec co.

tan co = dep/d'lat = 160.5 / 47.7 = 3.3648, thus co = 73°27'
course made good = N73°27'E, since d'lat is North and

departure is East.
Distance = d'lat x sec co. = 47.7 x sec 73°27'
Dist made good = 167.4 miles.

2. At 1200 hours on 25th June, 1992 a point of land in lat. 24°37'N, long. 047°12'W bore 057°(T), dist. off by radar 5.5miles. She then sailed the following courses and distances.

Gyro Co.	Gyro Error	Distance	Wind direction	Leeway
347°	1° High through out	111M	SW	3°
001°		47M	W	Nil
187°		27M	W	1°

Find the estimated arrival position. If the final position by observation was 26°27.5'N, 47°32.2'W, find the set and drift of the current experienced and the course and distance made good.

1st course	1st course	2nd course	3rd course
Gyro course	347°(G)	001°(G)	187°(G)
Gyro error	1°H	1°H	1°H
True course	346°(T)	000°(T)	186°(T)
Leeway	3°(+)	NIL	1°(-)
Course m.g.	349°(T)	000°(T)	185°(T)
Course m.g.	N11°W	NORTH	S5°W
Distance	111 miles	47 miles	27 miles

Note 1 : When the wind is on her port side, the vessel will make good a course to the right of the course steered. Therefore, when the course is expressed in three figure notation, the leeway should be added to the course steered to obtain the course made good. When the wind is on the starboard side, the vessel will make good a course to the left of the course steered and therefore the leeway should be subtracted from the course steered.

Note 2 : The bearing and distance given is that of the point of land from the ship. Therefore the bearing of the ship from the point of land will be the reverse of the given bearing, the distance off being the same. Thus the position of the ship at that instant can be found by applying to the position of the point of land, the d'lat and d'long obtained with the reversed bearing as course and the distance off as distance. Since in this example, the initial position is not required, the final estimated position could be obtained by applying to the position of land, the d'lat and d'long for the various legs of the traverse including the reverse bearing and distance off also as one of the legs.

It is important to note that, while finding course and dist. made good by the vessel, the d'lat and departure for the reverse bearing and distance off should be disregarded, as it is not part of the ship's run. If that d'lat and departure were also considered, the course and distance calculated would be erroneous.

T. Co.	Distance	d'lat		departure	
		N	S	E	W
Rev brg. S57°W	5.5	-	3.0	-	4.6
N11°W	111	109.0	-	-	21.2
N	47	47.0	-	-	-
S 5°W	27	-	26.9	-	2.4
		156.0	29.9	-	28.2

Resultant d'lat 156.0'N - 29.9'S = 126.1'N = 2°06.1'N
Resultant departure = 28.2'W
Latitude of point of land = 24°37.0'N
d'lat = 2°06.1'N
Lat.of estimated arrival position = 26°43.1'N
Mean latitude = 25°40.0'N

For dep. of 28.2 miles in mean lat. 25°40', d'long = 31.2'W
Long. of point of land = 047°12.0'W
d'long = 31.2'W
Long. of estimated arrival position = 047°43.2'W
Estimated position of arrival = 26°43.1'N, 047°43.2'W
Position by observation = 26°27.5'N, 047°32.2'W
d'lat and d'long from
estimated to observed position. :- d'lat 15.6'S, d'long 11.0'E

For d'long 11.0'E in mean lat. 26°35.3'N
dep = 9.8 miles E
tan set = dep / d'lat. = 9.8 / 15.6
set of current = S32°08'E
drift = d'lat x sec set = 15.6 x sec 32°08'
= 18.4 Miles.

To find course and distance made good :-
d'lat made good: 109.0'N + 47.0'N - 26.9'S - 15.6'S = 113.5'N

Departure made good : 21.2'W + 2.4'W - 9.8'E = 13.8'W
tan course made good = dep / d'lat = 13.8/113.5
Course made good = N6°56'W
Distance made good = d'lat x sec co
= 113.5' x sec 6°56'
= 114.34 M

3. Having reset the log. to zero, a ship steered the following courses from noon on 25th June, 1992.

Duration	**Course**	**Total dist.**
1200 - 1900	162°(T)	84
1900 - 2400	122°(T)	144
0000 - 0600	087°(T)	220
0600 - 1200	350°(T)	300

At 1730 hours, a point of land in 42°05'S, 118°28'E was observed to be 4 points on the port bow. At 1810, the point was abeam. Find the course and distance made good, and the DR position at noon on the 26th.

		d'lat		**departure**	
T. Co.	**Distance**	**N**	**S**	**E**	**W**
S18°E	84	-	79.9	26.0	-
S58°E	60	-	31.8	50.9	-
N87°E	76	4.0	-	75.9	-
N10°W	80	78.8	-	-	13.9
		82.8	111.7	152.8	13.9

Resultant d'lat : 111.7'S - 82.8'N = 28.9'S
Resultant dep. : 152.8'E - 13.9'W = 138.9'E

tan course made good = dep / d'lat = 138.9 / 28.9
Course made good S78°15'E.
Distance made good = d'lat x sec co = 28.9 x sec 78°15'
= 141.7M

To find arrival position :
Speed between 1200 and 1900 hours = 84 / 7 = 12 knots
Distance run between 4 points and
beam bearing (12 x 40) / 60 = 8 miles
Beam bearing 162° - 90° = 072°
Reverse bearing S72°W

Departure made good : 21.2'W + 2.4'W - 9.8'E = 13.8'W
tan course made good = dep / d'lat = 13.8/113.5
Course made good = N6°56'W
Distance made good = d'lat x sec co
= 113.5' x sec 6°56'
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N87°E	76	4.0	-	75.9	-
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		82.8	111.7	152.8	13.9

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To find arrival position :
Speed between 1200 and 1900 hours = 84 / 7 = 12 knots
Distance run between 4 points and
beam bearing (12 x 40) / 60 = 8 miles
Beam bearing 162° - 90° = 072°
Reverse bearing S72°W

(FIG.4.8)

D'lat & departure from point of land onwards (1810 to 1200)

Duration	Course	Distance	d'lat N	d'lat S	departure E	departure W
Reverse brg.	S72oW	8	-	2.5	-	7.6
1810-1900	S180E	10	-	9.5	3.1	-
1900-2400	S580E	60	-	31.8	50.9	-
0000-0600	N870E	76	4.0	-	75.9	-
0600-1200	N100W	80	78.8	-	-	13.9
			82.8	43.8	129.9	21.5

Resultant d'lat : 82.8'N - 43.8'S	=	39.0'N
Resultant dep. : 129.9'E - 21.5'W	=	108.4'E
Lat. of point of land	=	42°05.0'S
d'lat	=	39.0'N
Arrival latitude	=	41°26.0'S
Mean latitude	=	41°45.5'S
For departure 108.4', in mean latitude of 41°45.5'S, d'long	=	145.2'E
	=	2°25.2'E
Longitude of point of land	=	118°28.0'E
d'long	=	2°25.5'E
Longitude of arrival position	=	120°53.2'E

26th noon DR 41°26.0'S, 120°53.2'E

The above problems on day's work are only meant to show the principles involved in the solution of such problems, using the traverse table. To gain proficiency in such problems, it is advisable to do more such calculations from any text book on Practical Navigation.

5
NAUTICAL ASTRONOMY

Astronomical navigation requires some knowledge of astronomy.

The term Universe includes all the celestial bodies, as well as the intervening space between them. The Universe consists of innumerable galaxies separated from each other by immense distances. A normal galaxy is a large flattened system consisting of millions of stars, and gas clouds. The galaxies rotate about their centres and are also moving away from each other at phenomenal speeds. An average galaxy has a diameter of about 100,000 light years.

A 'light year' is the distance travelled by light in one year at the speed of 186,000 miles per second (approx. 6 million million miles).

The Sun is an average sized star and belongs to the Milky-Way galaxy. It is situated at a distance of about 30,000 light years from the centre of the galaxy. With the rest of the galaxy, the Sun revolves about the centre of the galaxy, completing one revolution in about 200 million years. From what has been stated above, it will be seen that stars including the Sun are not stationary. The Sun's motion is not apparent to us on the Earth, because the Earth and the other bodies of the solar system are also moving with the Sun. Due to their immense distances from the Earth, stars also do not exhibit any apparent motion. For our purpose therefore, we may consider the Sun and stars as stationary bodies. All the stars we see belong to the Milky Way galaxy. For convenience, we group them into different constellations. Apart from their proper names, stars may also be designated by the constellation to which they belong, pre-fixed by a greek letter, normally in the order of their apparent brightness in that constellation. Thus, apart from the Sun, the closest star situated at a distance of about 4.3 light years from us is called Rigel Kent or α Centauri.

5.1 STELLAR MAGNITUDE

The absolute magnitude of a star is a measure of the actual amount of light emitted by it. The apparent magnitude of a star is a measure of the brightness of that star as observed from the Earth.

The magnitude number of stars decreases as their apparent brightness increases. The increase in apparent brightness is in logarithmic proportion to the decrease in their magnitude number. For instance a second magnitude star is as much brighter than a third magnitude star as the third magnitude star is brighter than a fourth magnitude star and so on.

Stars faintly visible to the naked eye are of the 6th magnitude. 6th magnitude stars are used as the lowest reference for apparent brightness of other celestial bodies. A first magnitude star is 100 times brighter than a 6th magnitude star. Since $100 = (2.51)^5$ approximately, we can state that a 1st magnitude star is $(2.51)^5$ times brighter than a 6th magnitude star from which the first magnitude star is separated by 5 magnitude classes. It should be noted that the index of 2.51 is the difference in magnitude numbers of the two stars. Accordingly a star of magnitude 1.0 is 2.51 times as bright as a star of magnitude 2.0, $(2.51)^2$ times as bright as a star of magnitude 3.0 and so on.

There are some stars and other celestial bodies, namely the planets, Moon and Sun which appear brighter than first magnitude stars. Their magnitude numbers would obviously be less than 0.1 or even negative. The magnitude of Antares is 0.2, that of Canopus is -0.9, that of Sirius is -1.6, that of the Full Moon is -12.5 and that of the Sun is -26.7.

From what has been stated above it will be realized that we can calculate as to how many times one celestial body appears brighter than the other by using the following relation :

Relative brightness = $(2.51)^x$

where x = Magnitude number of less bright body MINUS the magnitude number of more bright body.

The following examples are given as illustration.

Star'A'(mag 4.0) is $(2.51)^2$ times brighter than star'B' (mag 6.0)

Star'C' (mag 0.3) is $(2.51)^{3.1}$ times brighter than star'D' (mag 3.4)

Star'E'(mag -1.6) is $(2.51)^{2.2}$ times brighter than star'F'(mag 0.6)

Full Moon (mag -12.5) is $(2.51)^{10.9}$ times brighter than star Sirius (mag -1.6).

The apparent magnitude of all stars and planets used for navigation are listed in the Nautical Almanac.

5.2 THE CELESTIAL SPHERE

To an observer on the Earth, the heavens appear to be an inverted hemisphere, with the Earth at its centre. The other half of the sphere, below his horizon, is not visible to him. All the celestial bodies appear projected on the inside of this sphere. Thus, the Earth appears to be at the centre of the Universe. This, we know is not true. For the purpose of navigation however, we may assume the Earth to be at the centre of a sphere of infinite radius, on the inside surface of which, all the celestial bodies are situated.

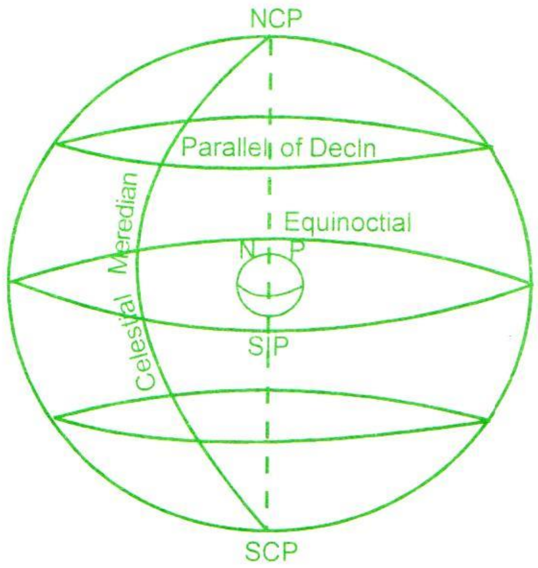
The Celestial Sphere is a sphere of infinite radius with the centre of the Earth as its centre.

In the following definitions, the similarity between the concepts and definitions pertaining to the celestial sphere and those pertaining to the Earth's surface should be noted.

Celestial Poles are the two points on the celestial sphere where the axis of the Earth produced would meet it.

Celestial Equator (Equinoctial)

is a great circle on the celestial sphere in the same plane as the plane of the Earth's Equator. Thus the Equinoctial is a projection of the Equator on the celestial sphere. Every point on the Equinoctial is 90° from the celestial Poles.



(FIG.5.1)

Parallels of declination

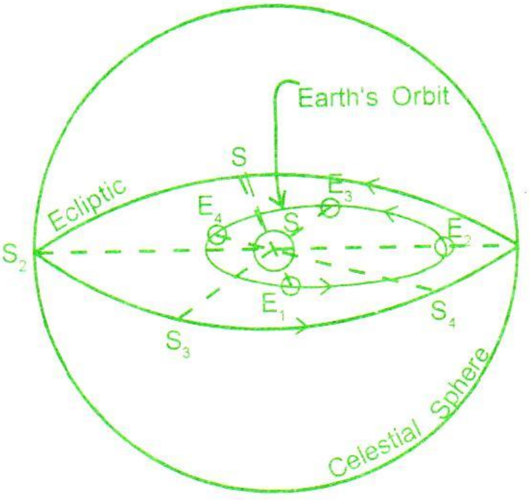
are small circles on the celestial sphere, the planes of which are parallel to that of the Equinoctial. These correspond to parallels of latitude on the Earth's surface.

Celestial meridians

are semi great circles on the celestial sphere , the planes of which pass through the celestial poles. These correspond to the meridians on the Earth.

Ecliptic

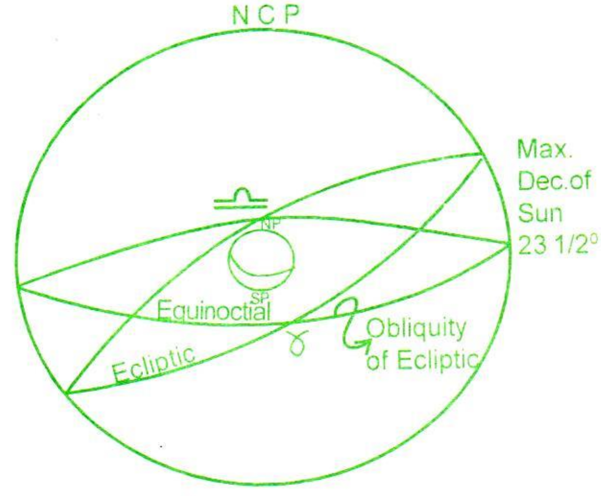
is a great circle on the celestial sphere in the same plane as the plane of the Earth's orbit around the Sun. Thus the Sun's apparent annual path on the celestial sphere is the Ecliptic. It is so called because the Sun, Moon and Earth must be on this plane for a solar or lunar eclipse to occur.



(FIG.5.2)

In Fig.5.2 when the Earth is at E_1 in its orbit around the Sun, the Sun appears to be at S_1 on the celestial sphere. When the Earth is at E_2, the Sun appears to be at S_2, and so on. The apparent path of the Sun around the Earth is therefore along a great circle called the Ecliptic, on the celestial sphere.

As stated later, in this chapter, the orbit of the Earth around the Sun and thus the Sun's apparent orbit around the Earth is an ellipse. The Ecliptic is a projection of this ellipse on to the celestial sphere. The plane of the Earth's orbit and therefore that of the Ecliptic is inclined at about 23½° to that of the Equinoctial. As the Sun appears to move along the Ecliptic, the maximum declination of the Sun, North and South is equal to this angle.



(FIG.5.3)

Obliquity of the Ecliptic is the angle between the plane of the Equinoctial and that of the Ecliptic. Its value is approx. 23½°.

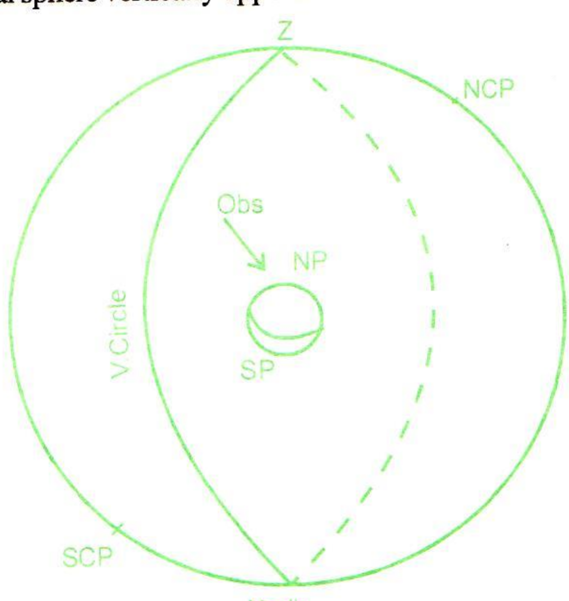
Zodiac is a belt on the celestial sphere extending 8° on each side of Ecliptic, within which the Sun, the Moon and the planets are always found. The belt of the zodiac is divided into 12 equal parts of the length 30° each. These are named after groups of stars or constellations within them. They are Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius and Pisces.

First point of Aries and First point of Libra

The two points on the celestial sphere, where the Ecliptic intersects the Equinoctial are called the Equinoctial points. On 21st March, at Vernal Equinox, the Sun appears to cross the Equinoctial from South to North. This point is known as the First point of Aries. It is denoted by the symbol γ. On 23rd September, at Autumnal Equinox, the Sun appears to cross the Equinoctial from North to South. This point is known as the First point of Libra, denoted by the symbol ♎.

The First point of Aries and the First point of Libra were named after the constellations in which they once lay. These points are however moving westward slowly, along the Ecliptic. Due to this, the 1st point of Aries is no longer in the constellation of Aries. It is now in the constellation of Pisces.

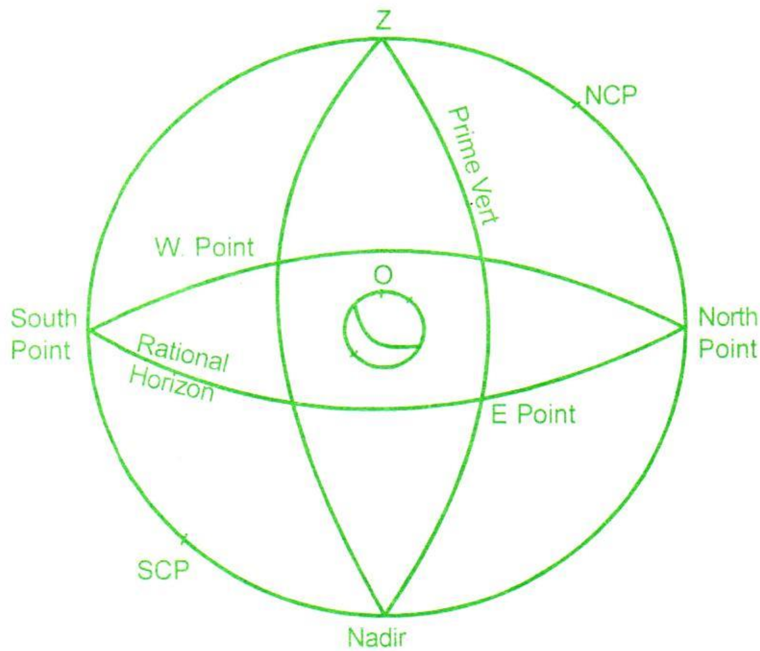
The Observer's Zenith is the point on the celestial sphere vertically above the observer i.e. the point at which a straight line from the centre of the Earth through the observer meets the celestial sphere. **The observer's Nadir** is the point on the celestial sphere vertically opposite his Zenith.



(FIG.5.4)

Vertical circles are great circles on the celestial sphere passing through the observer's Zenith and Nadir.

Prime vertical The observer's Prime vertical is the vertical circle passing through the East and West points of his rational horizon.



(Fig. 5.5)

Position on the celestial sphere A position on a sphere, may be defined by stating the angles at the centre of the sphere, or the great circular coordinates of that position, with respect to two reference great circles which are at right angles to each other.

Positions on the Earth's surface, for instance, are defined by stating such angles or coordinates with respect to two reference great circles, the Equator from which latitudes are measured, and the Prime meridian, from which longitudes are measured.

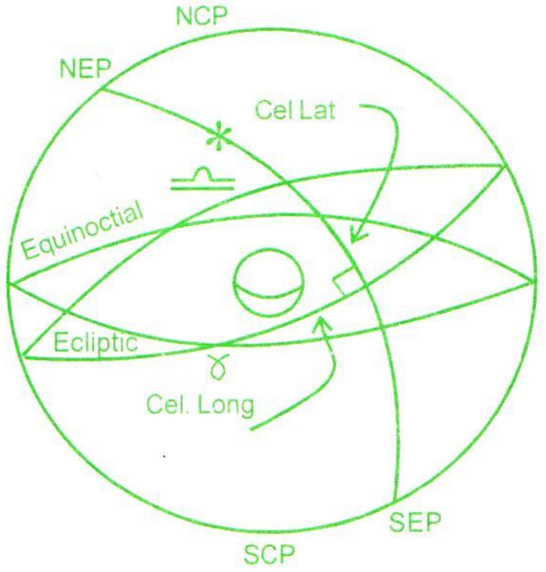
There are three main systems of defining a position on the celestial sphere.

1. The Ecliptic system

2. The Equinoctial system and

3. The Horizon system

In the Ecliptic system, (Fig.5.6) the coordinates used are celestial latitude, and celestial longitude, the reference great circles being the Ecliptic and the secondary to the Ecliptic passing through the First point of Aries. (Secondaries to a great circle are great circles passing through its poles).



(FIG.5.6)

Celestial latitude of a body is the arc of the secondary to the Ecliptic (passing through the body) contained between the Ecliptic and the body. Celestial latitudes are measured from 0° to 90°, North or South of the Ecliptic.

Celestial longitude of a body is the arc of the Ecliptic contained between the First point of Aries and the secondary to the Ecliptic through that body measured eastwards from Aries.

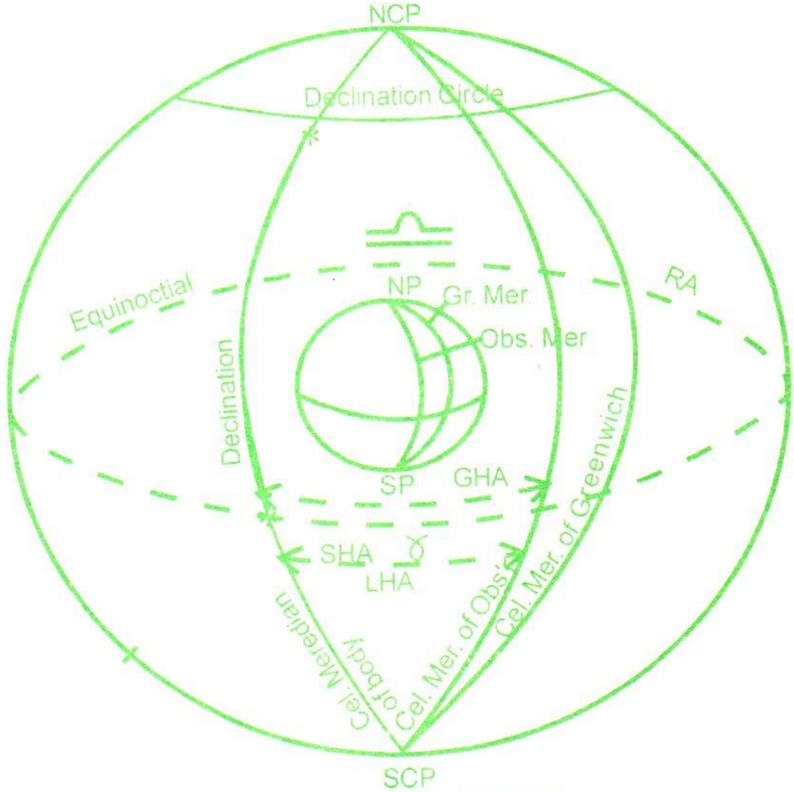
The Ecliptic system is not commonly used by navigators.

5.3 EQUINOCTIAL SYSTEM

In this system the reference great circles are (a) the Equinoctial and (b) the celestial meridian through the First point of Aries or the celestial meridian of Greenwich or the celestial meridian of the observer. The coordinates used are declination, and hour angle (Sidereal hour angle when measured from the celestial meridian of γ, Greenwich hour angle when measured from that of Greenwich and local hour angle when measured from that of the observer).

Declination of a celestial body is the arc of a celestial meridian or the angle at the centre of the Earth contained between the Equinoctial and the parallel of declination through that body. Declinations are measured from 0° to 90° N or S of the Equinoctial.

Sidereal Hour Angle (SHA) of a celestial body is the arc of the Equinoctial or the angle at the celestial pole contained between the celestial meridian of the First point of Aries and that through the body, measured westward from Aries.



(FIG.5.7)

Right Ascension (RA) of a celestial body is the arc of the Equinoctial or the angle at the celestial poles contained between the celestial meridian of the First point of Aries and that through the body, measured eastward from Aries. RA may also be expressed in hours, minutes and seconds, instead of, in arc.

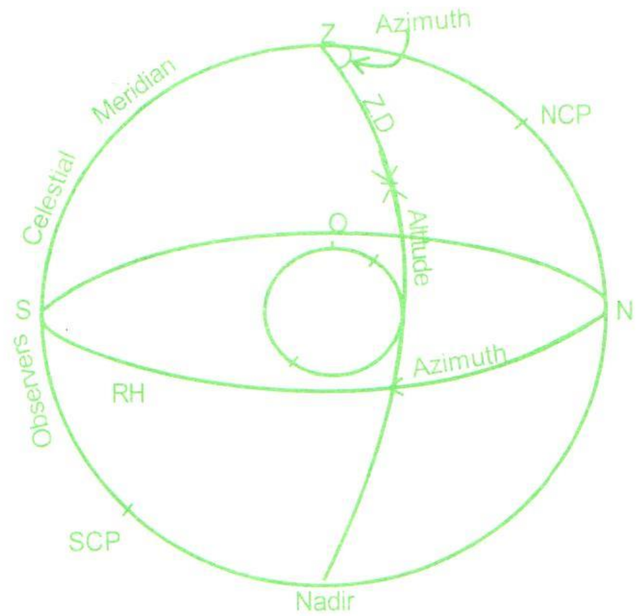
It should be noted that, since SHA is measured westward and RA eastwards from the same point, the SHA and RA of any body will together always add up to 360°.

Greenwich hour angle (GHA) of a celestial body is the arc of the Equinoctial or the angle at the celestial poles contained between the celestial meridian of Greenwich and that of the body, measured westward from Greenwich.

Local Hour Angle (LHA) of a celestial body is the arc of the Equinoctial or the angle at the celestial poles contained between the observer's celestial meridian and the celestial meridian through that body, measured westward from the observer. If the angle or arc is measured eastward from the observer, it is known as the Easterly Hour Angle (EHA) and not LHA. It can therefore be seen that the LHA of a body equals 360°-its EHA.

5.4 HORIZON SYSTEM

In this system the reference great circles are a) the observer's rational or celestial horizon and b) his celestial meridian. The coordinates used are a) **altitude** or **Zenith dist.** and b) **Azimuth.**



(FIG.5.8)

Celestial or Rational Horizon The observer's rational horizon is a great circle on the celestial sphere, every point on which is 90° away from his zenith.

True altitude of a body is the arc of the vertical circle through that body contained between the rational horizon and the centre of the body.

Zenith distance of a body is the arc of the vertical circle through the body contained between the observer's zenith and the centre of the body.

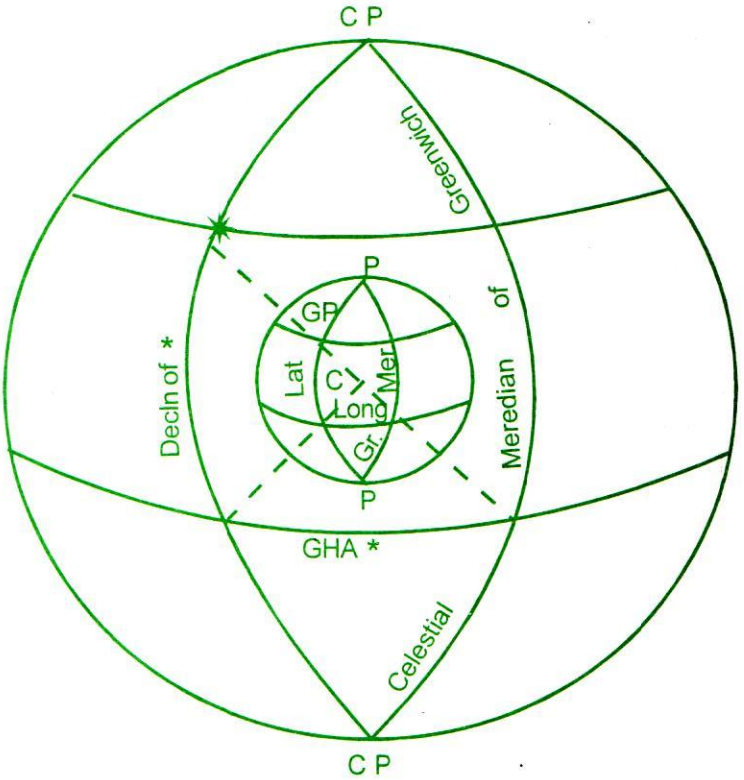
Since every point on the rational horizon is 90° from the observer's zenith, the zenith distance = 90° - altitude.

Azimuth The azimuth of a celestial body is the arc of the observer's rational horizon or the angle at his zenith contained between the observer's celestial meridian and the vertical circle through that body.

Amplitude of a celestial body is the arc of the observer's rational horizon or the angle at his zenith, contained between the observer's prime vertical and the vertical circle through the body, when the body is on the observer's rational horizon i.e. at theoretical rising or setting. Amplitude is therefore measured N or S from the observer's East point when the body is rising, and from his West point when setting e.g. E20°S or W15°N etc.

The coordinates of the position of a celestial body, defined using the horizon system, would vary depending on the observer's position on the Earth, because its altitude and azimuth at any instant would have different values when measured from different positions on the Earth. The nautical almanac therefore lists the position of celestial bodies using the Equinoctial system by tabulating the Declination and GHA or SHA of the celestial bodies.

In celestial navigation, where determination of the observer's position is the prime objective, the problem is solved by correlating the coordinates of a celestial body in the Equinoctial system, with those in the horizon system for the instant at which the altitude of the body was observed.



(FIG.5.9)

Geographical position of a celestial body is the point on the surface of the Earth, vertically beneath that body i.e. the point at which a straight line from the centre of the Earth to the celestial body meets the Earth's surface.

The GP being on the Earth's surface, is always expressed in terms of latitude and longitude. Since the centre of the celestial sphere is the Earth's centre and as the Equator and the Equinoctial are in the same plane, the latitude of a celestial body's geographical position is equal to the body's declination. The longitude of its GP corresponds to its GHA.

GHA is measured from 0° to 360°, westwards from Greenwich, while longitude is measured from 0° to 180°E and 0° to 180°W from Greenwich. The GHA of the body, if less than 180° will therefore be equal to the West longitude of its GP. If the GHA is more than 180° the long. of its GP will be (360°-GHA)East.

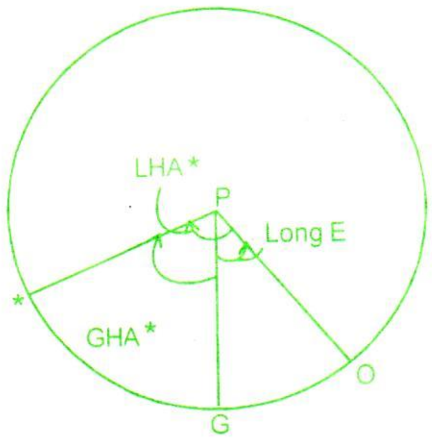
The d'long between the longitude of the GP of a body and that of the observer will be the body's hour angle from the observer. The great circle bearing of the GP of a celestial body from the observer's position corresponds to the azimuth of the body on the celestial sphere.

5.5 IMPORTANT RELATIONSHIPS

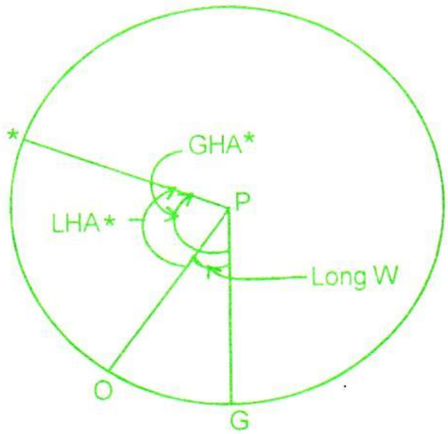
With the help of the figures below, the student should note some important relationships. He should also be in a position to draw such figures by himself and to prove similar relationships or to deduce required values.

LHA* = GHA* + Long.E
LHA* - GHA* = Long.E

LHA* = GHA* - Long.W
GHA* - LHA* = Long.W



(FIG.5.10)

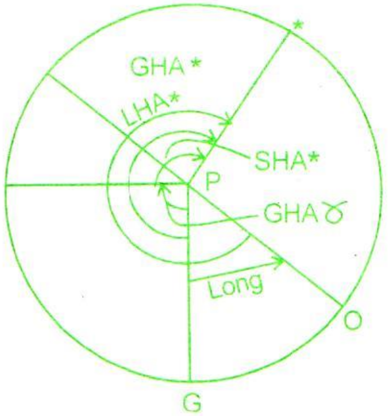


(FIG.5.11)

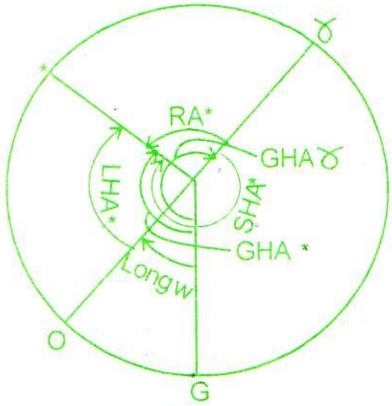
The figures (5.10 to 5.13) are drawn on the plane of the Equinoctial i.e. looking down on the celestial sphere from above the North celestial Pole. The outer circle therefore represents the Equinoctial. The celestial Pole appears at the centre. Celestial meridians radiate from the Pole. West-ward angles and arcs are measured clockwise. Eastward angles and arcs are measured counter-clockwise. The angle at the Pole, between any two meridians is equal to the corresponding arc on the Equinoctial.

GHA* = GHAγ + SHA*
LHA* = GHAγ + SHA* + Long.E

GHA* = GHAγ - RA*
LHA* = GHAγ + SHA* - Long.W



(FIG.5.12)

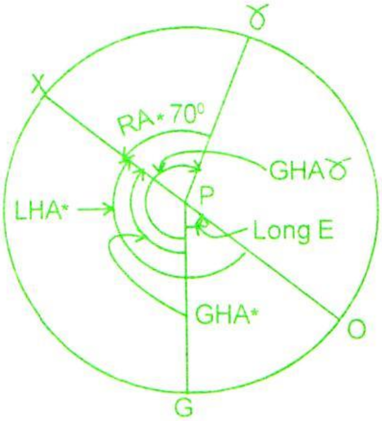


(FIG.5.13)

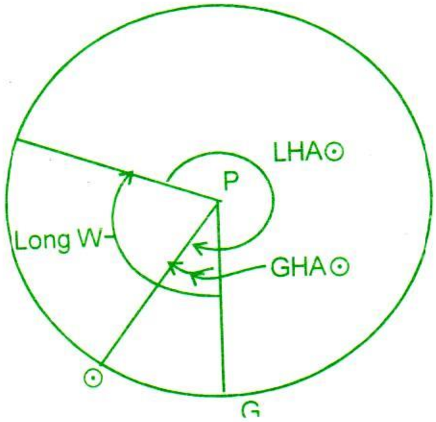
Examples :

1. Calculate the LHA of a star whose RA is 70°, for an observer in longitude 47°E, when GHAγ is 210°.

GHAγ	=	210°
RA*	=	70°
GHA*	=	140°
Long.(E)	=	47°
LHA*	=	187°



(FIG.5.14)



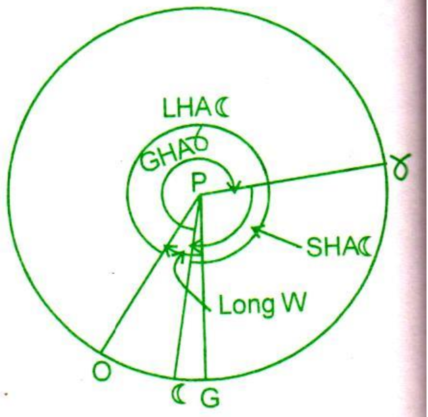
(FIG.5.15)

2. To an observer the Sun's LHA was 290°, when its GHA was 40°. Find the observer's longitude.

LHAS	=	290°
GHAS	=	40°
Major arc GPO	=	250°
Longitude	=	360° - 250°
	=	110°W

3. On a certain day in longitude 35°W, the Moon's LHA was 335°, when GHAγ was 263°. Find the SHA of the Moon.

LHA Moon	=	335°
Long.(W)	=	35°
GHA Moon	=	370°, (10°)
GHAγ	=	263°
SHA Moon	=	107°



(FIG.5.16)

EXERCISE V

1. The planet Venus was on the meridian of an observer in longitude 62°E. If the RA of Venus at that instant was 87°, find the GHA of a star, the SHA of which then was 162°.

2. State the GP of the Moon, when its GHA = 242° and dec 22°S.

3. What is the GP of the First point of Aries, when LHA γ was 112° for an observer in longitude 20°E.

Theory Questions

1. Define and illustrate by figures where necessary :
   (1) Celestial Sphere (2) Celestial Poles
   (3) Equinoctial (4) Celestial Meridian
   (5) Ecliptic.

2. Define
   (1) Equinoctial Points (2) Observer's Zenith
   (3) Vertical Circles (4) Prime Vertical
   (5) Declination (6) SHA
   (7) RA

3. Define and explain with the help of figure :
   (1) GHA (2) LHA
   (3) Rational Horizon (4) Zenith distance
   (5) Azimuth (6) Amplitude.

4. What do you understand by the term Geographical Position of a heavenly body ? What are the coordinates used to specify a Geographical Position?

6

SOLAR SYSTEM

The Solar system consists of the Sun, the planets, the planetary satellites, asteroids, comets and meteors. The most important member of the Solar system is the Sun. In mass, it is more than 700 times larger than all the other bodies of the Solar systems taken together. It has a diameter of about 865,000 miles. The Sun is the only body in the Solar system which radiates light. It rotates on its own axis, completing one rotation in about 25 days.

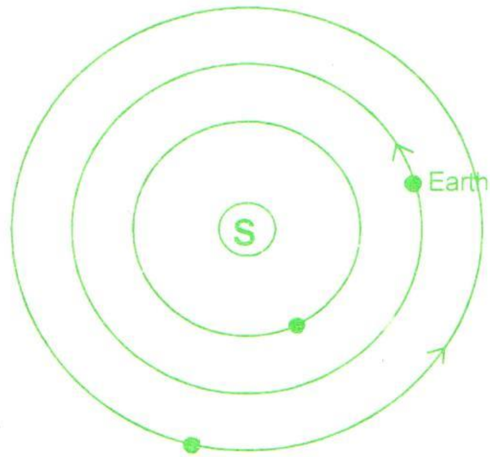
Next in importance to the Sun are the nine planets. Planets are not self luminous. We see them only because they reflect light from the Sun. Due to this fact, when viewed through a powerful telescope, it will be seen that they also exhibit phases like the Moon. In the order of their distance from the Sun, they are, Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto. Between the orbits of Mars and Jupiter, there are a large number of minor planets called asteroids.

Name	Mean dist. from Sun in miles	Diameter (in miles)	Period of rotation	Period of revolution round the Sun
Sun	-	865400	25.14 days	-
Mercury	$36x10^6$	3000	88 days	88 days
Venus	$67.3x10^6$	7848	not known	224.7days
Earth	$93x10^6$	7927	23 hrs.56 mins	365.25days
Mars	$141.7x10^6$	4268	24 hrs.37 mins	687 days
Jupiter	$483.9x10^6$	89329	09 hrs.50 mins	11.86 yrs
Saturn	$887.9x10^6$	75021	10 hrs.02 mins	29.46 yrs
Uranus	$1783.9x10^6$	33219	10.8 hrs.	84 yrs
Neptune	$2795.4x10^6$	27700	15.8 hrs.	164.8 yrs
Pluto	$3675.0x10^6$	3600	6.39 days	248.4 yrs

From the above table, it can be seen that the nine planets can be divided into two groups, the four small planets of the inner group (Mercury, Venus, Earth and Mars) and the five large planets of the outer group (Jupiter, Saturn, Uranus, Neptune and Pluto). The two planets Mercury and Venus which are closer to the Sun than the Earth are called the **Inferior planets.**

The six planets which are further away from the Sun than the Earth are called **Superior planets.**

All planets revolve about the Sun in a counter clockwise direction in elliptical orbits. They also rotate on their own axes in that direction.

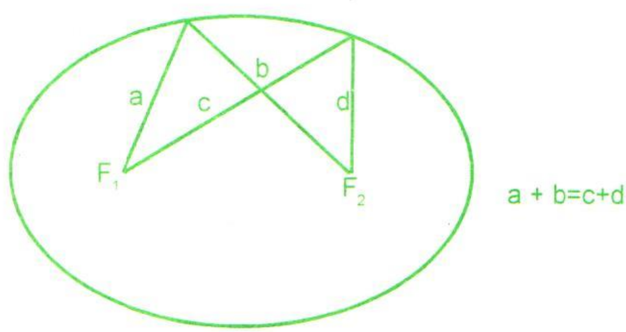


(FIG.6.1)

Fig.6.1 shows diagrammatically, the orbit of the Earth and those of an inferior and a superior planets. As shown, the orbital motion of all planets around the Sun is 'direct' or eastwards.

6.1 PLANETARY MOTION

Kepler's First law states that all planets revolve about the Sun in elliptical orbits with the Sun situated at one of the foci of the ellipse. An ellipse is a locus of a point, such that the sum of the distances from the point to the two foci of the ellipse is always constant. This is illustrated in the Fig.6.2.



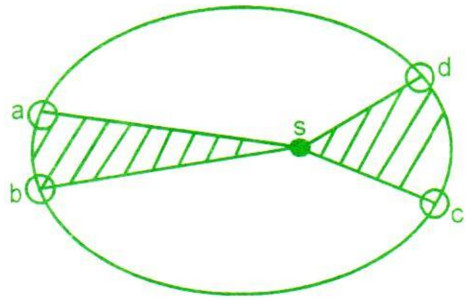
(FIG.6.2)

Though very correctly, the orbits of planets are elliptical, they are infact nearly circular. The ellipticity of the Earth's orbit is only about 1/7200.

The orbits of the various planets, except that of Pluto are also very nearly coplaner.

Kepler's Second Law

states that the radius vector of a planet (a line joining the centre of the Sun to the centre of the planet) sweeps out equal areas in equal periods.



(FIG.6.3)

For equal areas to be swept out in equal periods, the planets moves faster in its orbit when it is closer to the Sun and slower when it is further away.

A planet is said to be in **Aphelion,** when in its orbit, it is farthest from the Sun. It is said to be in **Perihelion,** when in its orbit, it is nearest to the Sun. Because the Sun is eccentric within the Earth's orbit, at aphelion, the Earth is 94.45 million miles and at perihelion, 91.35 million miles from the Sun. The average distance between Sun and Earth is 93 million miles. The eccentricity of the Earth's orbit is about 1/60. In the terms 'aphelion' and 'perihelion', we use the suffix 'helion' (for the Sun) as the distances were expressed from the Sun. If distances are expressed from the Earth, we use the suffix 'gee' (for geographic). Thus, when the Sun in its apparent orbit or the Moon in its orbit around the Earth, is nearest the Earth, they are said to be in **perigee**, and when farthest from the Earth, they are said to be in **apogee**.

Similarly when distances are expressed from the Moon, we use the suffix 'cynthion' or 'lune' (for the Moon) leading to the terms **apocynthion** or **apolune** and **pericynthion** or **perilune.**

Kepler's Third Law

gives the relationship between the distance of a planet from the Sun and the time it takes to complete one revolution around the Sun. According to this law, planets which are closer to the Sun have a greater angular orbital velocity than planets which are further away.

The planets used for celestial navigation are Venus, Mars, Jupiter and Saturn. Apart from the Sun and Moon, Venus is the brightest celestial body, visible in the mornings before sunrise or evenings after sunset. Mars is the reddish planet. Jupiter is the largest planet in the Solar system. When viewed through a powerful telescope 'Saturn' is distinguished by the rings around it.

Some of the planets have satellites or moons. Mercury, Venus and Pluto have no moons. The Earth has one, Mars and Neptune have two each, Jupiter has 12, Saturn has 9 and Uranus has 5 moons. Recent space probes have indicated more moons for some of the planets. The moons also rotate on their own axes and revolve around the parent planet in elliptical orbits, with the parent planet at one of the foci of the ellipse. In general the moons revolve about the parent planet in the same direction as the planets revolve about the Sun. Like our Moon, satellites are not self luminous. We see them due to the sunlight they reflect.

Comets

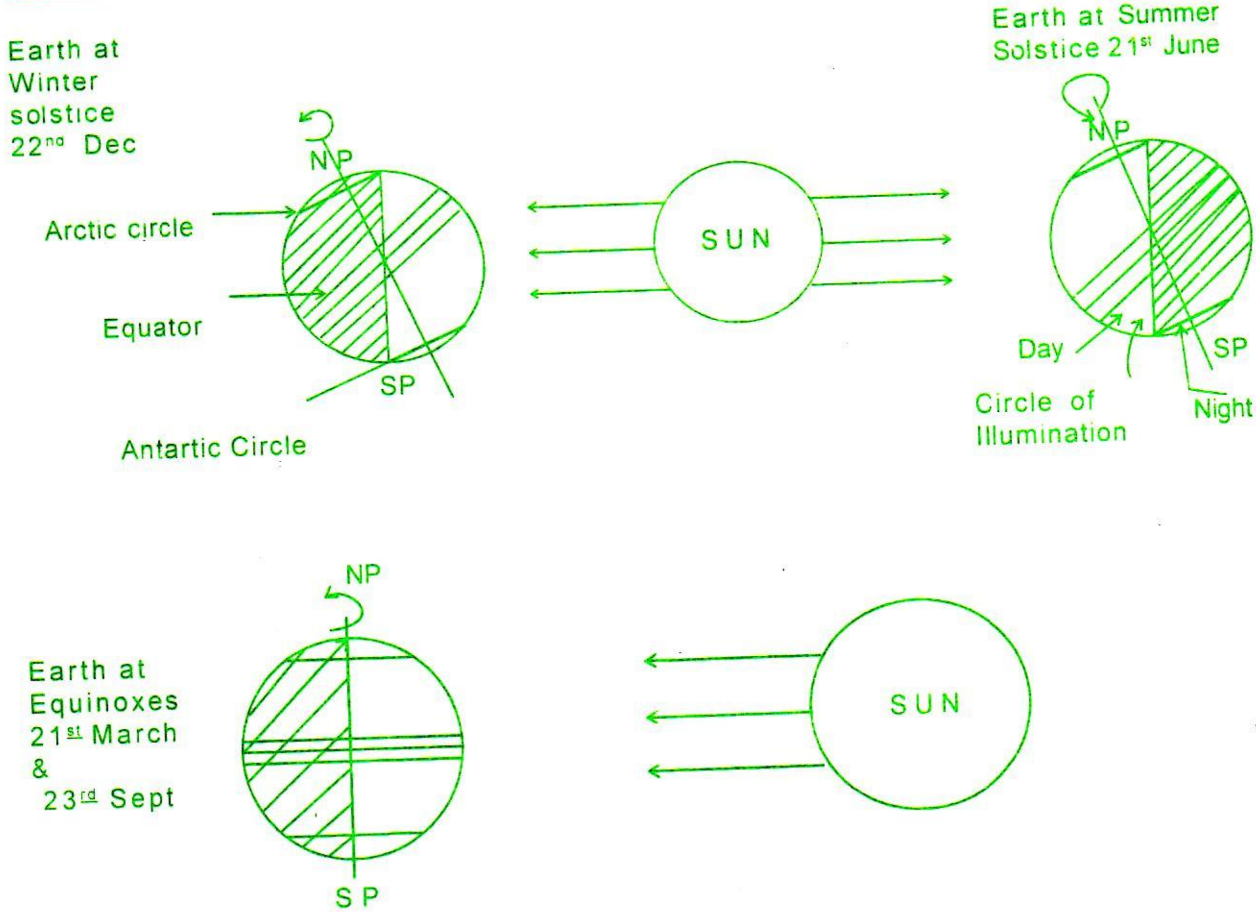
are made up of particles of meteoric matter, fine dust and frozen gasses, and water vapour. Therefore they have small mass. In general they orbit the Sun in very elongated elliptical orbits. We see them only when they approach close enough to the Sun to reflect sufficient Sun light to be visible from the Earth. It is thought that the radiation from the Sun causes the matter of the comet to stream away from its nucleus. Comets are therefore seen with their 'tails' generally in a direction away from the direction to the Sun. The orbital periods of the different comets vary from about 3 years to more than 1000 years. The most spectacular comet is the Halley's comet with a period of about 76 years.

Meteors

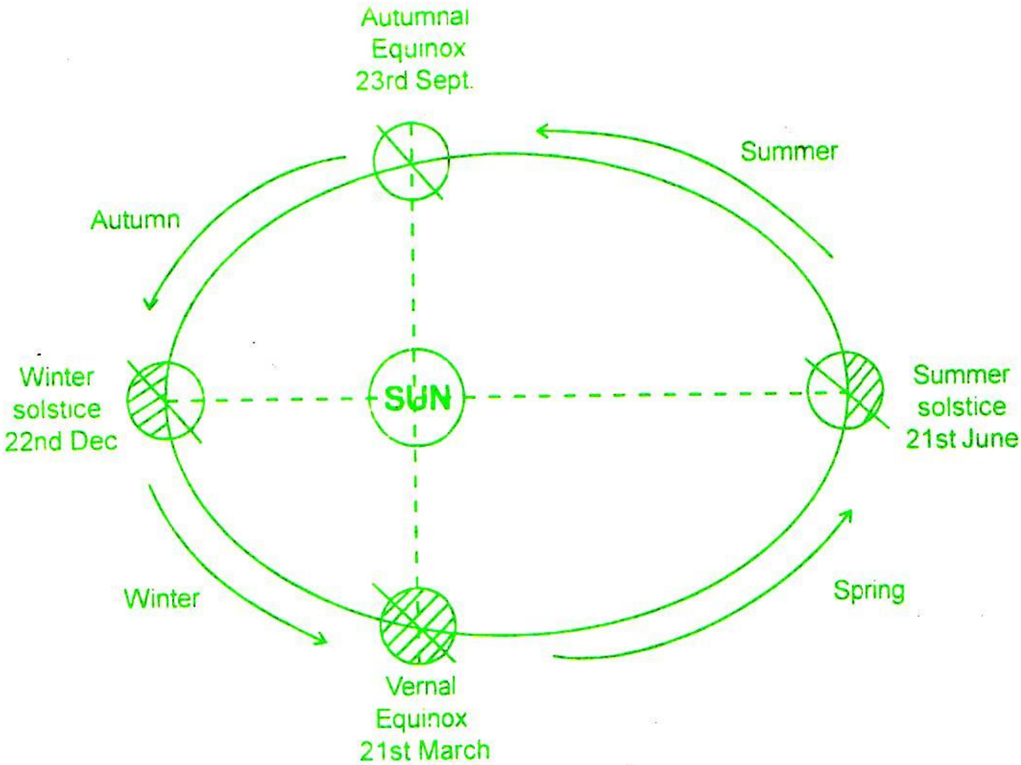
commonly called 'shooting stars' are small bits of space debris, frequently originating from comets. On passing close to the Earth, they are attracted by the Earth. When they pass through the Earth's atmosphere, they heat up and glow due to friction, appearing as a flash across the sky. Most meteors burn out in the atmosphere. Large ones may reach the ground, when they are called **meteorites.**

Day and Night and Seasons on the Earth.

The Earth revolves around the Sun in an elliptical orbit. At the same time, the Earth also rotates on its axis from West to East, completing a rotation in about in 24 hours. Since the Earth is nearly spherical, 50% of the Earth's surface is illuminated by the Sun's rays at any given time. The other 50% is in darkness. The circle bounding the illuminated hemisphere is known as the circle of illumination. As the Earth rotates, places on the Earth's surface successively pass through the illuminated zone and the zone of darkness, causing day and night respectively.


Earth at Winter solstice 22nd Dec
NP
Arctic circle
Equator
SP
Antartic Circle
SUN
Earth at Summer Solstice 21st June
NP
Day
SP
Circle of Illumination
Night
NP
Earth at Equinoxes 21st March & 23rd Sept
SP
SUN

(FIG.6.4)



(FIG.6.5)

In the discussion which follows, the student should refer to fig. 6.4 and 6.5. The axis of the Earth is inclined to the plane of its orbit at about 66½°. While the axis maintains its direction in space, its direction with respect to the Sun, changes according to the position of the Earth in its orbit. Let us consider the Earth at four important points in its orbit. On 21st June, when the North end of the Earth's axis i.e. the North Pole is tilted towards the Sun by the maximum amount of 23½°, the circle of illumination encloses the entire Arctic circle. On this date, the Sun attains its maximum declination North and the Sun's rays fall vertically over the Tropic of Cancer. The Sun is then said to be at the **Summer solstice**. All places in the Northern hemisphere then have the longest day and shortest night, while places in the Southern hemisphere have the shortest day and the longest night. Places within the Arctic circle have continuous daylight, while places within the Antarctic circle have continuous night.

On the 23rd of September, the tilt of the Earth's axis is in a direction at right angles to the direction from Earth to Sun. The Sun's rays then fall vertically over the Equator and the Sun's declination is 0°. The circle of illumination passes through the two poles. All places on the Earth have equal day and night of 12 hours duration each i.e. the Sun would rise at 6 a.m. and set at 6 p.m. through out the world. The Sun is now said to be at the **Autumnal equinox**.

On the 22nd of December, the South end of the Earth's axis i.e. the South Pole is tilted towards the Sun by the maximum amount of 23½°. On this date, the Sun is said to be at the **Winter solstice**, as it attains its maximum declination South. The Sun's rays then fall vertically over the Tropic of Capricorn. The circle of illumination now encloses the entire Antarctic circle. All places in the Southern hemisphere then have the longest day and shortest night, while places in the Northern hemisphere have the shortest day and longest night. Places within the Antarctic circle have continuous day light, while places within the Arctic circle have continuous night.

On the 21st of March, the Earth's axis is again titled in a direction at right angles to the direction from Earth to Sun. The Sun's rays again fall vertically over the Equator, and the declination of the Sun is zero. The Sun is then said to be at the **Vernal equinox.** On this date also the circle of illumination passes through the two poles, and all places on the Earth again have equal days and nights, of 12 hours duration each. The Sun once again rises at 6 AM and sets at 6 PM throughout the world.

From Vernal equinox (21st March) to Autumnal equinox, (23rd September), the North Pole of the Earth is tipped towards the Sun. Places in the Northern hemisphere, would therefore remain in the illuminated hemisphere for longer periods and in the zone of darkness for shorter periods. Therefore they would have longer periods of day light and shorter periods of night. It can be seen from the figure that the reverse would be the case in the Southern hemisphere.

From Autumnal equinox (23rd September) to Vernal equinox (21st March), the South Pole of the Earth is tipped towards the Sun, causing places in the Southern hemisphere to remain in the illuminated hemisphere for longer periods and within the zone of darkness for shorter periods. During these six months therefore, places in the southern hemisphere, have longer periods of day light and shorter periods of night. The reverse would be the case in the Northern hemisphere.

From the above discussion, it should be noted that, in latitudes of the same name as the Sun's declination, the period of daylight is longer than the period of night; while in latitudes contrary in name to the Sun's declination, the period of night is longer than the period of day light. As the Sun's declination increases, the inequality between the periods of day light and night in all latitudes (both North and South hemispheres) will increase because the circle of illumination would then divide the various circles of latitudes into more and more unequal,illuminated and dark segments.

From a reference to Fig. 6.4, it should also be apparent that for any declination of the Sun, other than nil, the illuminated and dark segments into which the circles of latitude are divided by the circle of illumination become more unequal as the latitude increases. The inequality between the period of daylight and the period of night therefore also increases as the latitude increases.

Whatever the declination of the Sun, the circle of illumination always divides the Equator into two equal halves, so that places on the equator have 12 hours of day light and 12 hours of night, throughout the year.

From Vernal equinox, to Summer solstice, i.e. the period when the Sun's declination is increasing from 0° to its maximum value of 23½°N, the Northern hemisphere is said to have Spring season. From Summer solstice to Autumnal equinox, when the Sun's declination decreases from a maximum of 23½°N to 0°, the Northern hemisphere is said to have summer season. From Autumnal equinox to Winter solstice, when the Sun's declination increases from 0° to the maximum of 23½°S, the Northern hemisphere is said to have Autumn season. From Winter solstice to Vernal equinox when the Sun's declination decreases from 23½°S to 0°, the Northern hemisphere is said to have Winter season. It should be noted that the 4 seasons are not of equal lengths.

The Earth is at Perihelion on 1st January and at aphelion on 4th July. The Earth moves faster in its orbit, when it is closer to the Sun and slower when it is further away. The varying speed of the Earth in its orbit causes the seasons to be of unequal lengths, approximately as follows. Spring : 93 days; Summer : 94 days; Autumn : 90 days; and Winter : 89 days.

(FIG.6.5a)

The Earth rotates on its axis from West to East i.e. counter clockwise as viewed from above the North Pole, completing one rotation in 23 hours 56 minutes 04.1 seconds of Mean Solar time. Thus the entire celestial sphere appears to rotate in the opposite direction i.e. from East to West completing an apparent rotation of 360° in about 24 hours. The GHA'S of celestial bodies which are measured westward from the celestial meridian of Greenwich, therefore increase by approximately 15° per hour.

This apparent rotation of the celestial sphere causes all celestial bodies to rise over the Eastern horizon. Thereafter, they appear to sweep across the sky, increasing in altitude, till they reach the observer's meridian bearing due North or South of the observer.

When a celestial body is on the observer's meridian, it is said to culminate. This is also referred to as the 'Meridian passage' or the 'Meridian transit' of the body. At culmination, a body of constant declination attains its maximum altitude for a stationary observer, and therefore, it attains its minimum zenith distance.

After culmination, the body appears to continue its westward motion reducing in altitude, till it sets below the western horizon. The apparent diurnal paths of celestial bodies on the celestial sphere, are along circles with the Celestial Pole as their centre.

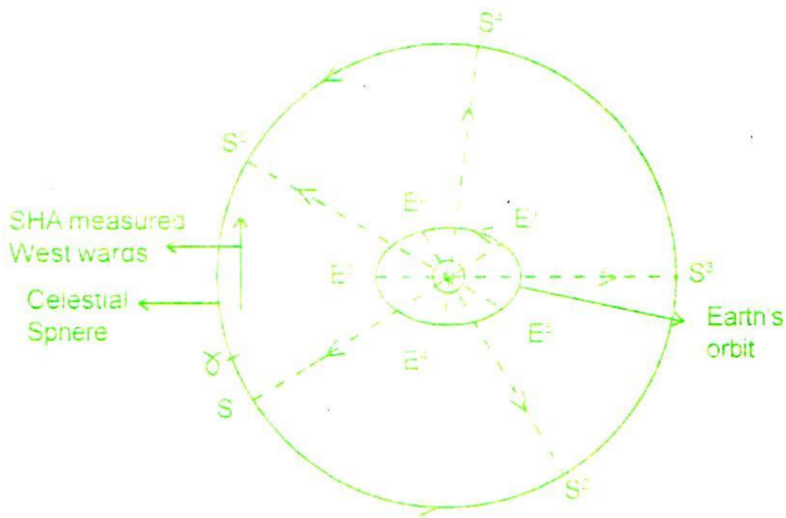
6.2.2. Apparent Motion of the Celestial Bodies due to orbital motion of the Earth.

Besides the apparent diurnal motion of the celestial bodies due to the Earth's rotation, the motion of the Earth in its orbit also causes an apparent change in the position of nearby celestial bodies on the celestial sphere.

The true orbital motion of the planets and the Moon further modifies the apparent motion of these nearby bodies caused by the movement of the Earth in its orbit. Because of the immense distances of the stars from the Earth, the motion of the Earth in its orbit does not produce any appreciable change in the directions to the stars as seen from the Earth. Thus, to an observer on the Earth, the stars appear as fixed objects on the celestial sphere. Similarly, the position of the First point of Aries also appears fixed on the celestial sphere. Since, we have a background of fixed stars, on the celestial sphere, we may study the apparent motion exhibited by the Sun, Moon and Planets, against the background of the Stars.

6.2.3 Apparent motion of the Sun

The Earth orbits the Sun in an eastward direction. Therefore, as observed from the Earth, the Sun appears to move eastwards on the celestial sphere, in the same plane as the plane of the Earth's orbit.



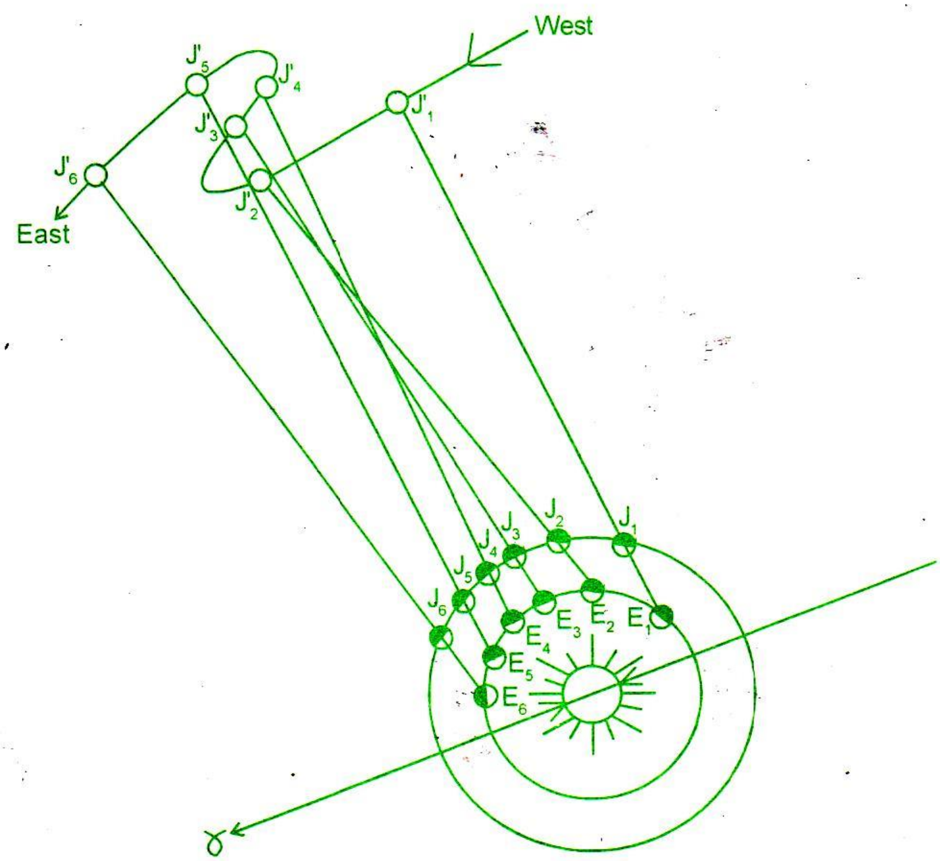
(FIG.6.6)

Fig.6.6 shows the apparent motion of the Sun along the Ecliptic on the celestial sphere, due to the Earth's orbital motion. The projection of the Sun on the Ecliptic from successive positions of the Earth in its orbit, appears to constantly move eastwards.

As stated earlier, the great circle on the celestial sphere, along which the Sun appears to move, is called the Ecliptic. In its apparent orbit around the Earth, the declination of the Sun, varies from 23½°N to 23½°S. Because the Earth completes a revolution of 360° around the Sun in about 365¼ days, the angular motion of the Earth around the Sun and therefore the apparent angular motion of the Sun among the stars is approximately 1° per day. Since SHA is a westward measurement from the First point of Aries, and since the Sun appears to move eastwards on the celestial sphere, the SHA of the Sun reduces constantly by about one degree per day.

6.2.4 Apparent motion of planets

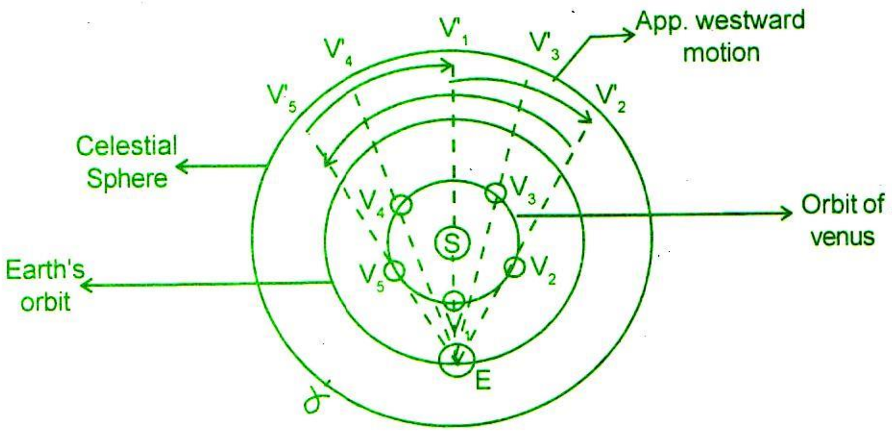
All planets revolve about the Sun, at different speeds depending on their distances from the Sun. As viewed from the Earth however, their motion appears very different because the Earth itself is not stationary, but is also moving in its own orbit around the Sun. Let us first consider the apparent motion of a superior planet such as Jupiter.



(FIG.6.7)

From the Earth, at position E_1, in fig.6.7, Jupiter at position J1 appears to be at position J_1' on the celestial sphere. Though both planets are moving eastwards in their orbits, the Earth moves faster according to Kepler's third law. Thus as viewed from the Earth, after Jupiter moves to position J_2 when it appears at J_2' on the celestial sphere, it appears to stop its apparent eastward motion and then appears to move westwards to position J_3' and J_4' on the celestial sphere. Thereafter, as the Earth continues to move in its orbit to position E_5 and E_6, Jupiter once again appears to stop and then move eastwards on the celestial sphere to positions J_5', J_6' and so on. It can thus be seen that superior planets exhibit a large apparent **direct** (eastward) motion followed by a small backward or **retrograde** motion westwards, once again followed by a large **direct** motion and so on. If the apparent position of the planet was plotted amongst the stars, over a period of many months, it would display an erratic motion as explained above. Depending on the change in declination of the planet during this period, the apparent path of the planet among the stars would appear to consist of loops or kinks as shown in the figure.

An inferior planet, such as Venus moves at a faster rate in its orbit than the Earth. Let us therefore initially consider the Earth to be stationary, while Venus moves in its orbit.

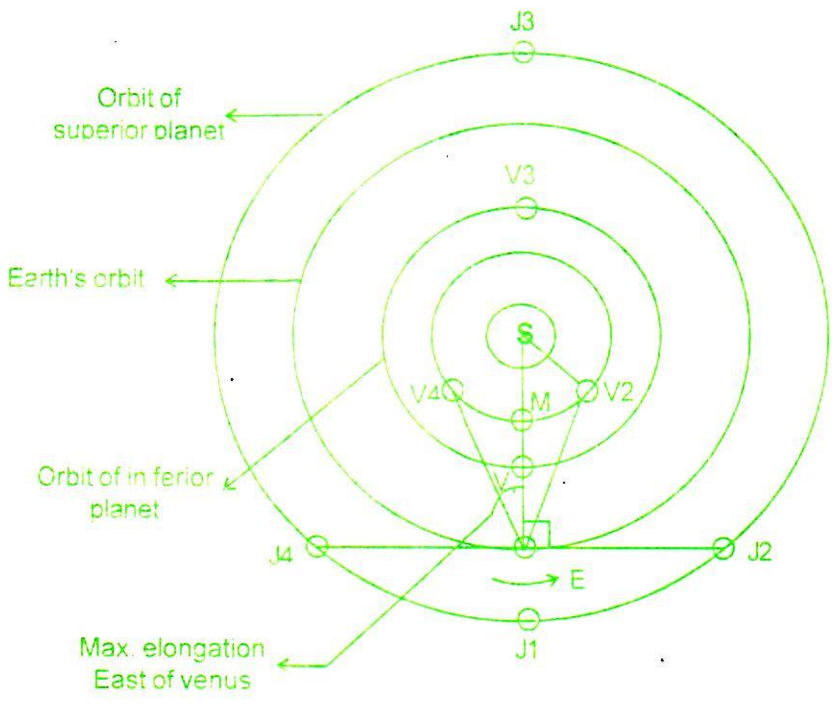


(FIG.6.8)

When Venus is at position V_1, it appears at V_1' on the celestial sphere. As it moves to position V_2, it appears to have moved westwards to V_2' on the celestial sphere. Thereafter as Venus moves through position V_3 and V_4 to V_5, it appears to move eastwards through V_3' and V_4' to V_5' on the celestial sphere. Thereafter, as Venus returns to position V_1 and then V_2, it again appears to move westwards on the celestial sphere. Thus, if the Earth was stationary, Venus would appear to swing forwards and backwards in the same sector of the sky. But since the Earth itself moves eastwards in its orbit, this whole sector continuously swings eastwards. Thus, inferior planets also exhibit a large apparent direct motion followed by a smaller retrograde motion, once again followed by a direct motion and so on. Unlike superior planets, the inferior planets Venus and Mercury appear to swing back and forth across the Sun. As stated earlier, the SHA of the Sun decreases continuously. The SHA of planets however some times decreases and at other times increases as explained above.

6.3 THE ELONGATION OF A PLANET OR THE MOON

Is the angle at the centre of the Earth contained between the centre of the Sun and the centre of the planet or the Moon, measured along the plane of the ecliptic.



(FIG.6.9)

It can be seen that inferior planets can never have a large elongation. In fact the maximum value of the elongation of venus is about 47° and that of Mercury is about 26°. Superior planets can have elongations upto 180° East and 180° West. Jupiter, at positions J_1 through J_2 to J_3 and Venus at positions V_1 through V_2 to V_3 are said to have westerly elongations, even though they appear to be 'eastward' of the Sun in the figure. A little thought will clarify the naming of the elongation.

Due to the rotation of the Earth, indicated by the arrow in the figure, to an observer on the Earth's surface, Venus would transit his meridian earlier than the Sun. It would therefore also set earlier than the Sun, and is thus obviously to the westward of the Sun. At positions J_3 through J_4 to J_1 and V_3 through V_4 to V_1, venus is said to have easterly elongations, as it would rise and set after the Sun and therefore is to the eastward of the Sun.

In the figure, V_2 indicates Venus at the position of its maximum elongation West and V_4, the position of Venus at its maximum elongation on East.

Conjunction

A planet or the Moon is said to be in conjunction with the Sun when as viewed from the Earth, it is in the same direction as the Sun (i.e. their celestial longitudes are the same).

Opposition

A planet or the Moon is said to be in opposition with the Sun when as viewed from the Earth, it is opposite in direction to the Sun (i.e. their celestial longitudes are 180° apart).

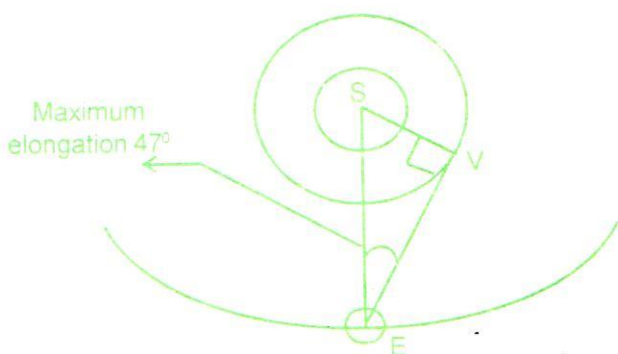
Quadrature

A planet or the Moon is said to be in quadrature when its elongation is exactly 90° East or West. In the figure, Jupiter is in quadrature at positions J_2 and J_4.

From the figure it can be seen that an inferior planet such as Venus may be in conjunction twice during one revolution around the Sun. i.e. at positions V_1 and V_3. To distinguish between these two conjunctions, the planet is said to be in inferior conjunction at position V_1, when it is closer to the Earth than the Sun, and in superior conjunction at V_3, when it is further away from the Earth than the Sun. It will be noticed from the figure that inferior planets can never be in opposition or in quadrature. Superior planets like Jupiter can only be in superior conjunction with the Sun. They can never be in inferior conjunction. They can however be in opposition and in quadrature.

Examples

1. If the greatest elongation of Venus is 47°, calculate the distance of Venus from the Sun, assuming planetary orbits to be circular and coplanar and that the Earth is 93 x 10^6 miles from the Sun.



(FIG.6.10)

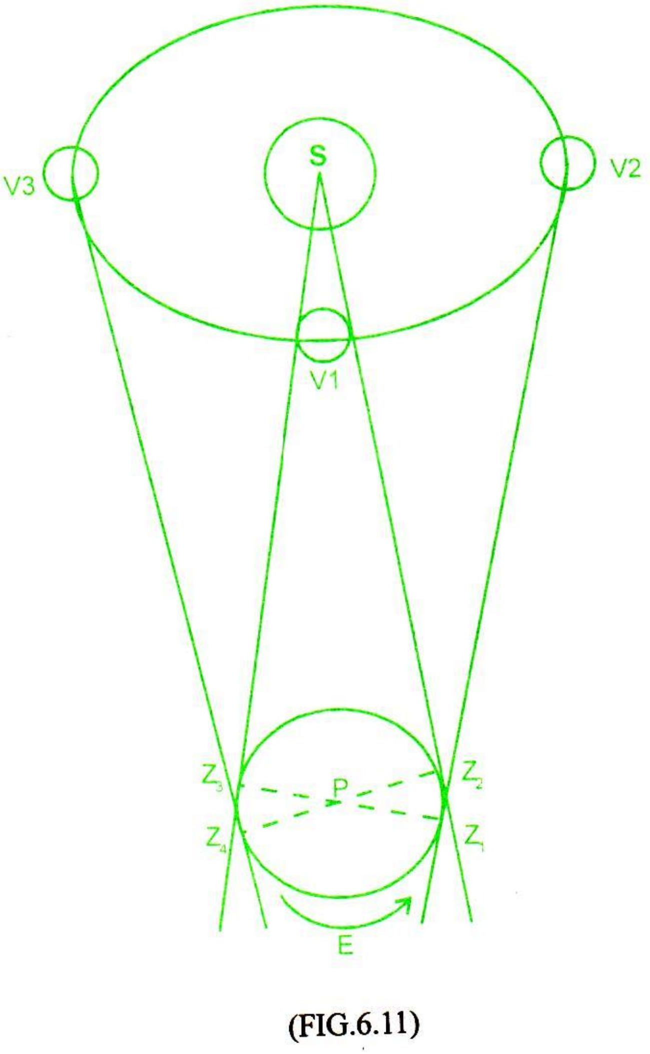
The angle at V is 90°, because the radius of a circle meets the tangent at 90°.

SE is the distance of Sun from the Earth = 93 x 10^6 miles.
SV = SE x sin angle E = 93 x 10^6x sin 47°=68.016 x 10^6 miles

EXERCISE VI

1. If the distance of planet Mercury from the Sun is 0.3871 of the distance between the Earth and Sun, find the maximum elongation of Mercury.

6.4 VENUS AS A MORNING AND EVENING STAR



(FIG.6.11)

The fig. shows the Sun, the Earth and Venus at three positions in its orbit.

When Venus is in conjunction with the Sun, as at position V_1, to an observer on the Earth, they wo appear to rise, culminate and set together, if Venus could be seen. When Venus has a westerly elong as at position V_2, a person on the Earth would see Venus rising, when he is at Z_1. The Sun would st below his horizon.

For the Sun to rise, the Earth would have to rotate further, till the observer is brought round to position Z_2. Thus Venus would be visible above the eastern horizon, for few hours before sun rise. Once the Sun rises, though Venus is above the horizon, it is not visible to the naked eye, because of the brilliance of the Sun. Having risen before the Sun, Venus would also set before the Sun and will therefore not be visible in the evening after sunset. At such times, therefore, Venus is said to be a **morning star**, as it is visible only in the mornings before sunrise.

When Venus has an easterly elongation, as at position V_3, a person on the Earth would experience sunset, when he is at position Z_3. Venus would still be above the horizon and will set only when the Earth rotates further, and the observer is brought round to position Z_4. Thus, Venus would be visible, for a few hours, over the western horizon, after sunset. Having set after the Sun, it will also rise the next morning, after sunrise, and therefore will not be visible during the day due to the Sun's brilliance. At such times, Venus is said to be an **evening star,** as it is visible only in the evenings after sunset.

At position V_2, Venus has a westerly elongation, because as stated earlier, Venus would set before the Sun and is therefore obviously to the westward of the Sun. At position V_3, Venus rises and sets after the Sun, it is therefore to the eastward of the Sun, and is said to have an easterly elongation.

From inferior conjunction to superior conjunction, Venus has a westerly elongation, and is a morning star. From superior conjunction to inferior conjunction, Venus has an easterly elongation, and is an evening star. Venus appears to swing forwards and backwards across the Sun. Due to the Sun's brilliance, it becomes invisible to the naked eye, when its elongation i.e. the angular distance from the Sun is small. Since the maximum elongation of Venus is about 47° only, it would be above the observer's horizon for approximately 3 hours only, before sunrise or after sunset.

6.5 APPARENT MAGNITUDE OF PLANETS

Planets are not self luminous. They are rendered visible only because they reflect light from the Sun. An inferior planet such as Venus would therefore exhibit phases just as the Moon does.

At superior conjunction, it appears full, while at inferior conjunction it is invisible to us as the illuminated hemisphere then faces away from the Earth. At intermediate positions, it would appear crescent shaped or gibbous. As Venus approaches inferior conjunction, the width of the crescent becomes less, but it is much closer to the Earth.

As it approaches superior conjunction it appears almost full, but it is much further away from the Earth. Venus therefore appears small and dim when full; and large and brilliant in crescent form. Venus appears brightest about 36 days before and after inferior conjunction.

(FIG.6.12)

Superior planets always appear nearly full. Their gibbosity is most noticeable when they are in quadrature. Therefore, their apparent magnitude depends mainly on their distance from the Earth. Since the light received from a source decreases inversely as the square of the distance of the source, superior planets appear brighter at opposition, and less bright at superior conjunction.

Theory Questions

1. Briefly describe the Solar system.
2. State the laws of planetary motion enunciated by Kepler.
3. Explain how seasons are caused on the Earth.
4. With the aid of suitable figures, explain the reasons for unequal duration of day and night.
5. How is the duration of daylight dependent upon
   (a) the observer's latitude ?
   (b) the Sun's declination ?
6. What do you understand by the terms
   (1) Equinox
   (2) Solstice
   When do they occur and what can be stated regarding the duration of day and night at such times ?
7. Distinguish between true motion and apparent motion of planets.
8. The SHA of the Sun decrease constantly, while that of a planet sometimes increases and some times decreases.
   Explain these phenomena for the Sun, a superior planet and an inferior planet.
9. Define the terms
   (1) Elongation
   (2) Superior conjunction
   (3) Inferior conjunction
   (4) Opposition
   (5) Quadrature.

10. Explain why Venus is sometimes referred to as a morning or an evening star.
11. Draw a figure showing the Earth in its orbit at the solstices and equinoxes. Using the figure, explain the yearly change in the Sun's declination.
12. What do you understand by the terms apogee, perigee, aphelion and perihelion ?
13. What are inferior and superior planets? Name them and state why the apparent magnitude of planets vary.
14. Under what conditions would planet Venus be visible before sunrise. Explain why Venus cannot be seen at midnight in navigable latitudes.

7

EARTH-MOON SYSTEM

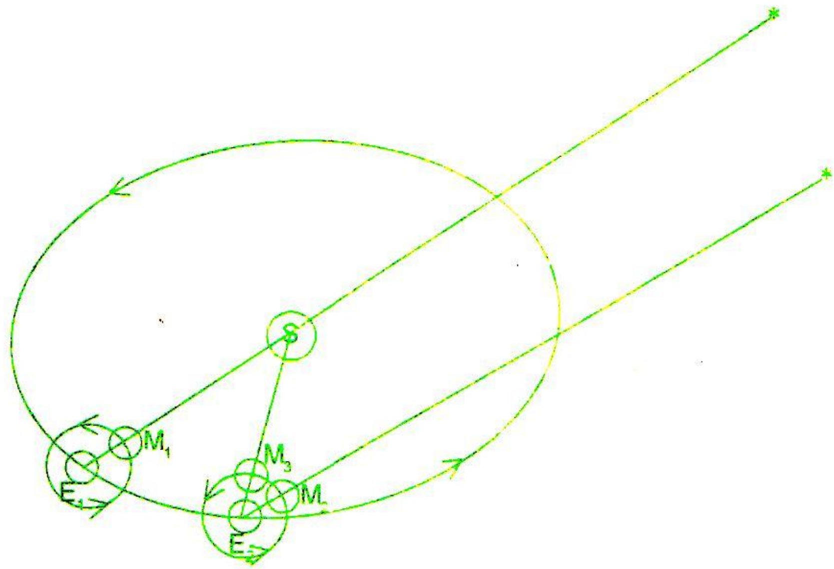
The Moon is the only natural satellite of the Earth. It has a diameter of about 2160 miles i.e. slightly more than a quarter of the Earth's diameter. It is interesting to note that the size of the Moon bears a larger ratio to its parent planet the Earth, than any other satellite in the Solar system to its parent planet.

The Moon revolves about the Earth. The motion is direct i.e. in the same direction as the Earth revolves about the Sun. Strictly, the Earth and Moon revolve about each other around the common centre of gravity of the Earth Moon system. This point, known as the "barycenter" lies about a thousand miles within the Earth. The orbit of the Moon around the Earth is elliptical with the Earth at one of the foci of the ellipse. At 'apogee' the Moon is about 253,000 miles from the Earth, and at perigee it is about 221,000 miles. The average distance of the Moon from the Earth may be taken as 240,000 miles.

Sidereal period of the Moon is the period of time taken by the Moon to complete one revolution of 360° around the Earth. The sidereal period is of constant duration, equal to 27 days 07 hrs. 43 minutes and 12 seconds i.e. approximately 27.33 days.

Synodic period of the Moon is the period of time between two consecutive New Moons or two consecutive Full Moons. The synodic period has an average length of about 29 days 12 hours 44 mins. This period may also be called a 'Lunar Month', a 'Lunation' or a 'Synodic Month'. It should be noted that the length of synodic period is not constant. It can have a maximum variation of about 13 hours from the mean value, due to the eccentricity of the Moon's orbit and that of the Earth's orbit. The variation is also caused by other disturbances, an investigation of which is beyond the scope of this book.

As the Moon revolves about the Earth, the Earth is also moving in its orbit around the Sun. When the Earth is at position E_1 in its orbit, and the Moon at position M_1, the Moon is in conjunction with the Sun and we have New Moon. Let us assume that as viewed from the Earth, the Sun and Moon are now in the direction of a star. This direction to the star is constant, irrespective of the Earth's motion in its orbit, as the star is at an infinite distance from the Earth. By the time Moon completes one revolution of 360° around the Earth, (it comes back in the direction of the same star), the Earth has moved in its orbit to position E_2.



(FIG.7.1)

One sidereal period has been completed but not a synodic period. To complete a synodic period, the Moon has to move further in its orbit till it is again in conjunction with the Sun (at position M_3). Thus, to complete a synodic period, the Moon has to revolve 360° + the angular motion of the Earth around the Sun, during that period.

The synodic period of the Moon is therefore of longer duration than its sidereal period. The amount of the angular motion in excess of 360°, required to complete a synodic period, varies depending on whether the Earth is then near aphelion or perihelion because, in the same interval the angular motion of the Earth around the Sun near perihelion will be larger, than that near aphelion. This is one of the reasons for the variation in the length of the Moon's synodic period. Due to this reason, the synodic period of the Moon is longer when the Earth is near perihelion and shorter when the Earth is near aphelion. The eccentricity of the Moon's orbit also causes a variation in the synodic period as the Moon would cover the angular motion in excess of 360° in a shorter period when at perigee, and in a longer period when at apogee.

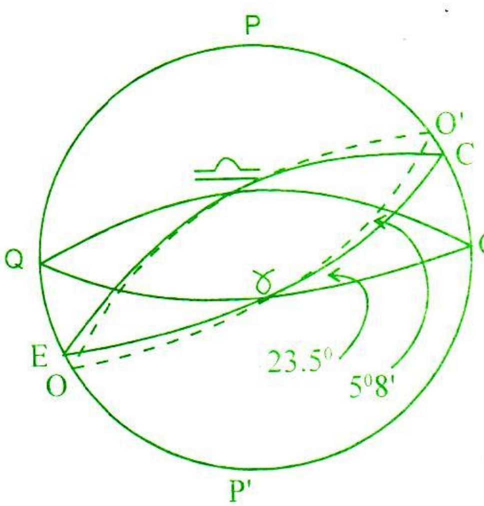
The Moon rotates on its own axis, completing one rotation in exactly in its sidereal period. This is the reason why the Moon always presents the same surface to us on the Earth. We therefore see the same features in the same position on the Moon.

The orbit of the Moon is inclined at an average of about 5°08'(varies from 5°18½' to 4°59½') to the plane of the ecliptic.

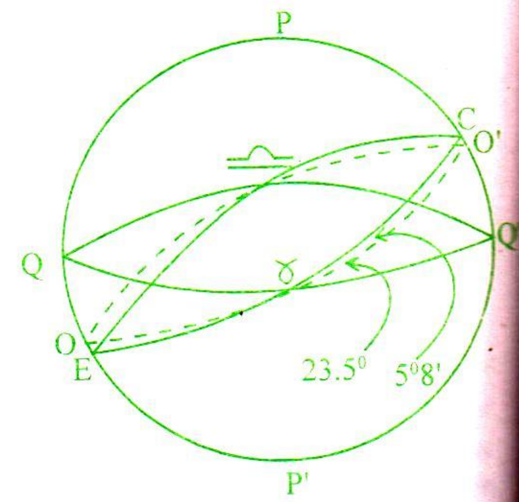
Nodes

The points at which the Moon's orbit intersects the Ecliptic are called the Moon's Nodes. That node at which the Moon crosses the Ecliptic from South to North is called the **Ascending Node** and the node at which it crosses the Ecliptic from North to South is called the **Descending Node**.

The nodal points are not fixed points on the Ecliptic. They move westward along the Ecliptic by about 19° a year. The nodes therefore complete a full cycle of motion around the Ecliptic in about 18.6 years. As a result of the nodal motion, the angle between the plane of the Moon's orbit and that of the Equinoctial and therefore the value of the maximum declination of the Moon varies from one lunation to the next.



(FIG.7.2)



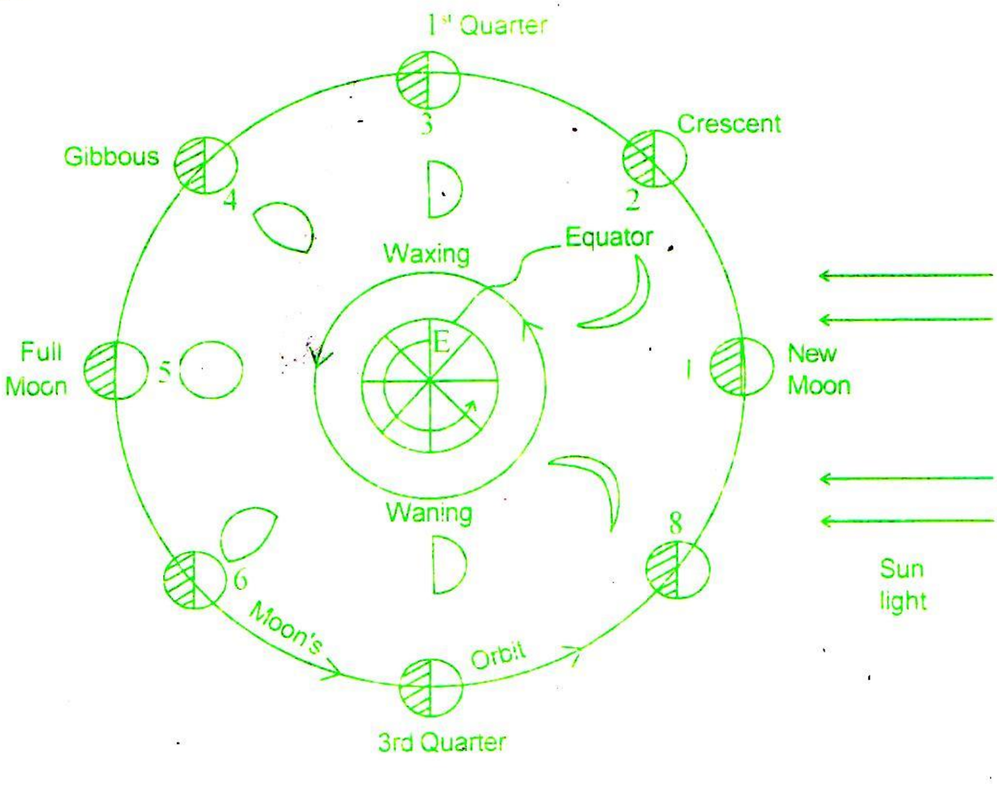
(FIG.7.3)

When the ascending node of the Moon coincides with the First point of Aries as shown in Fig.7.2, the inclination of the Moon's orbit with respect to the Equinoctial is equal to 23°30'+5°08' = 28° 38' (approx). For that lunation, the maximum declination of the Moon N and S would also be of the same value. About 9¼ years later, when the descending node, coincides with the First point of Aries, the inclination of the Moons orbit to the Equinoctial would be 23°30' - 5°08' = 18°22' (approx). As shown in Fig.7.3. the maximum declination of the Moon North and South for the lunation would also be of that value. At intermediate positions of the Moon nodes, the inclination of the Moon's orbit to the Equinoctial and therefore the maximum declination of the Moon, for those lunations will be of some intermediate value between the extreme limits stated above. It can thus be seen that unlike the Sun, whose maximum declination for each apparent orbit remains the same, that of the Moon varies from lunation to lunation.

7.1 PHASES OF THE MOON

The Moon is not self luminous. We see the Moon, as it reflects sunlight. Being spherical, 50% of the Moon's surface area is always illuminated by the Sun. The amount of the Moon's illuminated hemisphere, visible from the Earth, varies with the relative positions of the Sun and Moon with respect to the Earth.

The varying shapes of the illuminated portion of the Moon visible from the Earth is termed as 'the phases of the Moon'.



(FIG.7.4)

When the Moon is in conjunction (position 1 in Fig. 7.4), its entire illuminated hemisphere is turned away from the Earth. No part of its illuminated surface is visible from the Earth and the Moon is then said to be 'New'. At New Moon, the Sun and Moon rise and set at approximately the same time and they culminate at 1200 hours L.A.T.

As the Moon moves in its orbit to position 2 in the figure, a small part of the illuminated surface is visible from the Earth in the form of a crescent at the western side of the Moon's disc. About 7½ days from New Moon, when the Moon is in quadrature as indicated by position 3 in the figure, exactly half the illuminated disc of the Moon is visible from the Earth. The Moon appears dichotomised. This is the first quarter of the Moon.

As the Moon moves further in its orbit, to some position such as 4 in the figure, more than half the illuminated disc of the Moon is visible from the Earth. The Moon's appearance then is described as 'gibbous'.

About 14.75 days after New Moon, the Moon comes in opposition with the Sun (position 5 in the figure). The entire illuminated surface of the Moon now faces the Earth. We therefore see the entire disc of the Moon, illuminated. The Moon is then said to be Full. As the Sun and Moon are in opposition at Full Moon, the Moon would rise at about sunset, culminate at 0000 hours LAT and set at about sunrise.

During the second half of the lunation, the illuminated surface of the Moon visible from the Earth decreases so that the Moon appears gibbous at position 6 in the figure and dichotomised at position 7 (when the Moon is in the third or last quarter). This occurs about 22 days after New Moon.

At position 8, the Moon once again appears crescent shaped, and finally it returns to New Moon. The average duration of this cycle is about 29½ days, as stated earlier.

From New Moon to Full Moon, since the visible area of the Moon's illuminated surface is increasing, the Moon is said to be **waxing**. It is the western portion of the Moon's disc that is visible then.

From Full Moon to New Moon, the visible area of the illuminated surface of the Moon decreases and the Moon is then said to be **waning**. During this period, it is the eastern portion of the Moon's disc that is visible. At any time, the rounded, convex part of the Moon as seen from the Earth is always turned towards the Sun.

The Age of the Moon is the period of time elapsed, since the last New Moon.

Harvest Moon The Full Moon which occurs nearest the autumnal equinox is called the Harvest Moon. The following Full Moon is called the **Hunter's Moon**.

7.2 DAILY RETARDATION OF THE MOON

At New Moon, when the Sun and Moon are in conjunction, they would culminate at the same time. During the course of one day, the Moon would have moved eastwards by $360^0/29\frac{1}{2}$ i.e. about 12.2° in its orbit around the Earth, with respect to the Sun

Exactly one day after New Moon, when the Earth has completed one rotation of 360° with respect to the Sun, the Sun once again culminates. But, for the Moon to culminate again, the Earth would have to rotate a further 12.2°. Since the Earth rotates at 15° per hour, it takes about 49 minutes to rotate the further 12.2°.

Thus the Moon culminates about 50 minutes later each day. If the declination of the Moon remained unchanged, it would also rise and set approximately 50 minutes later each day. The average length of the 'Lunar day' is therefore about 24 hours and 50 minutes of Mean Solar time.

7.3 APPEARANCE OF THE MOON RELATIVE TO THE HORIZON

At New Moon, the Sun and Moon rise at approximately the same time. As the Moon rises about 50 minutes later each day, the Moon would rise after sunrise, on all days from New Moon to Full Moon. Since the rounded (convex) portion of the Moon always faces the Sun, during this period, the Moon would rise with the rounded portion upwards and set with the rounded portion downwards. Thus between New Moon and Full Moon the rounded portion of the Moon faces West.

At Full Moon, the Moon rises 12 hours after sunrise i.e. at about sunset. A day later, the Moon rises about 50 minutes later than it did at Full Moon. Thus by the time the Moon rises, the Sun has already set, about 50 minutes earlier. The Sun will therefore be about 11 hours 10 minutes behind the Moon for rising. Hence from Full Moon to New Moon, the Moon rises earlier than the Sun and as the rounded portion of the Moon always faces the Sun, the Moon would rise with the rounded portion downwards, and set with its rounded portion upwards. During this period, the rounded portion therefore faces East.

7.4 LIBERATION OF THE MOON

Since the Moon's rotational period is exactly equal to its sidereal period, the same area of the Moon's surface is always turned towards the Earth. It would therefore appear that the same 50% of the Moon's surface would be visible from the Earth at all times, while the other 50% which is turned away from the Earth would never be visible. This is however not true. Due to liberation, an additional 9% of the Moon's surface area becomes visible at different times. As a result, we can see a total of 59% of the Moon's surface area, though at any one time, only 50% of the area is visible, as the Moon is spherical. The remaining 41% of the area can never be seen from the Earth.

Liberation in latitude

The axis about which the Moon rotates is inclined at about 6½° to the perpendicular to its orbit. Thus during one revolution of the Moon, its North Pole and then its South Pole are alternately tilted a little, towards the Earth. When the North Pole of the Moon is tilted towards the Earth, we see about 6½° of its surface beyond its North Pole. In the opposite part of its orbit, we see about 6½° of its surface area beyond the South Pole.

Liberation in Longitude

Though the Moon rotates about its axis, with a uniform angular velocity, its angular motion in its orbit around the Earth is not uniform; being largest at perigee, and least at apogee. At apogee, its rotational velocity is greater than the orbital velocity i.e. greater than the rotational velocity necessary to present the same part of the Moon's surface towards the Earth. The Moon therefore appears to turn around slowly, and, we are then able to see more area of the eastern side of the Moon's surface. At perigee, the rotational velocity is less than the orbital velocity,and we therefore see a little more around the western side of its surface.

Diurnal liberation

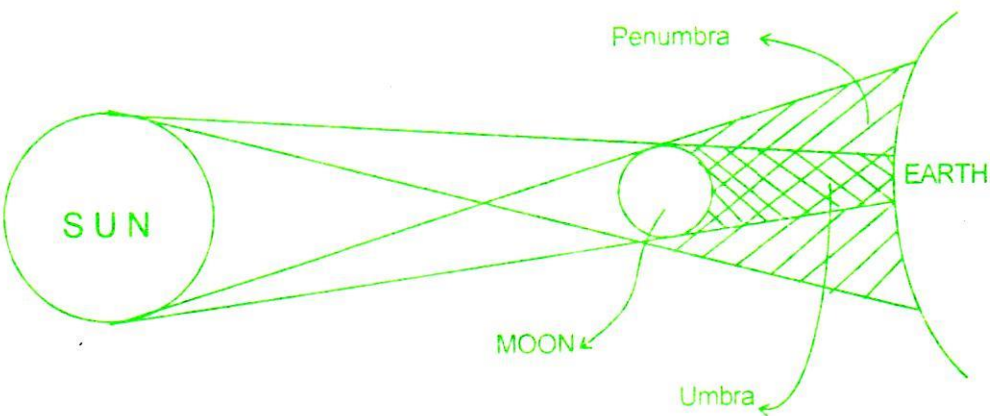
When the Moon is rising, we are able to see, a little over its top or western edge and when it is setting, we are able to see a little over its top edge then i.e. the eastern edge.

7.5 ECLIPSES

7.5.1 Solar Eclipse

When the Moon is in conjunction with the Sun and the centres of the three bodies are nearly in a line, the Moon appears directly over the Sun as viewed from the Earth, blocking off the Sun's disc, wholly or partly. Such an occurrence is called a 'Solar Eclipse'.

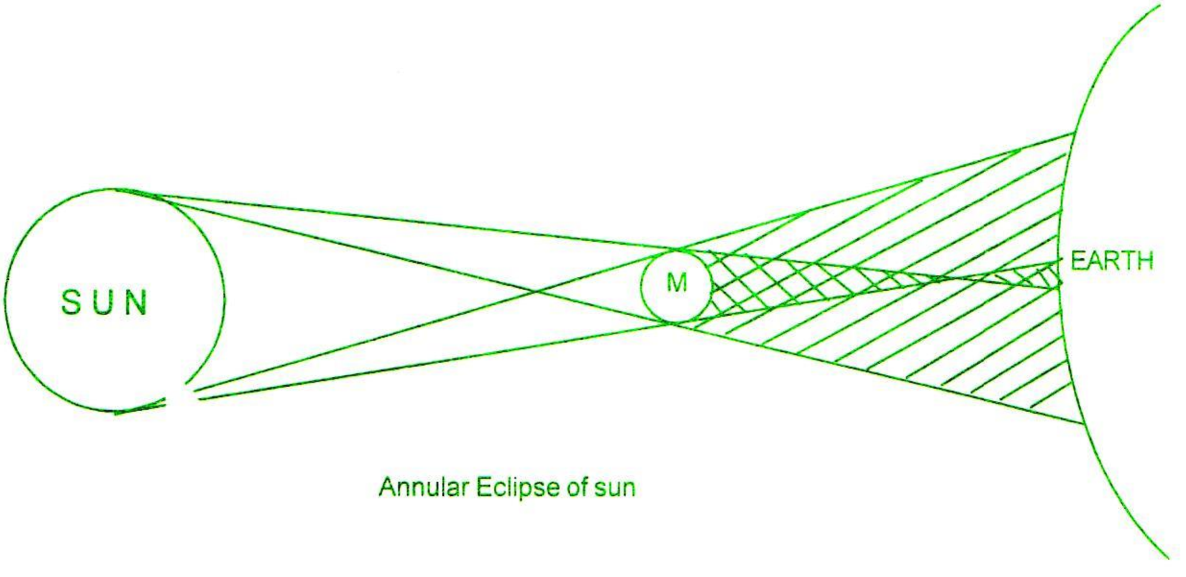
The shadow cast by the Moon is conical in shape. The tapering shadow cone within which no light from the Sun reaches is called the 'umbra'. The widening cone shaped region around the umbra, where a part of the Sun's rays reach, is called the 'Penumbra'. (Ref. Fig.7.5).



Solar Eclipse (FIG NOT TO SCALE)

(FIG.7.5)

Solar eclipses may be of three types **'Total'**,**'Partial'**, or **'Annular'**. People on the Earth within the area over which the umbra cone of the Moon falls, will have total darkness, because the Moon covers the entire face of the Sun and no light from the Sun reaches that area. Such an occurrence is termed a **'Total eclipse'** of the Sun. People on the Earth outside the umbra region of the Moon, but within the penumbra region, would be able to see a part of the Sun's disc with the remainder covered by the Moon. Such an occurrence is called as **'Partial eclipse'** of the Sun.



(FIG.7.6)

As the orbit of the Moon around the Earth is elliptical and eccentric, when the Moon is near apogee,it can happen that the umbra cone of the Moon does not reach the Earth's surface. (Fig. 7.6). People on the Earth, directly beyond the umbra cone would then see the Sun with the Moon obscuring the central portion of the Sun's disc, as the apparent diameter of the Moon then is smaller than that of the Sun. We then see the Sun as a narrow bright ring of light. Such an occurrence is called an **'Annular eclipse'** of the Sun. When the centres of the three bodies are exactly in a line as viewed from the Earth, whether a total or annular eclipse will occur, depends on whether the apparent diameter of the Moon is larger or smaller than the Sun's apparent diameter.

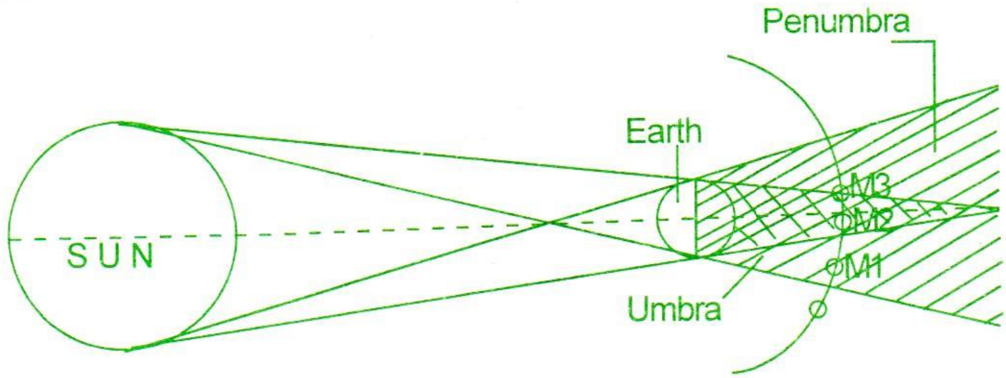
The maximum diameter of the area on the Earth, over which the umbra cone falls, is about 170 miles. The diameter of the Penumbra region on the Earth's surface may be upto about 4000 miles. A solar eclipse is therefore visible only over a very small portion of the Earth's surface, at any one time. As the Earth and the Moon move in their orbits, and as the Earth rotates on its axis, the umbra and penumbra cones of the Moon move over the Earth's surface and the eclipse becomes visible over a belt on the Earth's surface.

A total or annular eclipse always begins and ends as a partial eclipse. The period of totality can never exceed about 8 minutes at any one position.

For a total solar eclipse to occur the Moon must be in conjunction with the Sun. For the shadow of the Moon to fall on the Earth, the SHAs or GHAs of the Sun and Moon should be equal and their declination should be equal and of the same name.

A solar eclipse can therefore take place only on a New Moon day. However it is not necessary that it must take place on each New Moon day. This is so, because, though the condition regarding their SHA or GHA is fulfilled on each New Moon day, the condition regarding their declination may not be satisfied simultaneously, because the orbit of the Moon is inclined at 5¼° to that of the Earth. A Solar eclipse will take place, only if the Moon is on or near the ecliptic i.e. at or near its nodes on the day of New Moon.

7.5.2 Lunar Eclipse



(FIG.7.7)

The Earth casts a shadow behind itself. The shadow consists of a central cone shaped, tapering umbra, where no light from the Sun reaches, surrounded by a widening penumbra region where some sunlight does reach. The Moon is not self luminous and we see it only because it reflects sun-light. A lunar eclipse therefore takes place when the Moon passes through the Earth's shadow. This can happen only when the Moon is in opposition with the Sun.

Lunar eclipses may be of three types; **'total'**, **'penumbral'** or **'partial'**. When the Moon is entirely within the umbra of the Earth (M_2 in Fig.7.7), no light from the Sun reaches any part of the Moon. The entire Moon then becomes invisible. Such an occurrence is termed a **total eclipse** of the Moon. When the Moon is entirely within the penumbra of the Earth (M_1 in the figure), a part of the Sun's rays fall over the entire illuminated hemisphere of the Moon. We then see the Full Moon but with greatly diminished brilliance. Such an occurrence is termed a **penumbral eclipse** of the Moon. When the Moon is partly within the umbra and partly within the "penumbra" of the Earth (M_3 in the figure), that part of the Moon within the umbra becomes invisible while that part within the penumbra will be visible with very much diminished brilliance. Such an occurrence is termed a **partial eclipse** of the Moon.

Since the Moon must be in opposition with the Sun, for a lunar eclipse to occur, it can take place only on a Full Moon day. As the shadow of the Earth must fall on the Moon for a lunar eclipse to occur, the SHA or GHA of the Sun and Moon should differ by nearly 180°, and their declinations should be nearly equal but of opposite names.

A lunar eclipse need not take place on all Full Moon days, because, though the condition regarding their SHA or GHA is satisfied on each Full Moon day, the condition regarding their declinations may not be simultaneously satisfied, as the Moon's orbit is inclined to the plane of the ecliptic. A lunar eclipse will take place only if the Moon is on or near the ecliptic i.e. at or near its nodes on Full Moon day.

The maximum number of eclipses that can take place in a year is 7, of which four or five must be solar. The minimum number of eclipses that must occur each year is 2, both of which must be solar.

Though more solar eclipses take place, than lunar eclipses, more people on the Earth see lunar eclipses. This is so, because, during a lunar eclipse, the entire hemisphere of the Earth facing the Moon sees the eclipse. A Solar eclipse is however seen only over a comparatively small area of the Earth's hemisphere facing the Sun. Further, a Lunar eclipse caused by the Moon passing through the large shadow cast by the Earth lasts longer than a Solar eclipse, which is caused by the smaller shadow cast by the Moon.

7.6 OCCULTATION

Occultation is an occurrence somewhat similar to Solar eclipses. The Moon in its apparent motion in the sky frequently passes over stars and planets. The star or planet is then said to be occulted. For an occultation to occur, the SHA or GHA of the Moon and the occulted body should be equal and their declinations equal and of the same name.

On a certain day when the SHA of Sun and Moon were 185° and their declinations 2°N, the semi-diameters of the Sun and Moon were 16.1' and 15.9' respectively. What occurrence would take place?

Since their SHAs and declinations are the same, a solar eclipse would take place. Since the apparent diameter of the Moon is less than that of the Sun, it will be an annular eclipse.

EXERCISE VII

1. What sort of eclipse would occur, if the Sun's R A is 180° more than the Moon's R A and their declinations are equal but of opposite names ?

Theory Questions

1. Define the terms
   (1) Ascending node
   (2) Descending node
   (3) Age of the Moon
   (4) Barycenter
   (5) Lunar month.
2. Define
   (a) Sidereal Period of the Moon and
   (b) Synodic Period of the Moon.
3. Why does the duration of the Moon's Synodic period vary ?
4. During the course of a lunation the entire surface of the Moon is not visible from the Earth. Discuss this statement with reference to the Earth Moon System.
5. The maximum declination of the Sun, is constant, while that of the Moon varies from lunation to lunation. Explain this phenomenon.
6. With the aid of a suitable figure, explain why the Moon exhibits phases.
7. What do you understand by the term ' daily retardation of the Moon' ? Explain this phenomenon.
8. State with reasons, when you would expect the meridian passage of the Full Moon to occur.
9. Briefly describe liberation of the Moon.
10. Solar eclipses may be of three types. With the aid of suitable sketches, describe how they are caused.
11. What conditions are necessary for a Solar eclipse to take place ? Explain why a Solar Eclipse need not occur on all New Moon days.

12. What are the three kinds of lunar eclipses ? With the aid of a figure, explain how they are caused.
13. State the conditions necessary for a lunar eclipse to occur. Why is it that a lunar eclipse may not take place on each Full Moon day ?
14. Though more solar eclipses occur each year, more people on the Earth see lunar eclipses. Give two reasons for this.
15. What do you understand by the term "occultation" ? When a body occults another, what can be stated about their GHA's and declinations?

8

TIME

The West to East rotation of the Earth causes an apparent, opposite, East to West rotation of the celestial sphere, so that heavenly bodies continually cross an observer's meridian from East to West.

8.1 THE DAY

Is the interval in time between two successive meridian passages of a heavenly body over the same meridian.

From the above general definition of the day, we derive various definitions with reference to particular celestial bodies.

8.1.1 Sidereal Day

is the interval in time between two successive meridian passages of the First point of Aries over the same meridian.

The sidereal day is the true rotational period of the Earth. It has a duration of 23 hours, 56 minutes, 04.1 seconds of Mean Solar time.

The Sun makes an apparent revolution of 360° around the Earth in about 365¼ days. With reference to the stars and the First point of Aries, the Sun therefore appears to move eastwards by about 1° per day. From one meridian passage of the Sun to the next, the Earth has to therefore rotate approximately 361°. A solar day is therefore about 4 minutes longer than a sidereal day. If measured using the sidereal day as the unit of time, the Sun's meridian passage would occur about 4 minutes later each day. Since life on the Earth is governed by the Sun, it is essential to choose a unit of time that is closely related to the Sun, so that the Sun crosses the observer's meridian at about the same time each day, throughout the year. Therefore, the sidereal day is not used as a unit for measurement of time in civil life, though it is the true rotational period of the Earth and is of constant duration.

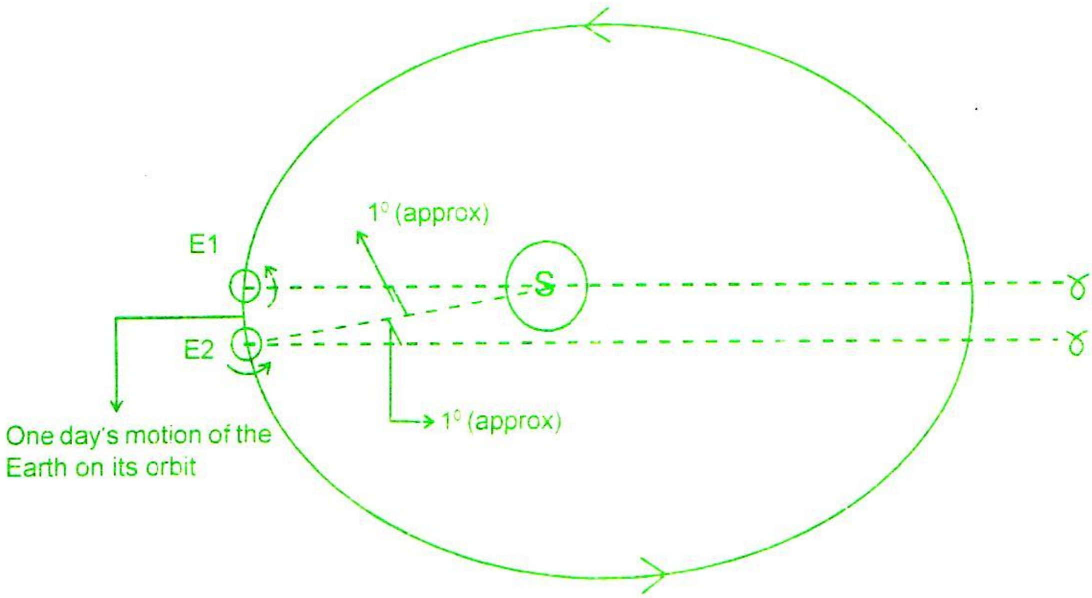
8.1.2 Apparent solar day is the interval in time between two successive transits of the True Sun, across the same meridian.

The apparent solar day is not of constant duration. The variation in the duration of the apparent solar day is caused due to

(a) the eccentricity of the Earth's orbit and

(b) the obliquity of the Ecliptic.

To complete an apparent solar day, the Earth has to rotate 360° with respect to the Sun. From fig. 8.1 it can be seen that the rotation necessary to complete an apparent solar day will be 360° + the angular orbital motion of the Earth during the day. The rate of angular motion of the Earth in its orbit is not constant, but is larger at perihelion and smaller at aphelion.

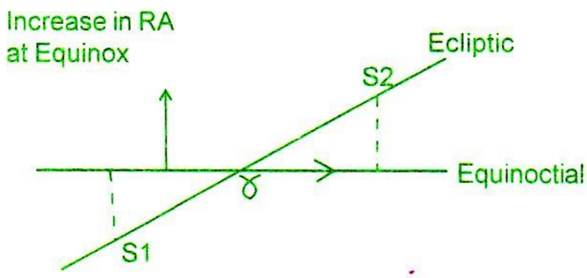


(FIG.8.1)

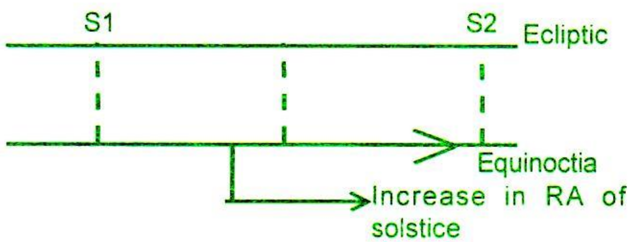
To complete an apparent solar day, the Earth would therefore have to rotate 360° + a larger angle at perihelion and 360° + a smaller angle at aphelion. The length of the apparent solar day is therefore larger at perihelion and smaller at aphelion, due to the Earth's eccentric orbit.

Even if the orbit of the Earth was circular, with the Sun at its centre, so that the rate of the angular orbital velocity of the Earth remains constant, there would still be a variation in the duration of the apparent solar day, caused due to the obliquity of the Ecliptic.

To understand this, let us assume that the Sun moves at a uniform apparent velocity on the Ecliptic. There would still be a non-uniform rate of increase of the Sun's RA (which is measured on the Equinoctial) due to the plane of the Ecliptic being inclined to that of the Equinoctial. At the equinoxes the Sun's track along the Ecliptic is at an angle of 23½° to the Equinoctial. A day's motion of the Sun on the Ecliptic, projected on the Equinoctial, would intercept a smaller arc on the Equinoctial than on the Ecliptic. At the solstices however, the Ecliptic and the Equinoctial are almost parallel to one another. A day's motion of the Sun on the Ecliptic when projected on the Equinoctial then, would intercept almost the same arc on the Equinoctial as on the Ecliptic.



(FIG.8.2)



(FIG.8.3)

To complete an apparent solar day, the Earth has to rotate 360° + the apparent angular eastward motion of the Sun during the day, (measured along the Equinoctial). As explained above, this angle is larger at the solstices and smaller at the equinoxes. The length of the apparent solar day would therefore be greater at the solstices and lesser at the equinoxes, due to the obliquity of the Ecliptic.

Thus, as a unit of time, the apparent solar day suffers from the serious disadvantage of not being of constant duration. If we measure time, using the apparent solar day as the unit, it would be necessary to have clocks showing 24 hours of different durations through the year. We cannot therefore, use the apparent solar day based on the True Sun, for measurement of time. Instead we use an imaginary body called the Mean Sun, to measure time.

The Mean Sun is an imaginary body assumed to move along the Equinoctial at a uniform rate, equal to the average rate of motion of the True Sun on the Ecliptic.

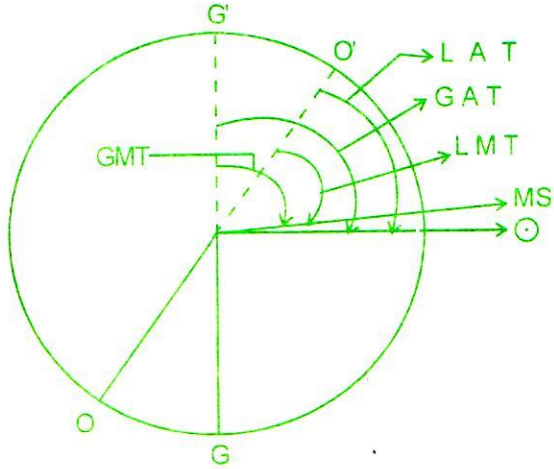
As can be seen from the definition, this imaginary body does not suffer from either of the two disadvantages of the True Sun, with respect to measurement of time, as it moves along the Equinoctial, and at a uniform rate. It should also be noted that the SHA's of the Mean Sun and True Sun would never be very different at any time through the year and that both these bodies complete one apparent revolution round the Earth in exactly the same time, i.e. one year.

8.1.3 Mean Solar day is the interval in time between two successive meridian passages of the Mean Sun across the same meridian.

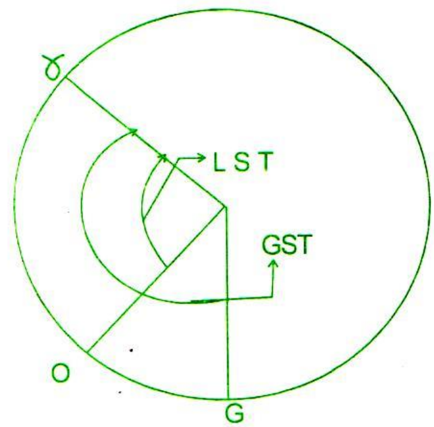
It is of constant duration, equal to 24 hours of Mean Solar time.

8.2 MEAN, APPARENT & SIDEREAL TIMES

Local Mean Time or Ship's Mean Time (LMT or SMT) is the westerly hour angle of the Mean Sun measured from the observer's inferior meridian. The observer's inferior or anti-meridian is the meridian 180° away from his own meridian.



(FIG.8.4)



(FIG.8.5)

Greenwich Mean Time (GMT) is the westerly hour angle of the Mean Sun measured from the inferior meridian of Greenwich.

Local Apparent Time or Apparent Time Ship (LAT or ATS) is the westerly hour angle of the True Sun measured from the observer's inferior meridian.

Greenwich Apparent Time (G A T) is the westerly hour angle of the True Sun measured from the inferior meridian of Greenwich.

Local Sidereal Time (L S T) is the westerly hour angle of the First Point of Aries measured from **observer's meridian.**

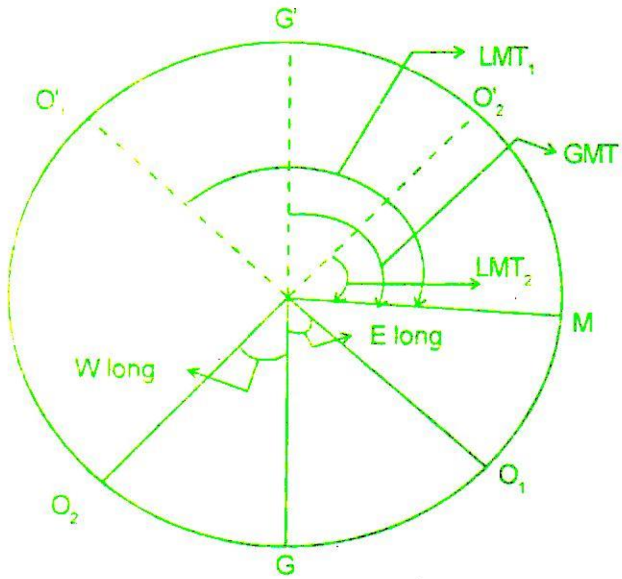
Greenwich Sidereal Time (G S T) is the westerly hour angle of the First point of Aries measured from the Greenwich meridian.

Sidereal time is measured from the meridian itself, while solar time, whether mean or apparent, is measured from the inferior meridian. By measuring solar time from the inferior meridian, we start with zero hours when the Sun is on the inferior meridian. Thus, a day starts at midnight. If solar time was measured from the meridian, the day would start at noon. The morning would then be one date and the afternoon the next date.

It has already been explained that a sidereal day is equal to 23 hours 56 minutes 04.1 seconds of Mean solar time. Thus the sidereal clock would gain about 3 minutes 56 seconds over a solar clock, each day.

8.3 RELATIONSHIP BETWEEN LONGITUDE AND TIME

The Mean Sun completes an apparent revolution of 360° with respect to a stationary point on the Earth, in 24 hours of mean solar time. The rate of motion of the Mean Sun is therefore uniformly 15° per hour or 1° in 4 minutes; that is 15' of arc in one minute of time, corresponding to 1' of arc in 4 seconds of time. Units of time may therefore be used as a measure of arc and vice versa.



(FIG.8.6)

Referring to the above figure, it may be seen that the difference in the Local times at two different places on the Earth is equal to the angle subtended at the Pole between the inferior meridians of the two places, which is equal to the difference in longitude between them. This is a very important relationship. When using this relationship the times being compared should both be LMTs or both LATs or both sidereal times.

By comparing the time at any place with the Greenwich time, we can therefore obtain the longitude of that place. Since time is a westward measurement, the time at places in East longitudes will be more than Greenwich time at that instant and the time at places in West longitudes will be less than Greenwich time at that instant, at the rate of one hour for every 15° of longitude. We can also find the local time in any longitude by **adding** to the Greenwich time the longitude converted to hours, in **East** longitudes, and **subtracting** that from Greenwich time in **West** longitudes.

8.4 STANDARD TIME

It is impracticable for each place on the Earth to measure time from its own inferior meridian. If this was done, each state, city and locality would maintain different local times, making civil life difficult. Nor is it convenient for all places on the Earth to measure time from one standard meridian. If all places on the Earth maintained time measured from the inferior meridian of Greenwich, for instance, places near the 180th meridian would have zero hours at noon. To obviate this difficulty, a system of standard times has been adopted by all countries of the world. The continents of the Earth are divided into several areas and each area keeps time, based on a some what central meridian through that area. Each of these areas is referred to as a 'Time Zone'.

The meridians on which the standard times of the various time zones are based are chosen so that the times based on them would differ from G M T by a convenient number of hours. For instance, Indian standard time used throughout India is based on 82½°East meridian, which differs from Greenwich time by 5½ hours of time. Generally an entire country has one standard time. Certain countries with a large east-west extent, like USA and Australia use different standard times over different areas. The standard time kept by the various countries are listed in the nautical almanac and in the Admiralty list of Radio signals Vol.II.

If the standard time of a country is 2 hrs behind GMT, it is listed as +2 hours, indicating that 2 hours are to be added to the standard time of that country to obtain G M T. Indian Standard Time is listed as - 5 hours 30 minutes.

8.5 ZONE TIME

Under the Zone time system, sometimes used by ships, when at sea, the Earth is divided into 24 zones, each zone being 15° of longitude in width. Ships in each of these zones, keep time based on the central meridian through that zone. Zone zero extends from 7½°E to 7½°W longitude. The central meridian of this zone being the Greenwich meridian, ships within this zone keep GMT. Zone + 1 covers the area from 7½°W to 22½°W. The time kept within this zone is based on the central meridian of this zone i.e. 15°W. Ships within this zone would have their clocks one hour behind GMT. Ships within zone (+) 2 covering the area from 22½°W to 37½°W keep time based on the central meridian 30°W i.e. 2 hours behind GMT. Similarly ships between 22½°E and 37½°E would be in - 2 zone keeping time, based on the 30° East meridian i.e. 2 hours ahead of GMT. Thus in addition to the zero zone, we have 12 zones with negative prefix and 12 zones with positive prefix. Zone 12 extending from 172½°E to 172½°W with the 180th meridian as its central meridian would obviously have both +ve and -ve prefixes. Zone +12 extends from 172½°W to 180°, and zone -12 from 172½°E to 180°. It should be noted that the zone time at any position will always differ from GMT by a full number of hours, because the central meridians used for measurement of zone time in the different zones, always differs from Greenwich meridian by multiples of 15°. A ship crossing the limiting longitude of a zone, would therefore advance or retard her clocks by one hour, at that instant.

To find the Zone Time, kept at any longitude, an easy method is to divide that longitude by 15°. The quotient converted to its nearest whole number would be the zone of that place. This should be added to or subtracted from the GMT depending on the longitude being East or West respectively, to obtain the zone time at that longitude. The difference between Time zones explained earlier and Zone Time explained above should be noted and clearly understood.

Examples :

1. Find the Zone time in longitude 50° West at 0700 LMT.

   To Calculate the Zone LMT
   0700
   50°W/15 = 3. 33 LIT(W)
   0320
   Zone = + 3 GMT
   1020
   Zone difference
   0300
   Zone time
   0720

2. Find the Indian standard time in longitude 85°E, at 1100 hours Zone time.

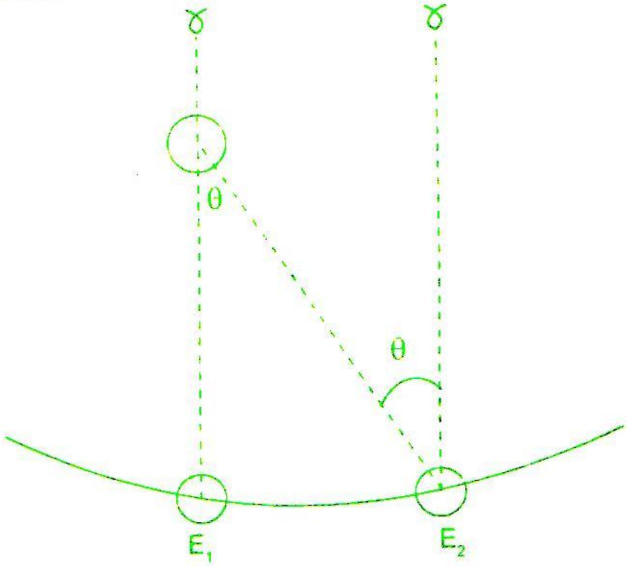
   To calculate the Zone Zone time
   1100
   85°E/15 = 5.67 Zone diff.
   0600
   Zone = -6 GMT
   0500
   Time zone of India 0530
   IST
   1030

8.6 INTERNATIONAL DATE LINE OR CALENDAR LINE

From what has been stated earlier, it will be realized that as a ship proceeds eastwards, she would have to advance her clocks at the rate of one hour for every 15° of d'long, and a ship proceeding westwards would have to retard her clocks at the rate of one hour for every 15° of d'long, if their clocks are to indicate the correct LMT.

Consider a ship circum-navigating the Earth in a westward direction, making a d'long of 12° per day. She would return to her original meridian in 360°/12° = 30 days. During this period, she would have retarded her clocks by 1 hour for every 15° of d'long i.e. a total of 360°/15° = 24 hours, or one day. By her calendar, she would therefore have returned to her original meridian in 29 days. If she had circum-navigated the Earth in an eastward direction, she would have advanced her clocks by 24 hours or one day and would return to the original meridian in 31 days according to her calendar. Thus compared to the date at a shore station, on the original meridian, the date of the ship which sailed westwards would be one day behind and the date of ship which sailed eastwards would be one day ahead. To obviate this anomalous situation, the **Date line** has been introduced by International agreement. The Date line roughly corresponds the 180th meridian. It deviates from this meridian so that islands in the same group and continuous land eas fall on the same side of the Date line. Ships crossing the Date line, on an easterly course retard their te by one day, while ships crossing the Date line on a westerly course advance the date by one day.

8.7 WHY STARS RISE, CULMINATE AND SET 4 MINUTES EARLIER EACH DAY



(FIG.8.7)

With reference to Fig. 8.7, when the Earth is at position E_1 in its orbit, if the Sun is in transit with the First point of Aries, the Sun and First point of Aries would culminate at the same time. Thereafter during the period the Earth completes one rotation of 360° on its axis, it moves to position E_2 in its orbit. This motion of the Earth does not alter the direction to the First point of Aries, as it is at an infinite distance from the Earth. Thus on completion of a rotation of 360°, the First point of Aries will once again culminate i.e. a sidereal day is completed. However, the motion of the Earth in its orbit, does make a difference in the direction to the Sun. Since the Earth completes a revolution of 360° around the Sun in about 365¼ days, the average daily angular motion of the Earth around the Sun is 360°/365¼ = approximately 0°59'.

To complete a solar day therefore the Earth has to rotate 360° + 59'. Thus the solar day is about 3m 56s longer than the sidereal day.

Since we measure time by the Sun, our clocks show 24 hours from one culmination of the Sun to the next. Measured by our clocks, therefore, the First point of Aries, and in fact all stars, would appear to culminate every 23h 56m 04s, that is about 4 min. earlier each day, than they did the previous day. Stars therefore rise and set also about 4 minutes earlier each day.

8.8 COMPARISON OF SOLAR AND SIDEREAL DAY AND TIME

We have seen that 24 hours of sidereal time is equal to 23h 56m 04s of Mean solar time. Using this relationship, we can convert durations in terms of sidereal time to those in terms of solar time and vice a versa.

To complete a solar day, the Earth has to rotate 360° + 360°/365¼. Thus, per day the Earth rotates 360°/365¼ in excess of one rotation. In one year therefore, the Earth would make 360°/365¼ x 365¼ = 360° or one rotation in excess of 365¼ rotations. Thus in one year, the Earth makes 366¼ rotations with respect to a fixed direction in space, say the First point of Aries. In one year of 365¼ solar days therefore, there are 366¼ sidereal days.

8.9 RELATIONSHIP BETWEEN ARC AND TIME

As explained earlier

15° of arc	=	1 hour of time
1° of arc	=	60m / 15 = 4 minutes
15' of arc	=	4 / 1 x 15 / 60 = 1 minute
1' of arc	=	60 / 15 seconds = 4 seconds
.25' of arc	=	1 second

Conversion tables are provided in nautical tables, and in the nautical almanac to facilitate converting time to arc or arc to time without actual calculations.

Examples

1. Convert 107°37' to time, without the use of the tables and verify the result using the conversion tables.

```
15)   107°  37'        (7h.
      105
      ----
        2
      x 60
      ----
      120
      + 37
      ----
15)   157              (10m.
      150
      ----
        7
      x 60
      ----
15)   420              (28s.
      300
      ----
      120
      120
      ----
        0              = 7h 10m 28s
```

2. Convert 9h 23m 14s to arc, without use of tables and verify the result using the conversion tables.

09h	x	15°	=					135°
23m	x	15'	=	345'	÷	60	=	05°45'
14s	x	15"	=	210"	÷	60	=	03'30"

								140°48'30"

8.10 RELATIONSHIP BETWEEN LONGITUDE AND TIME

GMT ~ LMT or GAT ~ LAT = long. in time (LIT)
Greenwich time (Best) Longitude West
Greenwich time (Least) Longitude East
Conversely, Local time (-) East LIT = Greenwich time
Conversely, Local time (+) West LIT = Greenwich time

Examples :

1. Find the GMT, when LMT in long.125°30'E was 5d 03h 15m 04s.

LMT	5d	03h	15m	04s
LIT(E)		08h	22m	00s
GMT	4d	18h	53m	04s

2. Find the LAT in long.86°34'West, when GAT was 5d 3h 04m 06s.

GAT	5d	03h	04m	06s
LIT(W)		05h	46m	16s
LAT	4d	21h	17m	50s

3. On 12th December, at 07h 33m 15s, GMT,the SMT at a place was 02h 45m 30s on the same date. Find the long. of that place.

GMT	12d	07h	33m	15s
SMT	12d	02h	45m	30s
LIT		04h	47m	45s

Converted to arc, Longitude = 71°56.25' West, as Greenwich time is greater.

4. Find the SMT at a place in longitude 88°E, at 0600 Zone time.
To find the zone : 88°E ÷ 15 = 5 x 13 /15
The place is situated in Zone -6

Zone time	0600
Zone	0600
GMT	0000
LIT	0552
SMT	05h52m

5. Find the LMT at a place in longitude 66°W, the Time Zone of which is + 04h 00m, at 18h 40m standard time.

Standard time	18h 40m
Standard time diff.	04h 00m
GMT	22h 40m
LIT(W)	04h 24m
LMT	18h 16m

6. A vessel sailed from longitude 173°E, at 14h 12m LMT on 15th July. She arrived in longitude 167° West at 04h 35m LMT on the 16th. Find the steaming time.

In such problems involving comparison of times in different longitudes, it is advisable to convert both the times to Greenwich times. The duration or interval can then be obtained by comparing the Greenwich times. Since Greenwich times are being compared finally, we do not have to concern ourselves with the ship's date having been adjusted on crossing the date line.

LMT Dep.	15d	14h	12m	00s		
LIT(E)		11h	32m	00s		
GMT Dep.	15d	02h	40m	00s	....	(i)
LMT Arr.	16d	04h	35m	00s		
LIT(W)	(+)	11h	08m	00s		
GMT Arr.	16d	15h	43m	00s	....	(ii)
	-15d	02h	40m	00s		
Steaming time	1d	13h	03m	00s		

In the calculations on time which follow, knowledge of conversion from one time to another, as well as arcs to time and vice a versa, will be assumed. The reader must make himself proficient in the above relationships before proceeding to them.

Further examples on Time

1. Star Regulus crossed the observer's meridian, at 2030 hrs LMT on a certain day. At what approximate LMT will it cross the observer's meridian 3 days later ?

As a star culminates approximately 4 minutes earlier each day, it will cross the observer's meridian 4 x 3 = 12 minutes earlier i.e. at 2018 LMT.

2. On a certain day, the Moon and a star culminated together. What will be approximate interval of time between their culminations next day ?

Since the star approximately 4 minutes earlier, and the Moon approximately 50 minutes later each day, the interval between their culmination will be approximately 54 minutes, the next day.

3. Convert 9h 30m of Mean Solar time to interval of Sidereal time. Assume length of a sidereal day to be 23h 56m of Mean Solar time.

   23h 56m of Mean solar time = 24h of sidereal time
   (1436m) (1440m)
   9h 30 m (MST) = (1440 x 570)÷1436=571.588m sidereal time
   (570m) = 9h 31m 35s of sidereal time.

4. Convert 4h 16m 12s of sidereal time into interval of Mean Solar time.

24 hours of sidereal time	=	23h 56m of Mean solar time
4h 16m 12s sidereal time	=	1436m / 1440m x 4h 16 m 12s
	=	4h 15m 29s of Mean Solar time

5. A sidereal clock and a solar clock show the same time. After 15h 36m of solar time, what will be the difference between the times on the two clocks ?

   In 23h 56m of solar time, the sidereal clock gains 3m 56s
   In 15h 36m of solar time it would gain
   (15h 36m) / (23h 56m) x 3m 56s = 2m 34s
   The sidereal clock will be ahead by 2m 34s

6. At 0600 hrs. GMT on 13th October, 1976, find
   (i) the Greenwich sidereal time
   (ii) the local sidereal time in longitude 170°W

GMT 0600, GHA γ	=	111°55.6'		
G.sidereal time	=	7h	27m	42s
LIT (W)		11h	20m	00s
Local sidereal time	=	20h	07m	42s

8.11 EQUATION OF TIME

Apparent time and Mean time have been defined as the time measured to the meridian of the True Sun and the time measured to the meridian of the Mean Sun respectively. Equation of time is the difference between the Mean time and the Apparent time, measured from the same meridian, at any instant. It is expressed in minutes and seconds of time. Navigators generally express Equation of time as Mean time minus Apparent time, though astronomers express it as Apparent time minus Mean time. In conformity with the practice in nautical text books, equation of time will be expressed as Mean time minus Apparent time in this book. Therefore, if Mean time is greater than Apparent time, equation of time is +ve and if Apparent time is greater than Mean time, equation of time is -ve.

From the above, it will be noted that equation of time is not a mathematical equation, but is an interval of time. In arc, equation of time is equal to the angle between the meridian of the Mean Sun and that of True Sun.

Since time is a westward measurement and as Greenwich and sidereal hour angles are also measured westwards, equation of time may also be considered as GHAMS - GHATS or LHAMS - LHATS or SHAMS - SHATS, expressed in minutes and seconds of time. Each of the above expressions would give the angle between the meridian of the Mean Sun and that of the True Sun. We may also obtain the equation of time as RATS - RAMS. (True - Mean, in this case, as RA is an eastward measurement).

The Mean Sun moves at a uniform rate along the Equinoctial, while the True Sun moves at a varying rate along the Ecliptic. However both the Suns complete an apparent revolution in exactly the same period (one year). The angle between their meridians at any instant is therefore never very large. In fact, the value of equation of time, never exceeds 16m 22s, corresponding to an angle of 4°05.5' between the meridians of the True Sun and Mean Sun. Equation of Time values are tabulated in the daily pages of the nautical almanac, for 00 hrs. and 12 hrs GMT on each day. The value for any intermediate time may be obtained by interpolation. The values tabulated in the almanac are the absolute values i.e. signs are omitted. Whether it is positive or negative may however be determined by inspecting the meridian passage time of the Sun in the adjacent column of the almanac. If the tabulated meridian passage time is in excess of 12 hours, say 12 04, it indicates that at 12 04 Mean time, the True Sun is on the meridian i.e. the Apparent time is 1200. Equation of Time is then obviously +ve. Conversely, if the tabulated meridian passage is less than 12 hours, equation of time is -ve. Infact, we can obtain the value of equation of time, correct to the nearest minute, by inspection of the tabulated meridian passage time of the Sun.
Meridian passage time - 12 hrs. = Equation of time (correct to the nearest minute).

1. Find the equation of time, at 1830 hours GMT on 13th October, 1976.

	Equation of time
13th 12hrs	13m 48s -ve
14th 00hrs	13m 55s -ve

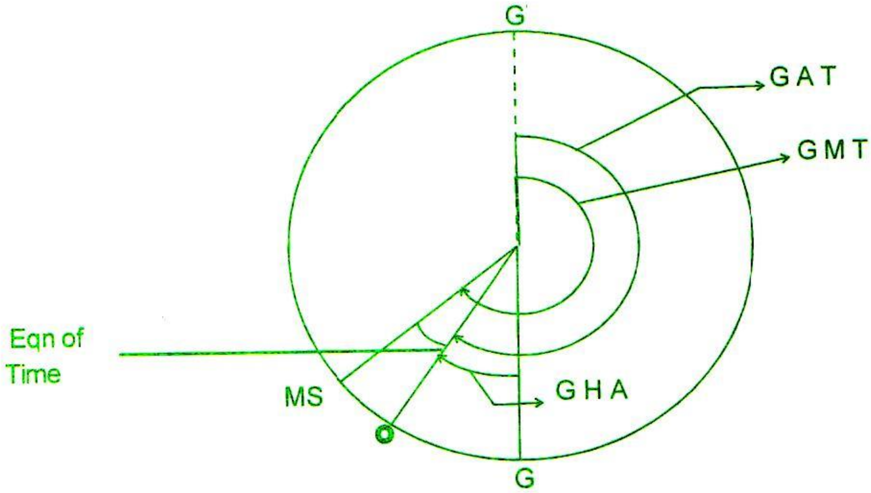
By interpolation, equation of time at 1830 on the 13th
= 13m 52s -ve

2. Find the equation of time at 1500 hrs. GMT when the GHA of the Sun was 42°04.7'

When GHATS = 42°04.7', its westerly hour angle from the inferior meridian of Greenwich will be 180° + 42°04.7'= 222°04.7'

GAT	=	14h 48m 19s
Equation of time	=	GMT - GAT
		15 00 00

	14	48	19
=	+	11m	41s



(FIG.8.8)

3. On October 14th, 1976, in longitude 120°E, find the LMT when LAT was 7h 12m 40s.

LAT	14d	7h	12m	40s
LIT(E)		8h	00m	00s
GAT	13d	23h	12m	40s
*Equation of time			-13m	54s
GMT	13d	22h	58m	46s
LIT(E)		8h	00m	00s
LMT	14d	06h	58m	46s

*Though equation of time is tabulated for GMT, we interpolated for its value using GAT, as GMT was not available. This is acceptable as GMT and GAT differ, only by the amount of equation of time, which as stated earlier, is never large.

Further theory on equation of time

We know that equation of time, is the difference between Mean time and Apparent time, that is, the angle between the meridians of the Mean and True Suns. There are two causes for their meridians differing.

(i) The True Sun moves at a varying rate on its elliptical apparent orbit around the Earth, while, by definition, the Mean Sun moves at a uniform rate along the Equinoctial.

(ii) The apparent orbit of the True Sun is inclined at about 23½° to the Equinoctial along which the Mean Sun moves.

Equation of time may therefore be considered as composed of two components :

(a) Component E_1, produced due to the eccentricity of the Earth's orbit, and

(b) Component E_2, produced due to the obliquity of the Ecliptic.

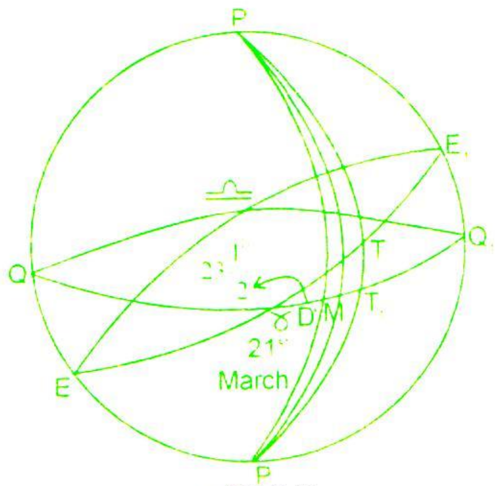
To discuss these two components of equation of time, it is necessary to introduce another body called the **Dynamical Mean Sun.** This is an imaginary body assumed to move along the Ecliptic at a uniform rate, equal to the rate of motion of the Mean Sun on the Equinoctial. When the True Sun is at **perigee,** its meridian is assumed coincident with that of the Dynamical Mean Sun.

We may now compare the movement of the Dynamical Mean Sun with that of the True Sun, both of which move in the same plane. The difference between their meridians at any time (RATS - RADMS) gives the component E_1 of equation of time then. We may also compare the movement of the Dynamical Mean Sun with that of the Mean Sun, both of which move at the same uniform rate, to obtain the component E_2 of equation of time, as (RADMS-RAMS). The algebraic Sum of E_1 and E_2 would then give RATS - RAMS, which is the equation of time -

	RATS	- RADMS	=	E1
+	RADMS	- RAMS	=	E2
	RATS	- RAMS	=	Equation of time

In Fig 8.9., T, D and M represent, the True Sun, the Dynamical Mean Sun and the Mean Sun respectively. Their meridians are also shown. RA being an eastward, measurement from the First point of Aries along the Equinoctial, γD', γM and γT' are the RAs of the Dynamical Mean Sun, Mean Sun and True Sun respectively. In the figure component E_1,which is RATS - RADMS is +ve, component E_2 which is RADMS - RAMS is -ve and Equation of time which is RATS - RAMS is +ve.

Let us first consider the component E_1 which is RATS - RADMS. Both True Sun and Dynamical Mean Sun complete one revolution in exactly the same period i.e. one year. True Sun will traverse the two symmetrical halves of the orbit (from perigee to apogee and from apogee to perigee in exactly equal periods i.e. a half year. Dynam.Mean Sun moving at a uniform rate will also traverse each half year of its orbit in half a year).

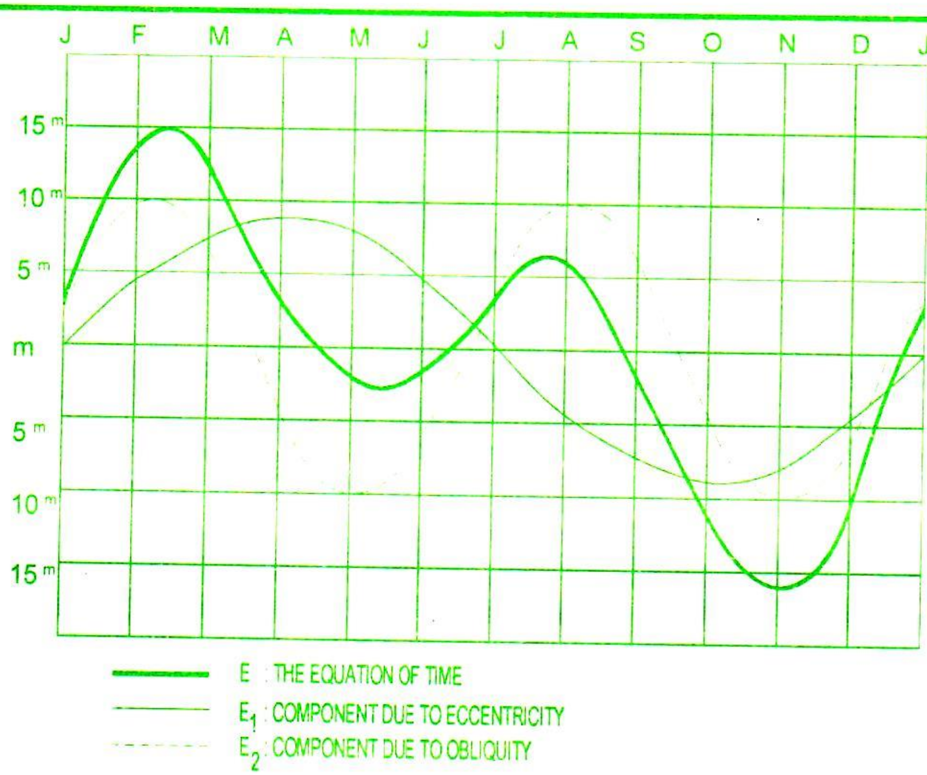


(FIG.8.9)

By definition, the meridians of the Dynamical Mean Sun and the True Sun coincide at perigee. E_1 is then nil. From perigee, the True Sun moves at a varying rate in its elliptical orbit while the Dynamical Mean Sun moves at a uniform rate along the Ecliptic. Being near perigee, the rate of motion of the True Sun is greatest. Its RA therefore increases faster than that of Dynamical Mean Sun. Thus E_1 will be +ve till they reach apogee, when their meridians once again coincide. Their RAs then become equal, and E_1, again becomes nil. At apogee the rate of motion of True Sun is least, while Dynamical Mean Sun continues moving at the same uniform rate. RADMS then increases faster than RATS. Thus from apogee, RADMS will be greater than RATS, and E_1 will be -ve till they reach perigee again. E_1 is thus nil at perigee, in early January and at apogee, in early July. It is +ve for the first half of the year, with a maximum value of about 7 minutes in early April and -ve for the second half of the year, with the same maximum value occurring in early October.

Let us now consider the component E_2 which is RADMS - RAMS. Dynamical Mean Sun and Mean Sun are coincident at the First point of Aries (Vernal equinox 21st March). E_2 is then nil. After a certain interval of time, they would have moved through the same arc on their respective tracks, because their rates of motion are the same. But their meridians will not be coincident, as Dynamical Mean Sun is on the Ecliptic, while Mean Sun is on the Equinoctial. Since RA is measured along the Equinoctial, RADMS will be less than RAMS, and E_2 will be -ve. At Summer solstice (June 21st), they have both travelled 90° on their respective great circle tracks, i.e. arrived at E_1 and Q_1 respectively, in fig. 8.9. They are then on the same meridian and E_2 is again nil. For the next quarter of the orbit RADMS is greater than RAMS and E_2 is then +ve. They would arrive together at the First point of Libra, at Autumnal equinox, on 23rd September, since they would both have travelled 180° in their respective tracks. E_2 is again nil then. For the next quarter of the orbit RADMS is again less than RAMS, and E_2 is -ve. At winter solstice, on 22nd December, when they have both travelled 270° from the First point of Aries on their respective tracks i.e. arrived at E and Q respectively their meridians again coincide and E_2 is nil then. For the next quarter of the orbit, till they return to the First point of Aries, RADMS is greater than RAMS and E_2 is +ve. Thus E_2 is nil at the equinoxes and solstices. It has maximum +ve and -ve values of about 10 minutes, midway between each equinox and solstice.

The values of E_1 and E_2 may be plotted for the entire year as shown in figure 8.10



(FIG.8.10)

We may then obtain a curve of equation of time for the entire year, as the algebraic sum of the two component curves. It may be noted from the curve of equation of time that its value varies throughout the year and that it attains zero value on four occasions during the year i.e. mid April, mid June, early September and end of December. The maxima are + 14m 21s, about mid February, -3m 45 s, about mid May + 6m 22s, about end July and - 16m 22s in early November.

Examples

1. On a certain day, the LMT meridian passage of Sun is tabulated as 11h 56m. What is the approximate value and sign of equation of time ?

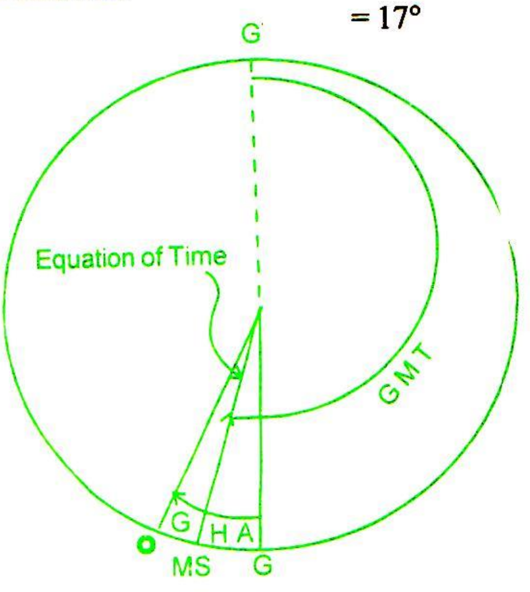
At meridian passage	LAT	1200
	LMT	1156
	Equation of time = -4m	

2. If equation of time is +6 m, what is the LMT of the Sun's meridian passage ?

LAT	1200
Equation of time	+06
LMT	1206

3. At 1300 hrs. GMT when equation of time is -8m. What is the Sun's GHA ?

At GMT 1300 hrs,	GHAMS	= 15°
Equation of time : 8m		= 2°
GHATS		= 17°



(FIG.8.11)

4. Find the LAT in longitude 80°E (Standard time zone -5h 30m) at 1000 hrs. standard time, when equation of time is -4m.

Standard time	10h	00m
Time zone	-5h	30m
GMT	4h	30m
LIT	5h	20m
LMT	9h	50m
Equation of time	0h	04m
LAT	09h	54m

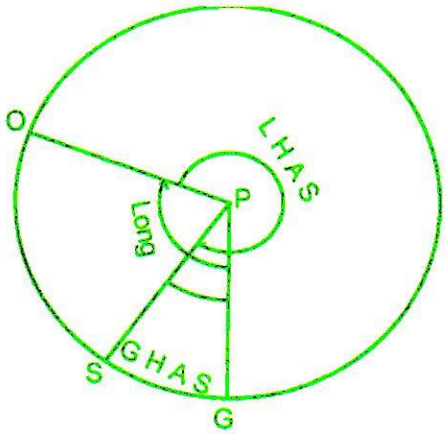
5. Find the GMT, when LAT in longitude 46°30' West is 17h 28m, if equation of time is +7m 15s.

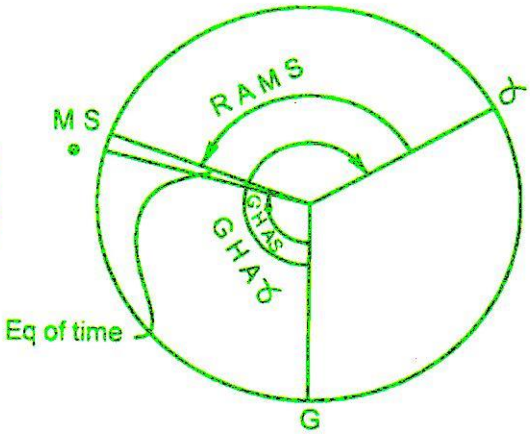
LAT	17h	28m	
Equation of time	+	7m	15s
LMT	17h	35m	15s
LIT(W)	3h	06m	00s
GMT	20h	41m	15s

6. At a place in 150°36'E (Time zone - 10), the Standard time of the Sun's meridian passage was 12h 02m 04s. Find the equation of time.

Standard time of meridian passage	12h	02m	04s
Time zone	(-)10h	00m	00s
GMT	02h	02m	04s
LIT	10h	02m	24s
LMT, meridian passage	12h	04m	28s
LAT, meridian passage	12h	00m	00s
Equation of time		+04m	28s

7. Find the observer's longitude, if GHAS was 40° and LHAS was 280°.





(FIG.8.12) (FIG.8.13)

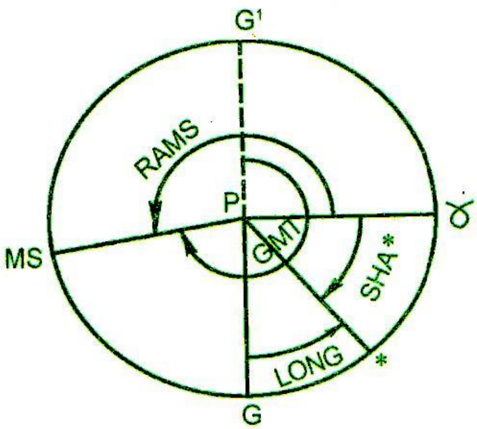
Longitude : 120°W.

8. Given GHAS 110°, GHA γ 240°, equation of time + 4m 16s. Find RAMS and SHAMS. (FIG. 8.13)

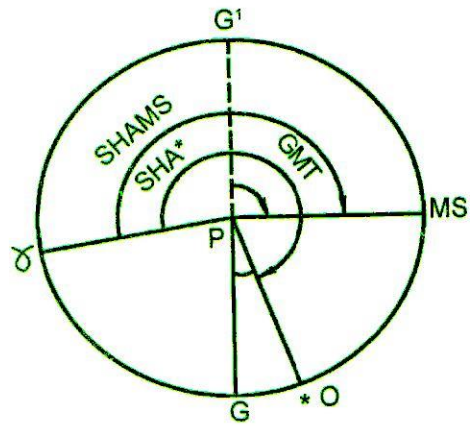
GHAS	110°
GHA γ	240°
RA Sun	130°
Equation of time (+)	-1°04'
RAMS	128°56'
SHAMS = 360°- 128°56' =	231°04.

9. At 16h 40m 30s GMT, RAMS was 13h 40m 20s and the longitude of a star's geographical position was 40°15'E. Find the SHA of the star.

SHAMS = 360°- RAMS = 360°- 205°05' = 154°55.0'
GHAMS = GMT - 12h = 250°7.5'- 180° = 70°07.5'
SHA of Greenwich meridian = SHAMS - GHAMS = 84°47.5'
Long. of GP of* = 40°15.0'E
SHA* = 44°32.5'



(FIG.8.14)



(FIG.8.15)

10. For an observer, in longitude 6°30'E, a star whose SHA was 276°15' was on the meridian at 07h 16m 12s GMT. Find SHAMS.

Angle GPMS = 12h - GMT	=	180° - 109°03'
	=	70°57'
Angle GPO	=	6°30'
Angle OPMS (EHAMS)	=	64°27'
SHAMS = SHA* - Angle OPMS	=	276°15' - 64°27' = 211°48'

(fig.8.15)

UNIVERSAL TIME (U.T) Mean solar time is based on the Mean Solar Day which is derived from the rotation of the Earth. Due to small variations in the rate of Earth's rotation, Mean Solar time and the (mean solar) second are not accurate enough for precise measurement of time and intervals, particularly for the present day scientific needs. It is therefore necessary to make allowances for the small irregularities involved in measurement of time in terms of Mean Solar Time.

$U.T._o$ In the concept of Universal Time, the Mean Solar Time at Greenwich, i.e. GMT is designated $U.T._o$ (Universal time as observed). This is based on the rotation of the Earth and is measured by timing the meridian transits of selected stars, the celestial co-ordinates of which are accurately known. When a star transits the observers meridian, the local sidereal time is equal to the stars R.A. By applying the Mean Sun's SHA ± 12 hrs, to the local sidereal time, the LMT at that instant can then be obtained. Such observations are made on the Greenwich meridian and other observatories. The Greenwich time, so observed provides $U.T._o$.

$U.T._1$

Since measurement of time and longitude are essentially inseparable, accurate fixing of longitude requires a time scale which allows precisely for any irregularities in the measurement of time. One cause of such irregularity in measurement of $U.T._o$ is the polar variation, caused due to the small wandering of the Earth's poles on the surface of the Earth. The poles appear to be describing approximately circles of diameter about 40m. A change in the position of the poles changes the plane of the Equator and those of the meridians. It, therefore introduces a variation in the time. This variation has a maximum value of ±30 milli seconds.

$U.T._o$ corrected for polar variation is designated $U.T._1$. For an accurate calculation of longitude, $U.T._1$ should be used instead of $U.T._o$ though the very small difference that exists between $U.T._o$ and $U.T._1$ will not produce any sensible difference in the result. Theoretically therefore, it is $U.T._1$ time signals which the navigator requires to correlate time, hour angles and longitude.

$U.T._2$

Since $U.T._1$ is based on the rate of rotation of the Earth, it does not provide a uniform time scale because of the variations in the rate of rotation of the Earth. There is a small secular retardation in the rate of the Earth's rotation in 100 years. This is probably caused due to tidal friction. Apart from the above, there are more significant variations in the rate of rotation of the Earth. These include an irregular variation due to internal disturbances in the Earth and a periodic (seasonal) variation due to meteorological effects on the Earth's atmosphere, causing the rate of rotation to increase slightly during summer and decrease slightly during autumn and winter. The amplitude of this variation is upto ±30 milliseconds. Although this variation follows the same general pattern annually, neither the amplitude nor the phase of the variation repeat exactly. $U.T._2$ is obtained by removing the effect of the seasonal variation from $U.T._1$.

The mean solar second was defined as (1/24) x 60 x 60 of a mean solar day. Due to the variations in the rate of rotation of the Earth, the second was redefined in 1956 in the terms of the orbital motion of the Earth, because of the necessity for much higher precision in scientific work. The second was then defined as 1 / 31556925.9747 of the tropical year 1900. Time so defined is called Ephemeris time. Astronomical determination of

U.T. and E.T. is a lengthy process as errors and irregularities have to be smoothed out over long periods - several years, in the case of E.T.

Co-ordinated Universal Time (U.T.C) : is a time scale based on the fundamental property of the atom. It has no relationship to observed astronomical events. The atom of the isotope of Cesium 133 can exist at two energy levels, or hyperfine levels of ground state. Its transition from one state to the other is accompanied by the emission or absorption of electromagnetic energy at a characteristic frequency. This frequency has been measured as 9,192,631,770 ± 20 cycles per second of Ephemeris time. In the atomic time scale, the second is therefore defined as equivalent to the period for 9,192,631,770 vibrations of the unperturbed hyperfine transition 4, 0-3, 0 of fundamental state 2s½ in C_s133. The atomic second was internationally accepted in 1964. Several investigations thereafter have shown that the atomic second and ET second agree to a degree much beyond what was originally anticipated.

Atomic Time was introduced on 1st Jan.1972 and was designated Co-ordinated Universal Time (U.T.C.).

The Atomic clock measures time intervals only and not the time of day as U.T. does. It merely provides a running total of seconds by summating the cycles of radiation. We may however measure atomic time from a zero specified in terms of an astronomical event. The error in an atomic clock is expected to be within 1 second in 5000 years. The atomic second has been adopted only temporarily, as further developments like the hydrogen maser gives promise of time accuracies of the order of 1 sec. in 33 million years.

Since U.T.C. is a uniform and accurate time scale, $U.T._1$ based on the rotation of the Earth, which is subject to irregularities, will differ from U.T.C. by small variable amounts. $U.T._1$ is required by navigators, astronomers, satellite tracking stations, for survey work etc. U.T.C. is required for precise measurement of time intervals in scientific work. It is also used in navigational systems like the GPS, Loran C etc.

Primary time signals are disseminated in accordance with U.T.C. The value of $U.T._1$ - U.T.C. in integral multiples of 0.1 sec. is also disseminated by code in the primary time signals. The correction, algebraically added to U.T.C. gives $U.T._1$. At present, the divergence between U.T.C. and $U.T._1$ is about 1 sec. annually. It is therefore necessary to make periodic adjustments to U.T.C. to ensure that it does not differ much from $U.T._1$ due to the accumulation of the divergence of long periods. It has been internationally agreed that the necessary corrections to U.T.C. will be made in steps of 1 sec. such that the difference between the two never exceeds 0.7 sec. Navigators therefore need not worry unduly if U.T.C. is used in place of $U.T._1$ for calculating positions at sea as the error in the calculated position is not likely to exceed acceptable limits within the parameters of accuracy attainable at sea.

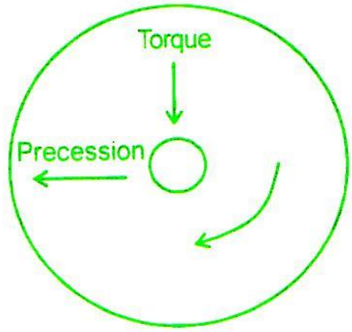
A step correction of 1 sec. is called a leap second. A positive or a negative leap second will be the last second of the U.T.C. month (31st Dec. and/or 30th June), 23h 59m 60s being followed by 00h 00m 00s or 23h 59m 58s being followed by 00h 00m 00s, as necessary.

An examination of the Earth as a time keeper since 1825 indicates that an average of one step adjustment per year, as at present, should suffice. In the early part of this century however, there was a period when more than one step adjustment would have been necessary.

8.12 PRECESSION OF THE EQUINOXES

Precession of the equinoxes can be better comprehended if the property of precession exhibited by free gyroscopes is first understood.

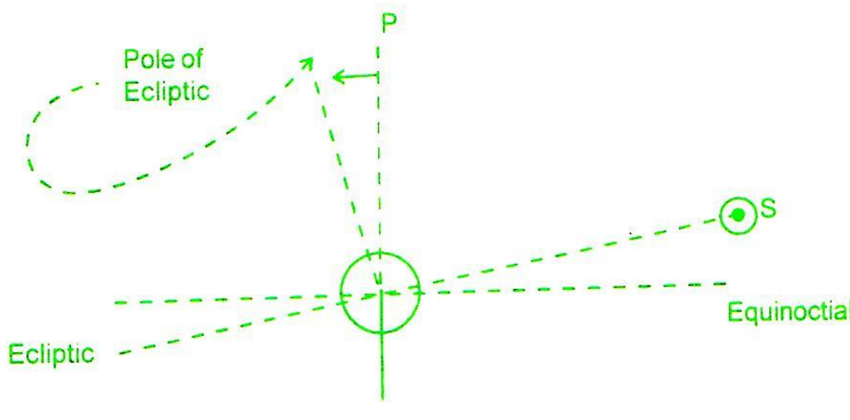
A free gyroscope consists of a heavy, well balanced wheel, rotating at a fast rate on frictionless bearings and having three degrees of freedom i.e. freedom to rotate about its spin axis,freedom to move the spin axis in azimuth and freedom to move the spin axis in altitude. Due to its property of precession, if a torque is applied on one end of the spin axis of a free gyroscope, the axis does not move in the direction of the applied torque. Instead it moves in a direction 90° away, as though the torque had been rotated through 90° in the direction of rotation of the wheel. For example, if the spin axis of a gyroscope was lying horizontal, in the North South direction and the wheel was spinning clockwise as viewed from the South end, as shown in Fig. 8.16, a down-ward torque applied on the South end, would cause that end to precess westwards.



(FIG.8.16)

The Earth satisfies all the conditions for a free gyroscope, since it is heavy, well balanced, rotating at a fast rate, it has no friction against its rotation, and it has all the three degrees of freedom. It will therefore exhibit the gyroscopic property of precession, if a torque is applied to its spin axis.

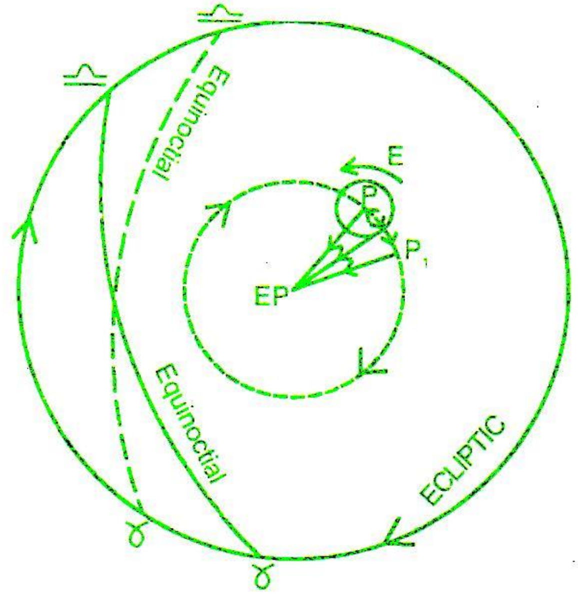
The gravitational forces between heavenly bodies is directly proportional to their masses. As stated earlier, the Earth is not a true sphere. It is flattened at the poles and bulging at the Equator. Because of the spheroid shape of the Earth, there is larger concentration of the Earth's mass at the equatorial zone than at the polar regions. The gravitational attraction of the heavenly bodies on the Earth is therefore greatest at the equatorial zone.



(FIG.8.17)

The Earth's Equator is in the plane of the Equinoctial. The plane of the Ecliptic is inclined at 23½° to that of the Equinoctial. The poles of the Earth, projected on the celestial sphere i.e. the celestial poles are therefore 23½° away from the poles of the Ecliptic. The Sun is always on the Ecliptic. The gravitational force of the Sun on the Earth, which is greatest at the equatorial zone, is therefore in a direction tending to align the plane of the Equator with that of the Ecliptic. This is equivalent to a torque being applied to the spin axis of the Earth in a direction tending to align its poles with the poles of the Ecliptic. But due to the

Earth's gyroscopic property of precession, its axis moves in a direction 90° away from the direction to the pole of the Ecliptic. As the pole of the Earth moves from its original position, the direction of the torque towards the pole of the Ecliptic also changes. As the direction of the torque changes continually, the direction of movement of the Earth's poles also continually changes. In effect, therefore, the pole of the Earth and therefore the celestial pole will describe a small circle of radius 23½° around the pole of the Ecliptic in a clock wise (westward) direction.



(FIG.8.18)

The Moon's orbit is inclined at about 5¼° to the plane of the Ecliptic. The mean position of the Moon may however be considered as being on the Ecliptic. The effect of the Moon's gravitational force on the Earth is therefore similar to that of the Sun. The combined effect of the two bodies together is termed "Luni Solar Precession".

As the direction of the Earth's axis changes due to precession, the plane of the Equator, and therefore the plane of the Equinoctial shifts in sympathy (refer figure 8.18.). Since the plane of the Ecliptic is unchanged, the change in the plane of the Equinoctial causes the points at which the Equinoctial intersects the Ecliptic to shift westward along the Ecliptic.

The slow westward motion of the Equinoctial points (First point of Aries and First Point of Libra) along the Ecliptic by about 50.2" of arc each year is termed as **Precession of the Equinoxes**. A full cycle of precession occupies about 25,800 years.

The direction of the Earth's axis at present is less than 1° from the direction to Polaris. In about 13,000 years, the Earth's axis would have precessed to a direction about 47° away from Polaris. It will then be pointing in a direction within 5° of Vega. Vega would then probably be used as the Pole star.

Also due to precession, the First point of Aries, is now no longer in the constellation of Aries. It has cessed into the constellation of Pisces. The signs of the zodiac, therefore no longer correspond to the stellations after which they were named.

ffects of Precession

1. As the First point of Aries moves to the westward, at an average rate of 50.2" of arc each year, the RA of fixed bodies like stars, increase by a corresponding amount, yearly.

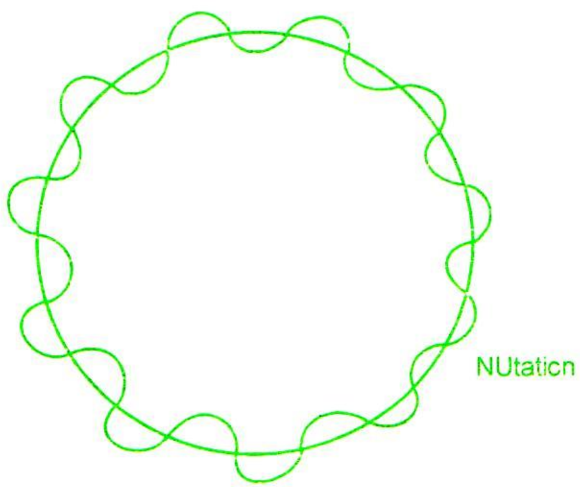
2. As the plane of the Equinoctial changes gradually, there is a corresponding change in the declination of stars, as declinations are measured N or S from the Equinoctial.

3. Due to precession the Tropical year is about 20 minutes shorter than a sidereal year.

   The value of precession is not uniform throughout the year, nor is its annual value constant.

8.13 NUTATION

The reason for the uneven rate of precession mentioned above is the change in direction of the gravitational force of the Sun and the Moon on the Earth, due to the change in their declinations. Under the topic, Earth Moon system, the 18.6 year cycle of movement of the Moon's nodes along the Ecliptic was described. Due to the motion of the Moon's nodes, the maximum declination of the Moon varies from a maximum value of about $28^3/_4{}^\circ$ to a minimum value of about 18¼°. This periodic variation in the value of the Moon's maximum declination causes the rate of precession to increase and decrease in an 18.6 year cycle. In addition, it also causes the Earth's axis to move inwards and outwards of the circle it would have described due to precession alone. This nodding of the Earth's axis as it moves around the pole of the Ecliptic is known as **Nutation.** The axis of the Earth therefore traces a wavy curve instead of describing a circle. The period of each wave is 18.6 years. (FIG. 8.19)



(FIG.8.19)

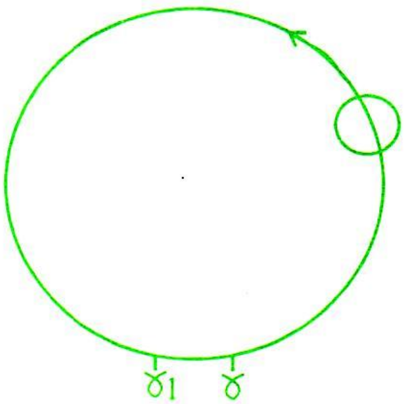
Effects of Nutation.

1. It makes the increase in RA of stars due to precession, uneven.
2. It produces a very small variation in the obliquity of the Ecliptic and also in the declination of stars.

8.14 THE YEARS

Sidereal year is the interval in time between two successive coincidences of the True Sun's centre with a fixed direction in space. In other words it is the time taken by the Earth to complete one revolution of 360° around the Sun. It is equal to 365.2564 Mean solar days.

Tropical year is the interval in time between two successive coincidences of the True Sun's centre with the First point of Aries i.e. the period between two successive vernal equinoxes. It is equal to 365.2422 Mean solar days. It is shorter than the sidereal year by 0.0142 mean solar days or about 20 minutes of time. This difference is caused due to precession of the equinoxes. If the First point of Aries was fixed, the Sidereal and Tropical years would have been of the same length. As explained earlier, due to precession of the equinoxes, the First point of Aries, has a westward motion of about 50" each year along the Ecliptic. Since the Sun's orbital motion is eastwards, it would return to the First point of Aries on completing 359°59' 10" of orbital motion, and not 360° as for completion of a Sidereal year. (FIG. 8.20)



(FIG.8.20)

Anomalistic year is the interval in time between two successive coincidences of the True Sun's centre with the point of perigee in its apparent orbit. It is equal to 365.2596 Mean solar days. Since the point of perigee moves eastwards, by about 11" of arc annually, the Anomalistic year is about 5 minutes longer than the sidereal year.

Civil year

Since the seasons recur at the interval of a Tropical year, that period would be the obvious choice as a civil year, because life on Earth is governed by seasons. It is also necessary that the calendar year should contain a full number of days. In an effort to combine these two requirements, Julius Caesar introduced the Julian calendar consisting of 3 civil years of 365 days each, followed by a 4th civil year of 366 days called a **leap year**.

For convenience, the years chosen to be leap years are the ones which are divisible by 4. Thus, according to the Julian calendar, the average length of the civil year, was exactly 365.25 Mean solar days. This is 0.0078 Mean solar days or about 11 minutes longer than the Tropical year. The slow accumulation of this difference over many years, would have put the calendar dates out of step with the seasons.

To allow for this, Pope Gregory XIII amended the Julian calendar in 1582. In the Gregorian calendar, 3 leap years of the Julian calendar are omitted in every 400 years. The leap years omitted are the century years in which the number of centuries are not divisible by 4. For example, 1700, 1800, 1900 and 2000 would have been leap years according to the Julian calendar. According to the Gregorian calendar, of these, only the year 2000 remains a **leap year.** Thus in 400 years, we now have 97 leap years and 303 years of 365 days. This makes the average length of the Civil Year equal to 365.2425 Mean solar days, which is very nearly equal to the length of Tropical year.

Exercise VIII

1. What will be the LHA of a star 6 hours after it was on the meridian of a stationary observer ?

2. On a certain day, at 0900 hrs, GMT, the Sun's GHA was 316°25'. Calculate the value of equation of time.

3. Given GHA Aries 300°50.8', SHAMS 132°10', LAT in long. 58°45'W, 13h 06m 12s. Find the value of equation of time.

4. Using the nautical almanac, find SHAMS and SHATS at 0735 GMT on 14th October, 1976.

5. A vessel sailed from 170°10'W, at apparent noon on 14th October, 1976. She arrived in longitude 170°05'E at LMT 14h 12m on 17th October, 1976. Find her steaming time.

6. On 14th October, 1976, find the equation of time at 14h 30m GMT without using the tabulated values of equation of time.

7. On 14th Oct.1976, find the LMT, at 1134 LAT in long.72°12'W.

8. At 16h 40m 30s GMT, RAMS was 12h 40m 40s. Find the SHA of a star whose GHA then was 40°05'.

9. Find the LAT when LHATS was 30°15'. If the observer's longitude was 15°15'E, what was the GAT ?

10. A vessel left port at 0910 GMT on a certain day. She steamed 445 miles till apparent noon the next day and was in long. 22°12'W. If at 1400 hrs GMT on that day, the Sun's GHA was 34°08', find her steaming time and average speed.

11. Express 8h 48m 28s of Mean solar time in sidereal time.

12. At 22h 08m 07s GMT, long. of the GP of a star was 24°32'W. If RAMS at that instant was 05h 46m 03s, find the star's RA.

13. At 04h 01m 30s GMT, to an observer, the Sun's LHA was 311°30.1. If equation of time was +11m 30s, find the observer's longitude.

14. At a certain pos'n a star, whose RA was 14h 06m 38s was at its max. altitude, when RAMS was 18h 38m 12s. If the GMT at that instant was 07h 48m 28s, find the long. of the pos'n.

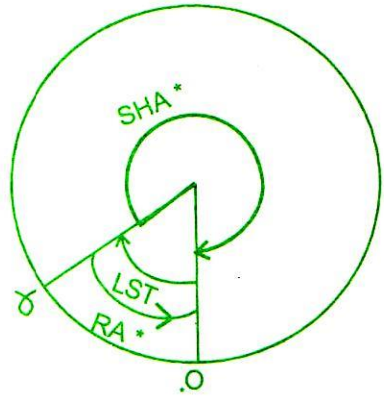
15. On a certain date, in longitude 15°W, LHA Sun was 320°, when GHA γ was 271°. Find the Sun's SHA.

Harder Problems

1. Star Aldeberan (SHA 291°21.6') was on the observer's meridian, when the observer's sidereal clock showed 4h 30m. Find the error of the clock. If the Sun was on the meridian at 13h 12m 10s by the same clock, find the Sun's SHA.

SHA *	=	291°21.6'			
RA*	=	68°38.4'			
Correct sidereal time	=	4h 34m		33.6s	
Time by clock	=	4h 30m		00.0s	
Error of clock	=	4m		33.6s	slow
Time of Sun's meridian passage by sid.clock	=	13h 12m		10s	
Error		4m		33.6s	slow
Correct sidereal time	=	13h	16m	43.6s	

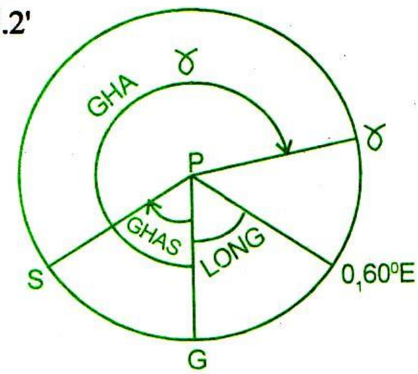
	=	RA Sun		
SHA Sun	=	10h	43m	16.4s



(FIG.8.21)

2. Find the True Sun's SHA at the instant when the First point of Aries crossed 60° East longitude, if on that day, GHA Sun was 62°47.2', when GHA Aries was 264°12'.

GHA γ	264° 12.0'
GHA Sun	62° 47.2'
RA Sun	201° 24.8'
SHA Sun	158° 35.2'



(FIG.8.22)

As the Earth turns from West to East, both Aries and Sun appear to move westwards, Aries at the rate of 15°02.46' per hour and Sun at the rate 15° per hour.

For Aries to come to 60°E, it has to move through 35°47.2'.

From increment tables for Aries in the nautical almanac, it is found that for GHA γ to increase 35°47.2', it takes 2 hours 22m 45 seconds. For the same period, it is found from the Sun's increment tables that its GHA would increase by 35°41.3'.

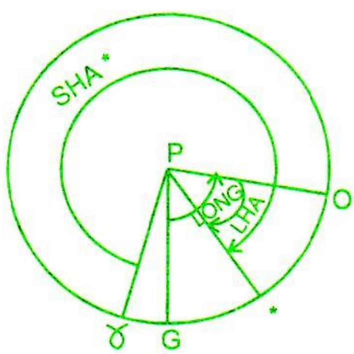
Aries therefore closes up on the Sun by :
(35°47.2' - 35°41.3') = 5.9'

Initial SHA of Sun 158°35.2'	
Reduction	5.9'
SHA Sun when Aries is on 60°E	= 158°29.3'

3. If at 1600 hrs. GMT, GHA γ was 357°12.2', find the GMT at which a star with SHA of 335°13' will have an LHA of 53°12' in longitude 074°50'E.

Longitude of observer	=	074°	50'
LHA *	=	53°	12'
Greenwich EHA *	=	21°	38'
SHA *	=	335°	13'
SHA Greenwich	=	356°	51'
GHA γ	=	03°	09'
GHA γ at 1600 GMT	=	357°	12.2'
Increase in GHA	=	5°	56.8'

From increment table for Aries in almanac, for 5°56.8', we get 23m 43s & hence GMT = 16h 23m 43s.



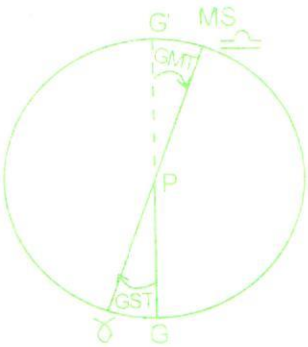
(FIG.8.23)

4. The Greenwich sidereal time of culmination of a star was 11h 14m 50s on October 14, 1976. What was the LMT then for an observer in longitude 162°12' West ?

Greenwich sidereal time	=		11h	14m	50s
GHA γ	= 168° 42.5'				
From nautical almanac, on 14th October					
GMT		14d	09h	42m	35s
LIT			-10h	48m	48s
LMT		13d	22h	53m	47s

5. At what time GMT and on what date in 1976, will two clocks one keeping GMT and the other GST, show the same ?

GMT is measured westwards from the inferior meridian of Greenwich, to the Mean Sun. GST is measured westwards from the Greenwich meridian to Aries. Since the two times are measured from meridians 180° apart, for the times to be equal, the points to which they are measured (MS and γ) respectively, should also be 180° apart i.e. GHAγ and GHAMS, should differ by 180° or MS should be at Libra.



(FIG.8.24)

We know that the True Sun is at Libra on 23rd September. Since the meridians of TS and MS differ only by the amount of the equation of time (which is never large), MS will also be at Libra on or about 23rd September.

Inspecting the nautical almanac, about that date, we should find the time when their GHAs differ by exactly 180°. When that occurs, the two clocks will show the same time as explained earlier.

		GHAMS	GHAγ	Diff.
On 21st Sept.	0000 hrs GMT	180°	359°59.8'	179°59.8'
On 22nd Sept.	0000 hrs GMT	180°	0°58.9'	180°58.9'
	24 hrs			0°59.1'

In 24 hours, the difference in GHA between MS and γ has changed by 59.1'. To change 0.2' it would take (0.2' x 24) ÷ 59.1 = 0.081 hrs = 4.9 m.
The two clocks will show exactly the same time at 00h 04.9m GMT on 21st September.

6. Calculate the equation of time, if the Sun rose at 0507 LMT and set at 1858 LMT at a certain position. Hence show the effect of equation of time on the lengths of the forenoon and afternoon. Assume the Sun's declination remained unchanged between sun-rise and sun-set.

LMT sun set	18h	58m	00s
LMT sun rise	05h	07m	00s
Length of 'day	*13h*	*51m*	*00s*
1/2 length of day	06h	55m	30s
LMT sun rise	05h	07m	00s
LMT meridian passage	12h	02m	30s
LAT meridian passage	12h	00m	00s
Equation of time		+02m	30s

Because the equation of time is +ve, Mean noon occurs earlier than apparent noon. Since Mean noon separates forenoon and afternoon, the forenoon will be shorter than afternoon. The effect will be opposite, if equation of time was -ve.

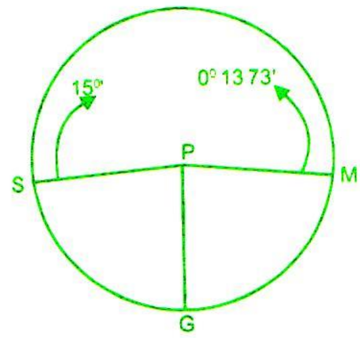
7. On a certain day, for a stationary observer, LMT sun rise was 06h 52m. The Sun's meridian passage occurred at 1151 LMT. Find LAT of sun set.

LMT meridian passage	11	51
LMT sun rise	06	52
Duration of morning	04	59
Duration of afternoon will also be	04	59
LAT meridian passage	12	00
LAT sun set	16	59

8. On 14th October, 1976 a vessel left Madras, lat. 13°06'N, Long. 080°18.0'E (Time zone - 5.30) at 1730 hrs, GMT and steered 048°(T), at 18 knots. Find the alteration to her clocks necessary so that they would show 12 00 hrs, at apparent noon, the next day.

Departure Madras GMT	14d 17h 30m 00s
GHA Sun	86°01.3'
Long.of Madras (E)	80°18.0'

LHA Sun	166°19.3'
	360°00.0'
Angular separation between Sun & ship on departure	193°40.7'
N48°E, 18M, dep = 13.38M	d'long 13.73'
The Sun moves westward	15°/hour
The ship moves eastward	0°13.73'/hour
They approach each other at	15°13.73'/hour



(FIG.8.25)

At apparent noon, the ship and the Sun are on the same meridian.

Steaming time till apparent noon next day

= 193°40.7' / 15°13.73' = 12h 43m 04s

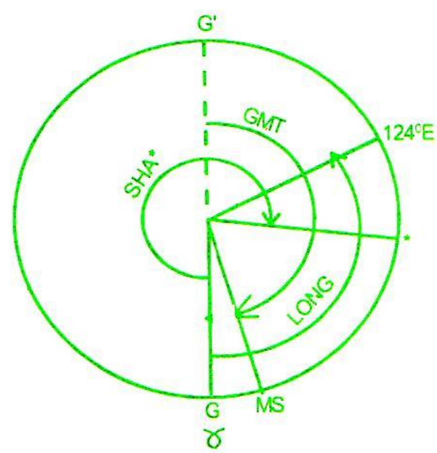
Departure GMT 00	17	30	
Time Zone	-5	30	
Departure IST 00	23	00	
Time shown by clock :-			
Steaming time	+12	43	04
Time shown by clock at apparent noon next day	35	43	04
	=11	43	04
The clock should show	12	00	00
Alteration necessary 16m 56s to advance			

9. On a certain day, the Greenwich transit of Aries was 11h 32m GMT. At what time LMT on the same day did star Betelguese (SHA 271°31') transit the meridian of 124° East.

SHA Betelguese	271°31'
Longitude (East)	124°00'

	395°31'
	-360°

LHA of Betelguese	035°31'



(FIG.8.26)

The star is 35°31' past the meridian of 124°E.

The rate of apparent motion of star = 15°2.46'/hour (as for Aries).

Time interval after star transitted 124° East			
(From increment table for Aries)	2h	21m	42s
Present GMT	11h	32m	00s
GMT of transit of star			
over 124°East	9h	10m	18s
LIT	8h	16m	00s
LMT transit of star	17h	26m	18s

10. On 14th October, 1976 in longitude 32°W the LHA of star Rigel was 44°12'. Find the GAT at that instant.

LHA * = GHAγ + SHA * - Long (W)

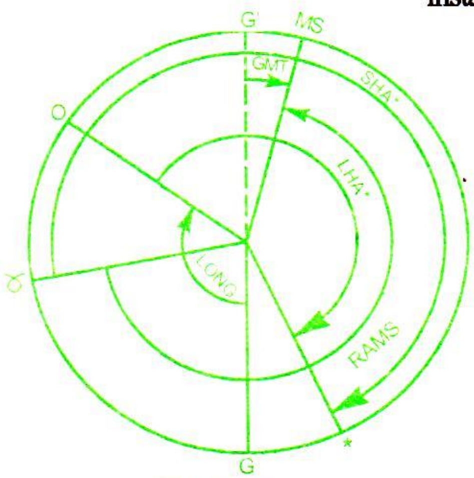
GHAγ = LHA * + Long (W) - SHA *

LHA *	=	44°12'
Long (W)	=	32°00'

GHA *	=	76°12'(+ 360°)
(-) SHA *		281°38.4'
GHAγ		154°33.6'

GMT for GHAγ of 154°33.6'	= 14d 08h 46m 06s
GHA of True Sun for this GMT	= 315° 02'
	- 180° 00
LHA of True Sun from	---------------
inferior meridian of Greenwich	135° 02'
GAT	= 14d 09h 00m 08s

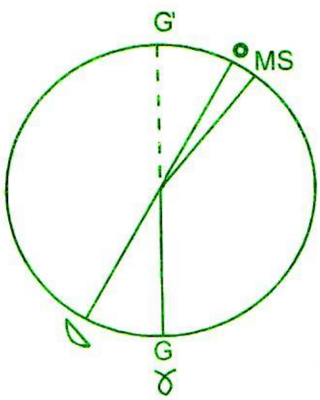
11. A star with SHA of 252°20' had an LHA of 212°45' in longitude 124°40'W. If RAMS then was 17h 06m 12s, find the GMT at that instant.



(FIG.8.27)

SHA *	252°20'
LHA *	212°45'
EHAγ	39°35'
Long (W)	124°40'
GHAγ	85°05'
SHAMS =360°-RAMS =	103°27'
GHAMS	188°32'
	-180°00'
	8°32'
GMT	00h 34m 08s

12. For an observer at Greenwich, the GMT transit of Aries, was 00h 34m 12s. Equation of time + 4m 15s. If at that instant, a total eclipse of the Moon occurred, find the Moon's RA. Also find the Moon's declination then, if the Sun's declination was 2°30'S.



(FIG.8.28)

As explained under the topic 'eclipse', for a lunar eclipse to occur, the GHAs of the True Sun and the Moon should differ by 180° and their declinations should be equal in amount and opposite in names.

GMT	=	00h	34m	12s
Eqn.of time	(+)	4m	15s	
GAT		00h	29m	57s
GHA Moon	=	00h	29m	57s
RA Moon	23h	30m	03s	

Since Sun's declination is 2°30'S, Moon's declination = 2°30'N

Theory Questions

1. Define 'Sidereal day', 'Apparent solar day' and 'Mean Solar day'.

2. Give the reasons for the variation in the duration of the apparent solar day.

3. Define 'Mean Sun', 'Dynamical Mean Sun', 'Local Mean Time', 'Greenwich apparent time' and 'Local sidereal time'.

4. Explain the terms Standard time and Zone time.

5. What is the International 'Date line' ? Why is it necessary and how is the date on a ship crossing the International Date line on an easterly course affected ?

6. Define 'equation of time' and explain the two components of 'equation of time.'

7. Explain how 'equation of time' becomes nil four times in the year. State the approximate dates on which it is nil.

8. What do you understand by the term 'Precession of the equinoxes' and what are its effects ?

9. Discuss Nutation and its effects.

10. What are Sidereal year, Tropical year and Anomalistic year ? Why are they not of the same length ?

11. Discuss the calendar in use at present.

12. Explain why a 'sidereal day' is about 4 minutes shorter than a 'solar day'.

13. What effect has the 'equation of time' on the length of 'fore- noon' and 'after-noon' ?

14. Why does a star appear to rise, culminate and set 4 minutes earlier each day.

15. With the aid of figures, show the following relationships.
    (1) GMT ~ LMT = Long
    (2) GHA* + Long.E = LHA*
    (3) GHAγ + SHA* - Long.W = LHA*
    (4) GHAγ - GHAMS = RAMS
    (5) LHA* + RA* = RA of observer's meridian.

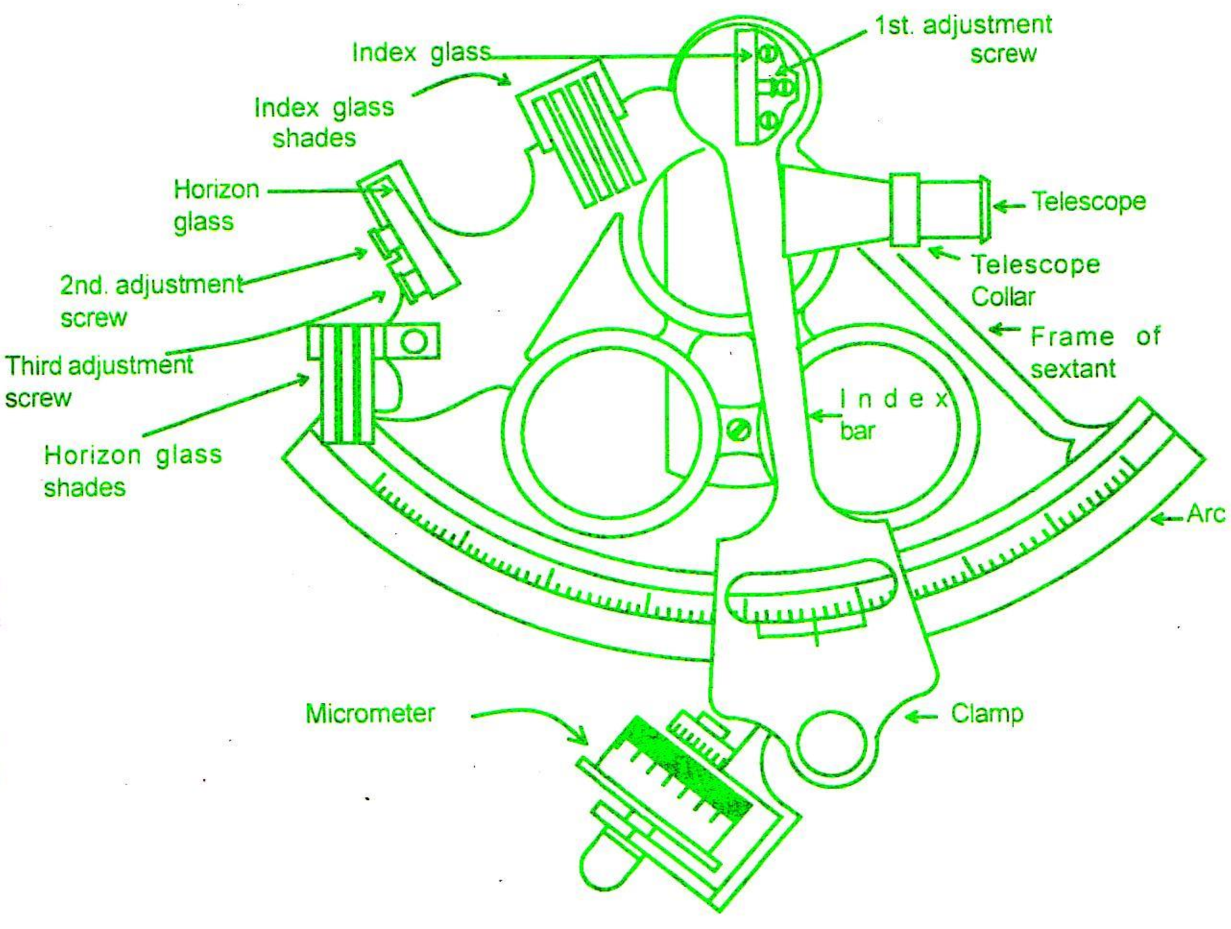
9

ALTITUDES

(Measurement and Correction)

9.0 SEXTANT

The Sextant is a precision instrument used at sea for measuring altitudes of celestial bodies and horizontal angles between terrestrial objects and also their vertical angles.



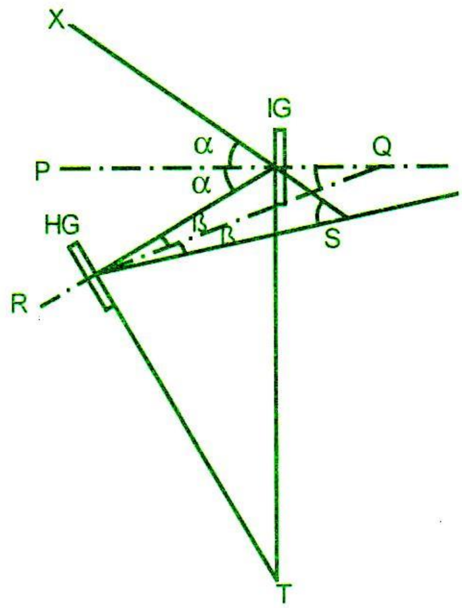
(FIG.9.1)

Principle of the sextant

1. When a ray of light is reflected by a plane mirror, the angle of incidence is equal to the angle of reflection, with the incident ray, reflected ray and the normal lying in the same plane.

2. When a ray of light, suffers two successive reflections in the same plane, by two plane mirrors, the angle between the incident ray and the final reflected ray is twice the angle between the mirrors.

The first principle is simple and requires no explanation. The proof of the second principle however is given below.



(FIG.9.2)

A ray of light from X is incident on the index glass at an angle α with the normal PQ. It reflects making the same angle with the normal. The reflected ray meets the horizon glass at an angle of incidence ß and is again reflected making the same angle with the normal QR.

The angle between the incident ray and the final reflected ray then is angle S. The angle T between the mirrors is equal to the angle between their normals i.e. angle Q.

To prove that angle S = twice angle Q

$$\alpha = Q + \beta$$

$$Q = \alpha - \beta$$

Multiplying by 2

$$2Q = 2\alpha - 2\beta \text{ .... (i)}$$

Again $2\alpha = 2\beta + S$

(ext. angle = sum of interior opposite angles)

Substituting in (i) $2Q = 2\beta + S - 2\beta = S$

When the sextant reads zero, the index and horizon glasses are parallel to each other. When the Index bar and therefore the Index glass is rotated through any angle, the angle between the incident ray and the final reflected ray is twice the angle through which the index bar was rotated. The arc of the sextant is only 60° in extent, but due to the principle of double reflection, we are able to mark the arc and measure angles upto 120°. Sixty degrees being a sixth of a circle, the instrument is known as a sextant.

The present day sextants are provided with micrometers which enable easy and accurate reading of angles to an accuracy of 0.1' of arc. The vernier sextant is now practically obsolete.

Errors and adjustments

The errors on a sextant may be classified as :

(i) Adjustable errors and

(ii) Non-adjustable errors

The adjustable errors are :-

(a) **Error of perpendicularity** is produced by the index glass not being perpendicular to the plane of the instrument. To check for this error, clamp the index bar at about the middle of the arc, and holding the sextant horizontally, with the arc away from you, look obliquely into the index mirror till the arc of the sextant and its reflection in the index mirror, are seen simultaneously. If they appear in alignment, error of perpendicularity is not present. If not, turn the first adjustment screw at the back of the Index glass, until they appear in alignment.

(b) **Side error** is caused by the horizon glass not being perpendicular to the plane of the instrument.

To check for side error, by day, clamp the index bar at zero, hold the sextant horizontally and observe the horizon through the telescope. If the true horizon and its reflection in the mirror half of the horizon glass, appear in alignment, side error is not present.

If they do not, side error exists. It can be removed by turning the second adjustment screw (the top screw at the back of the horizon glass), until the true and reflected horizons appear in the same line.

To do this at night, clamp the index bar at zero and holding the sextant vertically, observe a star through the telescope. If the star and its reflection are not displaced horizontally, side error is absent. If they are displaced horizontally, the error exists and can be eliminated by adjusting the 2nd adjustment screw till there is no horizontal displacement between them.

(c) **Index error** is caused by the index glass and the horizon glass not being parallel to each other, when the index bar is at zero. To find the index error, by day, using the horizon, clamp the index bar at zero and holding the sextant vertically, view the horizon through the telescope. If the true horizon and its reflection appear in the same line, Index error is not present. If they appear displaced vertically, turn the micrometer drum till they are in the same line. The micrometer reading then is the index error, which is **on the arc** if the micrometer reading is more than zero, and **off the arc** if it is less than zero.

To eliminate Index error, clamp the index bar at zero and looking through the telescope, turn the third adjustment screw, till the true horizon and its reflection appear in alignment. The third adjustment screw is located below the 2nd adjustment screw, and towards the side, at the back of the horizon glass.

The index error can also be found using the Sun. With the Index bar clamped at zero, using the necessary shades, view the Sun through the telescope, holding the sextant vertically. Turn the micrometer, 'on the arc', till the upper limb of the reflected Sun touches the lower limb of the True Sun. Note the reading 'on the arc'. Turn the micrometer 'off the arc' till the lower limb of the reflected Sun touches the upper limb of the True Sun. Note the reading 'off the arc'. If the two readings are the same, index error is not present. If not, the amount of the Index error is half the difference between the two readings and named 'on the arc' or 'off the arc' respectively according to whether the 'on the arc' reading or the 'off the arc' reading was larger. This method allows a check on any observational errors as the sum of the two readings divided by 4 should give the semi diameter of the Sun for that day.

To find the Index error at night, clamp the Index bar at zero and holding the sextant vertically, view a star through the telescope. If the star and its reflection are not displaced vertically, index error is not present. If they are displaced vertically, adjust the micrometer till they appear with no vertical displacement. The micrometer reading then is the Index error. To correct this, with the index bar clamped at zero, turn the third adjustment screw, till any vertical displacement between the star and its reflection is eliminated.

The second and third adjustments being carried out on the horizon glass itself, one adjustment may affect the other. It is therefore advisable after adjusting one, to check for the other. When the Index error is not large, it is usually left uncorrected, as frequent adjustments may cause the adjusting screw to become slack. The error if left uncorrected should be allowed for when correcting the measured angle.

(d) **Error of collimation** is due to the axis of the telescope not being parallel to the plane of the instrument.

Old sextants were provided with an adjustable telescope collar so that this error if present, could be removed by adjusting the collimating screws on the telescope collar. In present-day sextants, the telescope is attached to the body of the sextant in such a manner that it cannot tilt. These sextants are therefore not provided with any collimating screws.

Non adjustable errors

(a) **Graduation error** is due to inaccurate graduation of the main scale on the arc or of the micrometer/vernier.

(b) **Shade error** is due to the two surfaces of the coloured shades not being exactly parallel to each other.

(c) **Centering error** is caused due to the pivot of the index bar not being coincident with the centre of the circle of which the arc is a part.

(d) **Optical errors** may be caused by prismatic errors of the mirrors or aberrations in the telescope lenses.

(e) Wear on the rack and worm, which forms the micrometer movement would cause a **back-lash**, leading to inconsistent errors.

9.1 CORRECTION OF ALTITUDES

The Observer's Zenith has been defined earlier as the point on the celestial sphere, vertically above the observer i.e. the point at which a straight line from the centre of the Earth, through the observer, would meet the celestial sphere.

Observer's Rational or Celestial horizon is a great circle on the celestial sphere, every point on which is 90° away from his zenith. The plane of the observer's rational horizon, therefore, passes through the centre of the Earth.

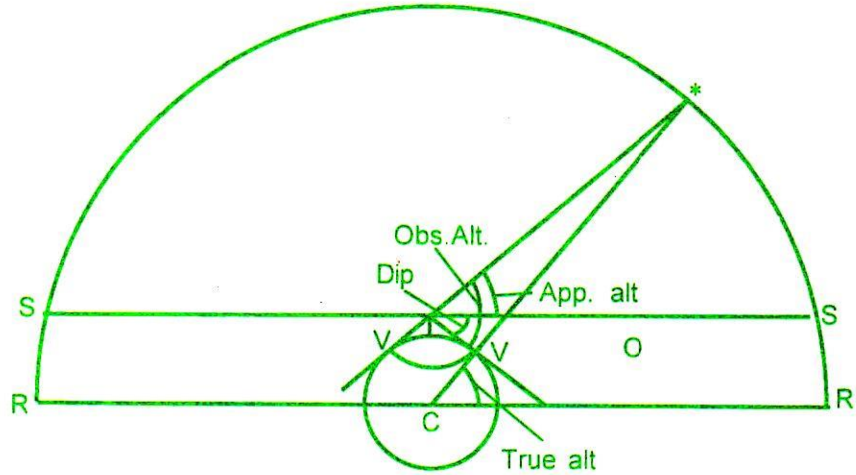
Visible horizon is the small circle on the Earth's surface, bounding the observer's field of vision at sea.

It should be realized that the radius of the visible horizon increases as the observer's height of eye increases.

Sensible horizon is a small circle on the celestial sphere, the plane of which passes through the observer's eye, and is parallel to the observer's rational horizon.

Sextant altitude is the altitude of a body, above the visible horizon, as read off from the sextant.

Observed altitude of a celestial body is the angle at the observer between the body and the direction to the observer's visible or sea horizon. The observed altitude is therefore, the sextant altitude corrected for any index error.



(FIG.9.3)

True altitude of a heavenly body is the arc of a vertical circle, or the angle at the centre of the Earth contained between the plane of the observer's rational horizon and the centre of the body.

To obtain the true altitude of a celestial body, various corrections have to be applied to its altitude measured by the sextant.

The corrections, in the order they are to be applied, follow : Index error, Dip, Refraction, Semidiameter and Parallax.

Index error is the instrumental error of the sextant used in measuring the altitude. The sextant altitude is therefore corrected first for Index error. I. E. is **added** if it is **off the arc** and **subtracted** if it is **on the arc.**

Dip is the angle at the observer between the plane of observer's sensible horizon, and the direction to his visible horizon.

Dip occurs because the observer is not situated at sea level. The value of dip increases as the observer's height of eye increases. The values of dip are tabulated on the cover page of the nautical almanac and in nautical tables, as a function of the height of eye.

Dip correction tables are arranged as critical tables. No interpolation is required, as a single correction value applies for an interval of heights of eye. At a critical entry, the upper value of the correction is to be taken.

Dip is applied to the observed altitude to obtain the altitude of the body above the sensible horizon. The altitude of a body above the sensible horizon is known as its apparent altitude.

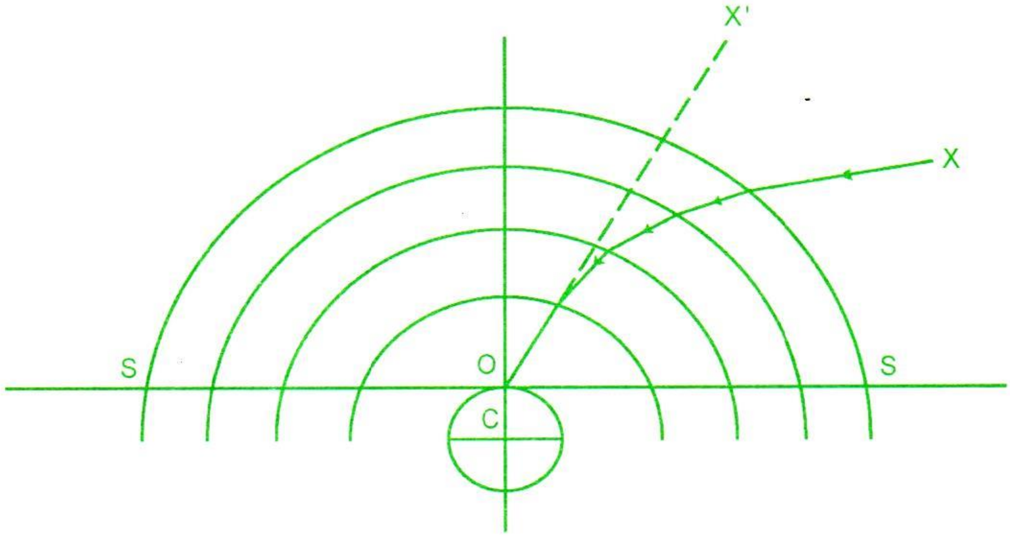
As can be seen from the figure, dip should always be subtracted from the observed altitude.

Apparent altitude is the sextant altitude corrected for Index error and dip.

Refraction is the deviation of light rays passing from one medium to another. When passing from a rarer medium into a denser medium,the ray refracts towards the normal to the surface of separation between the two media.

The atmosphere of the Earth is most dense at the Earth's surface and becomes rarer as the height above the surface increases. It may therefore be considered as being composed of various layers, each layer being rarer than the one below it. A ray of light from a celestial body, passing through the Earth's atmosphere, is continuously refracted until it reaches the observer. Due to this, the apparent direction in which the ray finally reaches the observer is larger in altitude than the true direction to the body.

Since refraction increases the apparent altitude of the body, refraction correction is always negative.



(FIG.9.4)

The value of refraction varies with the angle which the ray makes with the normal to the surface of separation between the two media. Refraction has a maximum value of about 34.5' when the body is on the horizon and it decreases as the altitude increases. It is nil when the body is at the zenith, as no refraction can take place when the ray is coincident with the normal.

Refraction correction is tabulated as a function of the altitude. Tables of correction for refraction are available, both in the nautical tables and on the cover page of the nautical almanac. In the almanac, they are tabulated under the head 'Total correction for stars and planets'. Besides index error and dip, refraction is the only correction necessary for star altitudes. The value of refraction correction for a particular altitude holds good for all celestial bodies.

Refraction occurs, not only in the case of light rays from celestial bodies, but also of those from the visible horizon. The actual angle of dip is therefore less than the theoretical angle at the observer. This difference caused due to terrestrial refraction is also allowed for in the tables of dip corrections.

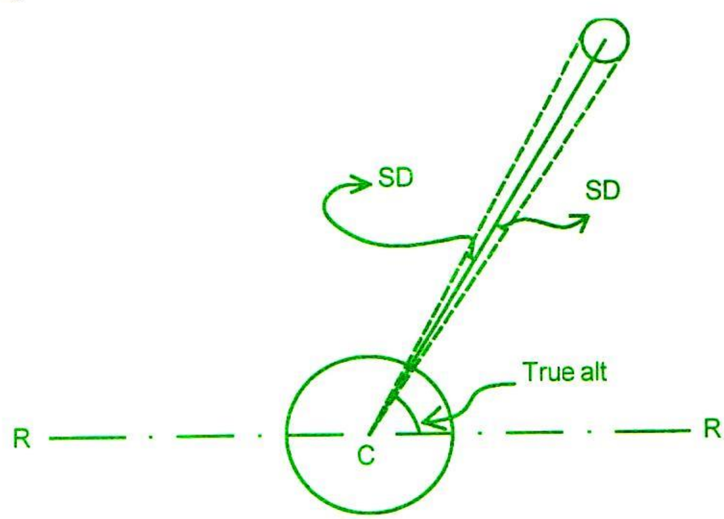
When the temperatures and/or pressure are abnormal, a further correction may become necessary due to the abnormal refraction, particularly when the measured altitudes are small. A table of corrections for such conditions is also provided in the nautical almanac.

Semidiameter

The values of declination and GHA of the various celestial bodies tabulated in the almanac are those of their centers. Since these parameters used in working a sight refer to the centre of the body, it is essential that the body's zenith distance used in the working and therefore the true altitude should also refer to the centre of the body.

Stars and planets appear as point sources of light. The altitude of these bodies, when observed is therefore, directly that of their centers.

The Sun and Moon present visible discs to the observer. It is not possible to measure the altitude of their centers, as it is difficult to judge their exact centers, by sight. Therefore, we measure the altitude of either their upper limb or lower limb, to which we apply half the apparent diameter of the body to obtain the altitude of their centers. It is obvious that the semi-diameter should be added to an altitude of the lower limb and subtracted from the altitude of the upper limb to obtain the altitude of the centre of the body.



(FIG.9.5)

The semi-diameter (SD) of the Sun is tabulated, once for every 3 days, in the daily pages of the nautical almanac. For the Moon, it is tabulated for each day.

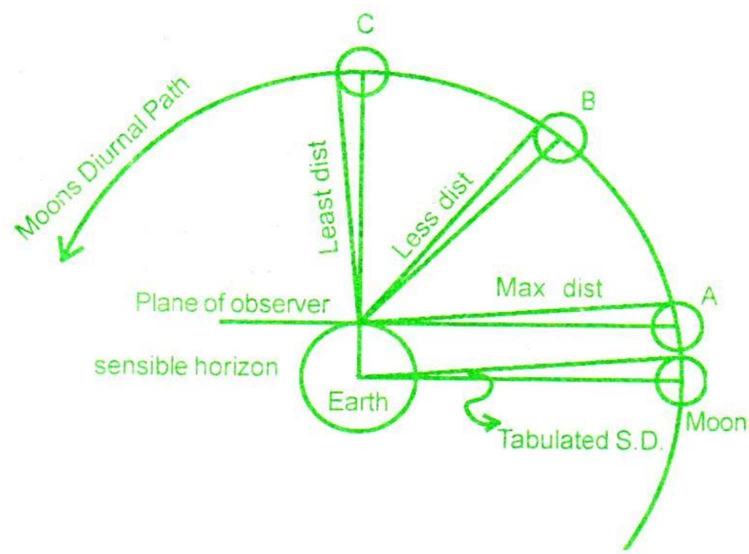
The apparent semi-diameter of these bodies depend upon their distance from the Earth. They are maximum when the bodies are closest to the Earth and minimum when they are farthest. In the case of the Sun, the SD varies from 15.8' at the beginning of July when the Sun is at apogee to 16.3' at the beginning of January when the Sun is at perigee. Similarly the SD of Moon varies from about 14.8' to about 16.7'.

It may be seen from the figure that :

Sin SD = radius of body / dist. of body from the Earth

Augmentation of the Moon's SD

The semi-diameter values tabulated in the almanac are those as would be apparent from the centre of the Earth. As the observer on the Earth's surface is closer to the Moon than the Earth's centre, the SD of the Moon as observed by him would be larger than the tabulated SD value.



(FIG.9.6)

When the Moon is on the horizon, its distance to the observer is about the same as its distance to the centre of the Earth. As the Moon rises in altitude, its distance to the observer becomes less than its distance to the Earth's centre. When at the zenith, the Moon is closer to the observer by the amount of the Earth's radius about 4000 miles. The observed SD of the Moon therefore increases as its altitude increases. Augmentation of the Moon's SD is the increase in the observed SD of the Moon caused due to its distance to the observer reducing with increase in its altitude.

Augmentation is nil when the Moon is on the horizon. It increases as the Moon's altitude increases and reaches a maximum value of 0.3' when the Moon is at the zenith. To allow for this, it is necessary to augment or increase the tabulated value of the Moon's SD by the amount of the augmentation correction. The augmented semidiameter is then applied to correct the altitude.

Augmentation corrections are available in the various nautical tables. It is tabulated as a function of the altitude.

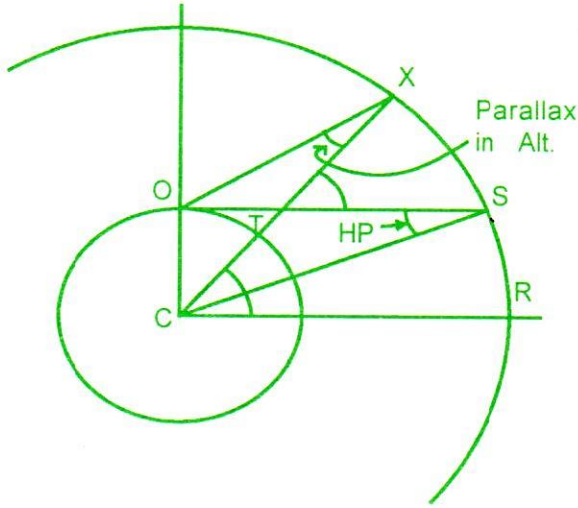
As the average distance of the Moon from the Earth is only about 240,000 miles, the radius of the Earth, which is approximately 4,000 miles does make a significant difference between the distance to the Moon from the Earth's centre and that from an observer situated on the Earth's surface. The Sun being about 93,000,000 miles away, the radius of the Earth does not cause any significant reduction in the distance of the Sun from the observer. Augmentation correction is therefore not necessary in the case of the Sun.

Parallax :

Having applied the various corrections, mentioned above the altitude of the centre of the body, above the observer's sensible horizon, is obtained. To obtain its altitude above the observer's rational horizon, it is necessary to apply a further correction known as **parallax in altitude** or simply **parallax.**

Horizontal Parallax

(HP) of a body is the angle at the centre of the body contained between the centre of the Earth and observer at the surface of the Earth, when the body is on the observer's sensible horizon.



(FIG.9.7)

Parallax in altitude is the angle at the centre of the body contained between the centre of the Earth and the observer on the surface of Earth, when body is at any altitude. It may be seen from the figure, that parallax is maximum when the body is on the sensible horizon, and it reduces as the altitude increases, till it becomes nil when the body is at the observer's zenith.

In fig.9.7, angle SOX is the altitude of the body above the sensible horizon, obtained by applying the various corrections other than parallax. The true altitude above the rational horizon is angle RCX.

The True Alt. (angle RCX)=angle XTS (corresponding angles) but angle XTS = angle SOX + angle OXC (being ext.angle of triangle XOT)

Angle RCX = angle SOX + angle OXC
Thus true alt. = altitude above sensible horizon + parallax in altitude.
Parallax correction is therefore always additive.

It is obvious that the parallax of bodies will reduce as their distance from the Earth increases. Parallax is therefore largest in the case of Moon, lesser in the case of planets, still lesser in the case of the Sun and nil in the case of stars, as the radius of the Earth will not subtend any measurable angle at the centre of stars, which are immensely distant.

Parallax correction for the Sun is available in the various nautical tables. It is tabulated as a function of the altitude. It varies from 0.15' when the altitude is nil to zero when the Sun is at the zenith.

For planets, this correction combined with a correction for phase is given as an additional correction under the head "Stars and Planets" on the cover page of the nautical almanac. For the Moon the horizontal parallax is tabulated for each hour. Its value is around 60'. The Moon's parallax in altitude may be obtained by multiplying the horizontal parallax by the cosine of its apparent altitude, as proved below.

In fig. 9.7, by the sine rule applied to triangle OCX

$$\frac{\text{sine parallax in alt}}{OC} = \frac{\text{sin angle COX}}{CX}$$

$$\text{sin parallax in alt.} = \frac{OC}{CX}\sin(90 + \text{angle SOX})$$

$$= \frac{OC}{CS}\cos \text{apparent alt.}$$

$$= \sin HP \times \cos \text{apparent alt.}$$

As the sine of a small angle is equal to the angle itself (in radians) and as both parallax in alt. and horizontal parallax are small angles,

Parallax in alt. = Horizontal parallax x cos app.alt.

Where extreme accuracy is required, the Moon's HP should be reduced by the amount of the 'reduction' tabulated in the various nautical tables, as a function of the observer's latitude. This correction allows for the equatorial radius of the Earth being about 13½ miles larger than its polar radius. Due to the variation in the Earth's radius the HP of the Moon is largest for an observer at the Equator. It reduces as the observer's latitude increases. The tabulated values of HP in the nautical almanac are those for an observer at the Equator.

The corrections to be applied to the sextant altitudes of the various celestial bodies are listed as follows :-

Stars	Planets	Sun	Moon
a) IE	a) IE	a) IE	a) IE
b) Dip	b) Dip	b) Dip	b) Dip
c) Refraction	c) Refraction	c) Refraction	c) Refraction
-	d) Correction for parallax and phase (Venus and Mars only)	d) SD	d) Augmented SD
		e) Parallax in alt.	e) Parallax in alt.

Total Correction

In Practical Navigation, it is usual to apply the corrections listed at 'c', 'd' and 'e' above as a "Total Correction". Total correction tables for the various celestial bodies are available on the front and back cover pages of the nautical almanac and in the nautical tables. After applying the Index error, if any, the dip correction is subtracted to obtain the apparent altitude. The apparent altitude is used as the argument to obtain the 'total correction' for the various bodies.

Tables of total correction are provided separately for stars and planets together, for the Sun, and for the Moon. The total correction for stars consists of the refraction correction alone. A further small correction is provided for Venus and Mars, due to their proximity to the Earth, to allow for their parallax and phase. The total correction table for the Sun is provided separately for lower limb and upper limb observations; for two periods of the year.

The total correction table for the Moon is provided in two parts. In the upper part, the corrections are tabulated as a function of the apparent altitude. The correction is additive to lower limb observations. In the case of upper limb observations also the correction is to be added, but 30' is to be subtracted thereafter. The lower table gives the correction for parallax, separately for lower and upper limbs, as a function of the Horizontal parallax obtained from the daily pages. This correction is also additive.

9.2 BACK ANGLES

When the near horizon is not available for a sight due to fog or intervening land, it is possible to measure the altitude of a celestial body to the opposite point of the horizon. The altitude thus measured to the far horizon would be over 90°, and is called a 'back angle'. Such an observation is possible only when the body is fairly close to the observer's zenith, as the sextant cannot measure angles greater than 120°. To correct a back angle observation, index error, dip and SD are applied initially. The angle so obtained is subtracted from 180°. Refraction and parallax for the angle so obtained are then applied to it. All corrections are applied with the normal signs, as for an observation to the near horizon.

9.3 COMPUTING THE SEXTANT ALTITUDE

If the approximate sextant altitude of a star could be pre-computed and set on the sextant, it would help greatly in locating that star for a sight during twilight, when the sky is still fairly bright. This method is often used to obtain altitude of stars on the meridian,during that period. To compute the altitude, the true altitude of the body is calculated for that time, using the ship's DR position. The various corrections are then applied to the calculated altitudes in the reverse manner and in the reverse order, to obtain the sextant altitude of the body.

Examples :

1. Correct the following sextant altitudes applying each correction separately. Verify the results using the total correction method.

   (a) Capella, 23°12.7'; IE 1.2' off the arc; HE 11.0 m
   (b) Mars ; 42°54.3'; IE 0.7' on the arc; HE 9.0 m; on 2nd Feb.76
   (c) Sun's UL; 35°19.1'; IE nil; HE 12.8m, on 14th Oct.1976
   (d) Moons LL 60°12.0'; IE 1.5' off the arc, HE 14m, on 14th October, 1976 at 1730 GMT.

(a) Capella

Sext alt.	23° 12.7'
I.E.	+ 1.2'
Observed alt.	23° 13.9'
Dip	- 5.8'
Apparent alt.	23° 08.1'
Refraction	- 2.3'
True alt.	23° 05.8'

(b) Mars

Sext. alt.	42° 54.3'
I.E.	- 0.7'
Observed alt.	42° 53.6'
Dip	- 5.3'
Apparent alt.	42° 48.3'
Refraction	- 1.0'
Parallax	+ 0.1'
True alt.	42° 47.4'

(c) Sun's U.L.

Sext. alt.	35° 19.1'
I.E.	Nil
Observed alt.	35° 19.1'
Dip	- 6.3'
Apparent alt.	35° 12.8'
Refraction	- 1.4'
	35° 11.4'
SD	- 16.1'
	34° 55.3'
Parallax	+ 0.1'
True alt.	34° 55.4'

(d) Moon's L.L.

SD = 15.0'
Aug. = + 0.2'

HP 55.1
Parallax = 55.1' Cos $60^0 07'$ = 27.5

Sext. alt.	60° 12.0'
I.E.	+ 1.5'
Observed alt.	60° 13.5'
Dip	- 6.6'
Apparent alt.	60° 06.9'
Refraction	- 0.6'
	60° 06.3'
Aug. SD	+ 15.2'
	60^0 21.5'
Parallax	+ 27.5'
True alt.	60^0 49.0

Note

Examples 2 to 10 are given after the reader has gained proficiency in altitude correction by solving Exercise IX

Exercise IX

Obtain the true altitudes of the following bodies and verify your results by the total correction method.

(1) Star Spica; Sext.Alt. 54°27.4'; IE 1.4' on the arc HE 15.5m

(2) Venus; Sext.Alt.40°16.1',IE nil HE 11.6m on the arc, on 2nd July, 1976.

(3) Sun's LL; Sext.Alt. 27°03.2', IE 1.4' off the arc, HE 10.1m on 13th October, 1976.

(4) Moon's UL; Observed alt. 31°12.0', IE 2' on the arc, HE 13.2m at 1421 GMT on 13th October, 1976.

Examples

2. Find the true altitude of the Sun at visible sunrise on 14th October, 1976. HE 16m.

Note : At visible sunrise, the observed altitude of the Sun's UL is 00°00'

Observed alt.	00° 00.0'
Dip	- 7.0'
Apparent alt.	- 00° 07.0'
Refraction	- 00° 34.5'
	- 00° 41.5'
SD	- 16.1'
	- 00° 57.6'
Parallax	+ 0.2'
True altitude	- 00° 57.4'

3. Find the true altitude of the Moon when its LL just touches the visible horizon. HE 17m, on 14th October, 1976 at 02.30 GMT.

Observed alt.of Moon's LL	00° 00.0'
Dip	- 7.3'
Apparent alt.	- 00° 07.3'
Refraction	- 00° 34.5'
	- 00° 41.8'
Aug SD	+ 15.0'
	- 00° 26.8'
Parallax(HP)	+ 54.8'
True altitude	00° 28.0'

4. What should be the observed altitude of the Sun's LL at the time it is observed for amplitude.

Note : When observed for amplitude, its true altitude should be nil.

True alt.of Sun	00^0 00.0'
Parallax	- 0.2'

Apparent alt.	- 00^0 00.2'
Mean SD (LL)	- 16.0'

	- 00^0 16.2'
Refraction	+ 00^0 34.5'

Obs. alt.of Sun's LL	00^0 18.3'

Since HE is not given, dip is not applied. When observing the Sun's amplitude from sea level, its LL should appear 18.3' or slightly more than its SD above the visible horizon.

5. Compute the altitude to be set on the sextant for an observation of the Moon's lower limb at 1200 hrs. GMT on 13th Oct. 1976, when the Moon's true altitude was calculated to be 41°21.8'. HE 6.5m IE 2.2' on the arc.

True altitude	41^0 21.8'
Parallax in alt.	- 40.9'

	40° 40.9'
Augmented SD	- 15.1'

	40° 25.8'
Refraction	+ 1.1'

	40° 26.9'
Dip	+ 4.5'

	40° 31.4'
IE	+ 2.2'

Sext.altitude	40° 33.6'
	-----------

6. Sextant alt. of Sun's UL by back angle was 116°52.5', IE 2.5' off the arc. HE 6.2m, SD 16.2'. Find the true altitude of the Sun.

Sext. alt.		116°52.5'
IE	+	2.5'

Observed alt.		116°55.0'
Dip	-	4.4'

		116°50.6'
SD	-	16.2'

		116°34.4'
Subtract from 180°		180°00.0'

		63° 25.6'
Parallax in alt	+	0.1'

Refraction		63° 25.7'
	-	0.5'

True altitude		63° 25.2'
		-----------

7. Sext alt of Star Canopus by back angle was 120°5.8 IE 1.6' on the arc. HE 10.8m. Find the true altitude.

Sext altitude		120° 5.8'
IE	-	1.6'

Obs. altitude		120° 4.2'
Dip	-	5.8'

		119°58.4'
		180°00.0'

		60° 01.6'
Refraction	-	0.6'

True altitude		60° 01.0'
		-----------

8. Sextant altitude of Mars by back angle was 110°1.5'. IE nil. HE 14m. Date 14th October, 1976. Find the true altitude.

Sext.altitude		110° 1.5'
IE		nil

Obs.altitude		110° 1.5'
Dip	-	6.6'

		109°54.9'
		180°

		70° 05.1'
Parallax & Phase	+	0.1'

		70° 05.2'
Refraction	-	0.4'

True altitude		70° 04.8'
		-----------

9. On 14th October, 1976 (1330 GMT) sextant altitude of Moon's UL by back angle was 118°30.2' HE 11m IE 1.2' on the arc. Find the true altitude.

SD = 15.0	Sext. altitude		118°30.2'
Aug.+ 0.2'	IE	-	1.2'
Aug.SD 15.2'			-----------
			118°29.0'
	Dip	-	5.8'

			118°23.2'
	Aug.SD	-	15.2'

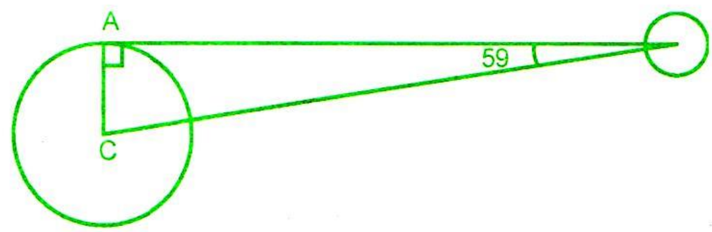
			118°08.0'
			180°

			61° 52.0'
	Parallax in alt. = HP x cos 61°52'	+	25.9'

			62° 17.9'
	Refraction	-	0.5'

	True altitude		62° 17.4'
			-----------

10. Calculate the distance from the centre of the Earth to the centre of the Moon when the Moon's HP is 59' assuming the radius of the Earth to be 3990 miles.



(FIG.9.8)

Distance = radius x cosec 0°59'
= 3990 x cosec 0°59'
= 232, 494.8 miles

Further problems for practice on this topic may be obtained from any practical navigation book.

Theory Questions

1. State the optical principles of the sextant and show how the sextant measures double the angle through which the Index bar is moved.

2. How are the following sextant errors caused? How would you find and correct them ?

   (i) Error of perpendicularity
   (ii) Side error
   (iii) Index error

3. What are the non adjustable errors in a sextant and how are they caused ?

4. Describe the method of determining the Index error of the sextant by means of the Sun. If, when doing so, the two readings obtained were 35.7' on the arc, and 28.3' off the arc, what is the Index error of the sextant and what is the Sun's semidiameter ? (3.7' on the arc; SD 16')

5. Give reasons why the second and third adjustments of a sextant are better made using a star rather than the horizon.

6. What corrections are necessary to a horizontal sext. angle ? Why is refraction correction not necessary in obtaining the true horizontal angle ?

7. Define and illustrate, visible horizon, sensible horizon, rational horizon, observed altitude, apparent altitude and true altitude.

8. What is dip ? Why is dip correction necessary and on what does the amount of the correction depend ?

9. Explain with the aid of a sketch, why refraction correction is to be applied to observed altitudes of heavenly bodies.

10. Define Semi-diameter. Is this correction necessary for all bodies. If not, why?

11. What do you understand by 'augmentation of the Moon's S.D.'? Why is augmentation correction not necessary in the case of the Sun ?

12. Define Parallax in altitude and Horizontal Parallax. With the aid of a figure, show why this correction is always additive.

13. Prove that Parallax in altitude = Horizontal Parallax x cos app altitude.

14. List the corrections to be applied to a sextant altitude of
    (a) stars
    (b) planets
    (c) Sun
    (d) Moon

    Also state where each of these corrections are available.

15. When observing the Sun for an amplitude, what should be the observed altitude of the Sun's lower limb ? Explain your answer.

10

NAUTICAL ALMANAC

The nautical almanac is a compilation of astronomical data for an entire year. It provides the various information required for astronomical calculations on ships. The contents of the almanac are listed below, in the order in which they appear, together with brief notes on their layout and use. When studying the layout and contents of the almanac, it is essential to keep an almanac handy and to refer to it regarding each item mentioned below :

(i) The inside of the front cover page contains tables for correction of altitudes of the Sun, stars and planets. The facing page provides similar tables for low altitudes observations.

(ii) The next page contains additional 'refraction corrections' for non-standard temperatures and pressures.

(iii) This is followed by the list of contents of the nautical almanac, a calendar of the phases of the Moon, the calendar for the year, and notes and maps giving information on eclipses occurring in that year.

(iv) Provided thereafter, are the planet notes, and the planet diagram for the year showing the LMT of meridian passage of the Sun, and the five planets, Mercury, Venus, Mars, Jupiter and Saturn. This diagram indicates the period when each planet is too close to the Sun for observation and when the planets are visible. It also indicates whether they are available for morning or evening sights. It further gives an indication of the position of the planets at twilight.

(v) The above information is followed by the 'ephemeris' for the entire year,tabulated against Greenwich mean times and dates. Each pair of facing pages provides information for three days regarding the following :

Aries — GHA of Aries is given for each hour, and the GMT of its Greenwich meridian passage time for the middle day. The GMT of Greenwich meridian passage for the preceding and succeeding dates can be obtained by adding or subtracting respectively, 23h 56m 04s.

Planets

The GHA and declination of the planets Venus, Mars, Jupiter and Saturn are given for each hour. Also listed are their magnitudes as well as their 'v' and 'd' applicable on all the three days. Their SHA's at 0000 hrs. GMT on the middle date, and the GMT of their Greenwich meridian passage on that date are also given immediately below the star tables.

Stars

The SHA's and declinations of 57 selected stars are provided . They are valid for all the three days.

Sun

The GHA and declination are provided for each hour. The SD for the middle day and 'd' applicable on all the three days are also listed. To the right of the page, at the bottom, the "equation of time" is tabulated for 00h and 12h GMT on each of the three days. Next to it, is the GMT of Greenwich meridian passage of the Sun on each of the three days. This time may also be taken as LMT meridian passage of the Sun over any longitude as the rate of increase of the Sun's GHA is almost exactly 15° per hour.

Moon

GHA, declination, 'v', 'd' and 'horizontal parallax' values are provided for each hour. The Moon's SD is given for each of the three days. Also listed at the bottom right of the page are the GMT of upper and lower meridian passages of the Moon over Greenwich meridian on each of the three days; the age of Moon and its phase.

At this point, we will diverge a little, for an explanation on 'v' and 'd'. The increment tables provided at the end of the almanac are based on the assumption that the hourly increase in the GHA of Sun and planets is 15°00', that of Aries is 15°02.46' and that of the Moon 14°19'. The values of 'v' tabulated in the daily pages of the almanac are the actual hourly increase in the GHA of these bodies in excess of the assumed values stated above. 'v' is generally positive, except sometimes in the case of Venus, when its hourly increase in GHA is less than 15°. At such times, 'v' for Venus is tabulated with a negative prefix. This happens in the case of Venus alone, due to its proximity to the Earth causing the apparent direct motion of Venus to be more rapid than those of the other planets. Though the Moon is closer than Venus, its 'v' is never negative, because the assumed value of 14°19', is lesser than the least actual hourly increase in the Moon's GHA.

'v' is not tabulated for Aries, as its actual hourly increase in GHA never differs from the value of 15°02.46' used for its increment tables. 'v' is not tabulated for the Sun either because its rate of increase of GHA per hour is always very nearly equal to the assumed value of 15°. Any small difference is made up in the next tabulated hourly value of the Sun's GHA. 'v' is tabulated once on each page for each of the four planets. It is applicable for all the three dates on the page. For the Moon, it is tabulated hourly, as its rate of change of GHA varies from hour to hour.

'd' is the hourly change in the declination of the various bodies. Whether it is an increase or a decrease can be found by inspection of the almanac around that time. 'd' is not tabulated for Aries as it is always on the Equinoctial, with a constant nil declination. For the Sun and planets, the 'd' listed is the mean value of their hourly change of declination for the three days on the page. For the Moon it is tabulated hourly due to the rapid change in its rate of change of declination.

The actual 'v' or 'd' correction for any duration of minutes and seconds of time is obtained from the increment and correction tables, towards the end of the almanac.

Returning to the ephemeris tables of the almanac, on the right side of the page, are listed the sunrise and sunset times, the times of beginning of nautical and civil twilights in the morning and those of the end of civil and nautical twilights in the evening,for the middle day. Moon-rise and moon set times are given for four days. Each of the above is given for a range of latitudes from 72°N to 60°S. All times given are the GMT of the phenomenon over Greenwich meridian.

The Greenwich mean times of the solar phenomena may be used as the LMT of the phenomena in any longitude without appreciable error. Interpolation is however necessary for latitude and for the required date.

To obtain LMT of moonrise or moonset, interpolation is required for latitude and for longitude, between the dates concerned.

In these tables, there are three symbols used. The white box indicates that the Sun or Moon remains continuously above the horizon, the black box indicates that they do not rise, and the strokes indicate that twilight lasts all night.

(vi) After the ephemeris, explanations are provided giving the principle and arrangement of the nautical almanac together with examples to show the correct use of the information provided in it. The explanation also gives the procedure for using the current nautical almanac in the following year.

(vii) The tables of standard time gives the time difference between GMT and the standard time of the different areas of the world.

(viii) Star charts are provided separately showing the northern stars, southern stars and equatorial stars. These charts help in the identification of the important stars.

(ix) The table of 173 stars gives their magnitude, their constellation names (on the left hand page), proper names (on the right hand page),and their SHA's and declinations, for each month. The stars are listed in ascending order of SHA. Though the 57 selected stars are also included in this table, their SHA's, and declinations, are obtained more accurately from the daily pages.

(x) The 'Polaris' tables provide the corrections 'a_0', 'a_1' and 'a_2' to be applied to the true altitude of Polaris, to obtain the latitude. It also gives a table for obtaining the azimuth of Polaris. Explanations regarding the use of these tables are also provided, together with an example.

(xi) The table for conversion of arc to time is useful for converting arc to its equivalent in time and vice versa.

(xii) Using the increment and correction tables, we can obtain the GHAs of the Sun, planets, Aries and Moon as well as the declinations of Sun, planets and the Moon accurately, for any second of time during the entire year.

(xiii) The table for interpolation is divided into two. Table I is used for interpolating, LMT of sunrise, sunset, twilight, moonrise, moonset and Moon's meridian passage for the required latitude. Table II is for interpolating the times of the above phenomena for longitude.

(xiv) The index to selected stars gives the number, magnitude, SHA and declination (to the nearest degree) of the 57 selected stars, both in alphabetical and numerical order. The same information is also provided on the book mark.

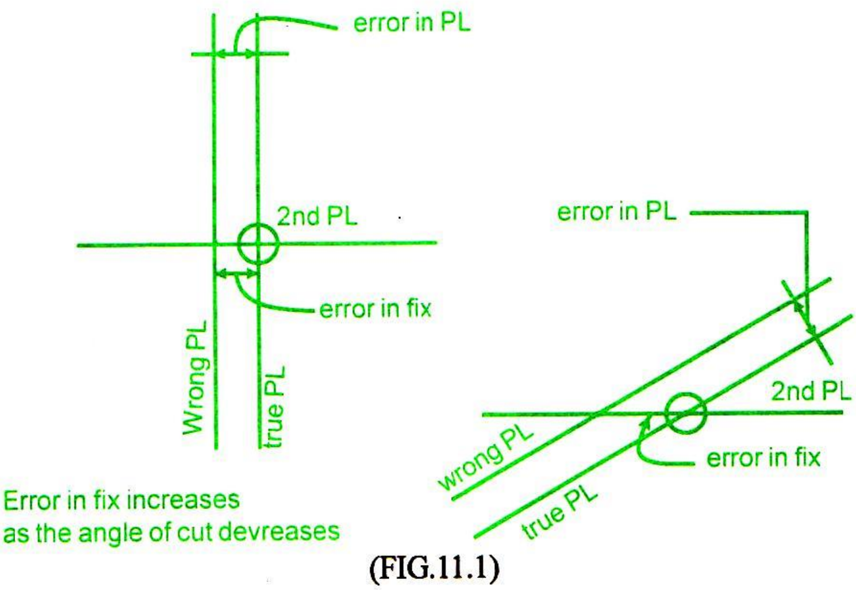
(xv) Altitude correction tables for the Moon contain the corrections to be applied to observed altitudes of Moon's lower or upper limbs. The use of this table has already been explained in the chapter on 'Altitudes'.

11

POSITION LINES

11.1 TERRESTRIAL POSITION LINES

A line, somewhere on which, the ship must be situated is called a position line. The true bearing of a terrestrial object provides a position line, drawn from the object in a direction opposite the bearing. The ship must be somewhere on this line, as no other line on which the object will have that bearing can be drawn from that object. It is not possible to obtain the ship's position from a single position line. To fix the ship's position, at least two position lines are necessary. The intersection of the two position lines gives the position of the ship. The accuracy of the position, so obtained increases, as the angle of cut between the two position lines approaches 90°, since the error produced in the position so obtained due to the displacement of one of the position lines, is least when the angle of cut is 90°.



(FIG.11.1)

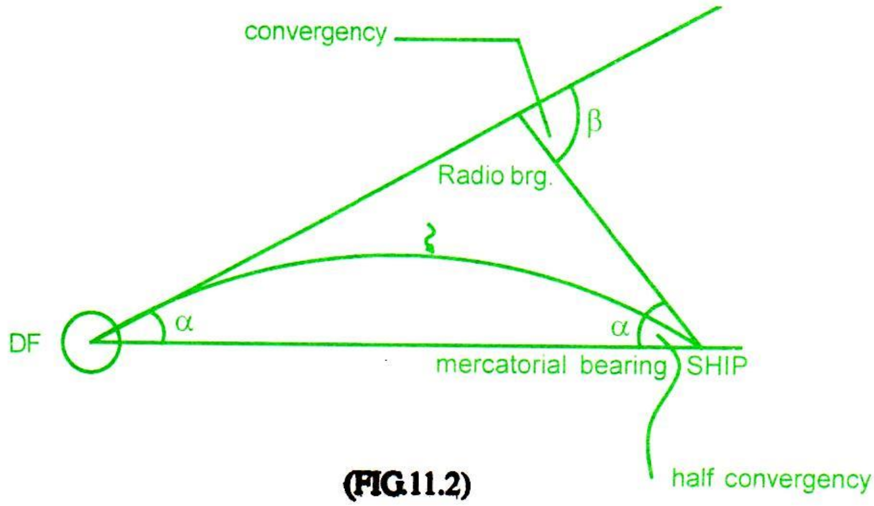
If the two conspicuous objects are observed in transit an accurate position line is obtained, as the ship would then be on the line joining the objects, produced. The direction of this line is not affected by any error on the compass.

11.2 POSITION CIRCLES

From the vertical sextant angle of an object of known height, it is possible to calculate the distance off from that object. A circle could then be drawn with the object as centre and radius equal to the distance off. The position of the ship must obviously be somewhere on the circumference of that circle. Such a circle is known as a position circle. A position circle can also be obtained from an observation of the horizontal angle between two terrestrial objects, as the ship must lie somewhere on the arc of the circle passing through the objects and containing the measured angle.

A radar bearing gives a position line while a radar range gives a position circle.

DF bearings are great circle bearings, as radio signals travel along great circles. Prior to plotting them on a Mercator Chart, they should be converted to mercatorial bearings by applying the half convergency.



(FIG.11.2)

The difference between the initial direction of the radio signal and the final direction in which it reaches the ship is equal to the angle between the tangents (at the ship and at the radio station) to the great circle between the ship and the station. This angle is known as the convergency of the great circle between the DF station and the ship. It can be seen from the figure, that the angle at the ship between the great circle, DF bearing and the Mercatorial bearing is half the convergency.
Half convergency is obtained by the formula;

Half convergency = ½ d'long x sin middle lat.

It may also be obtained from the nautical tables, using the arguments d'long and mid lat.

This correction is to be always applied towards the Equator is towards South in North hemisphere and towards North in South hemisphere. This is so because, the mercatorial bearing will always lie towards the equatorial side of the great circle bearing, since great circles always curve towards the pole of the hemisphere.

Examples

1. A ship in DR position 32°12'S, 170°14'E, obtains the DF bearing of a station in 34°05'S, 172°49'E, as 131°. Find the half convergency correction and thence the mercatorial bearing.

Half convergency	=	½ d'long x sin mean lat
	=	½ x 155' x sin 33°08.5' = 42'
	=	- 0.7°
GC bearing	=	131.0°
Mercatorial bearing	=	130.3°(T)

Since the positions are in Southern hemisphere, correction is applied towards the Equator i.e. northwards)

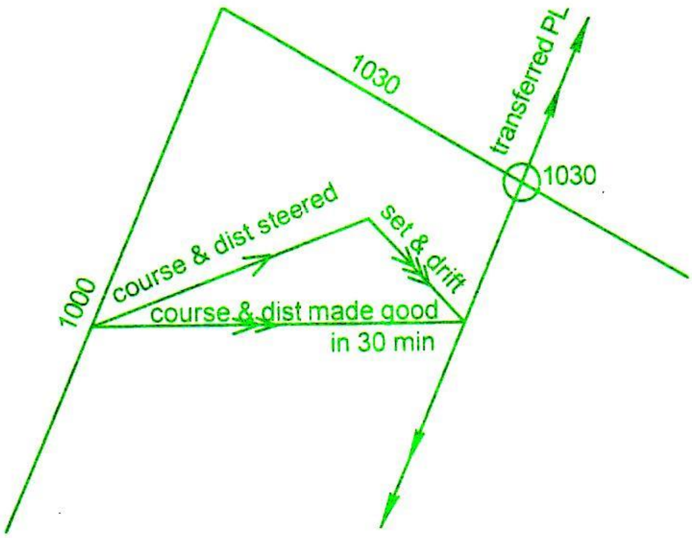
2. Ship and DF station are in North latitude. If DF bearing is 240° and ½ convergency correction 1.2°, find the mercatorial bearing.

GC bearing	=	240.0°
½ convergency corr'n	=	- 1.2° (southwards, hence negative)
Mercatorial bearing	=	238.8°

Half convergency is the product of ½ d'long between ship and station, and the sine of the mean latitude. Therefore, if the d'long or mean latitude is nil or very small, the ½ convergency correction will also be nil or negligible. Thus, when the DF bearing is near 0° or 180°, since the d'long will be negligible, the correction will also be negligible. Similarly, when the ship and station are close to the Equator or lie on either side of the Equator, the mean latitude and therefore its sine will be negligible and so also the correction.

11.3 TRANSFERRED PL

Provided the course and distance made good since obtaining a PL are accurately known, that PL may be transferred through the course and distance made good and the ship will then be somewhere on the transferred PL. If any set and drift or leeway is experienced during that interval, they should also be allowed for as part of the run. Using this principle, it is possible to obtain the position of the vessel from bearings of a single object obtained at two different times, by transferring the first PL through the course and distance made good between the bearings.The fix so obtained is popularly known as a running fix.



(FIG.11.3)

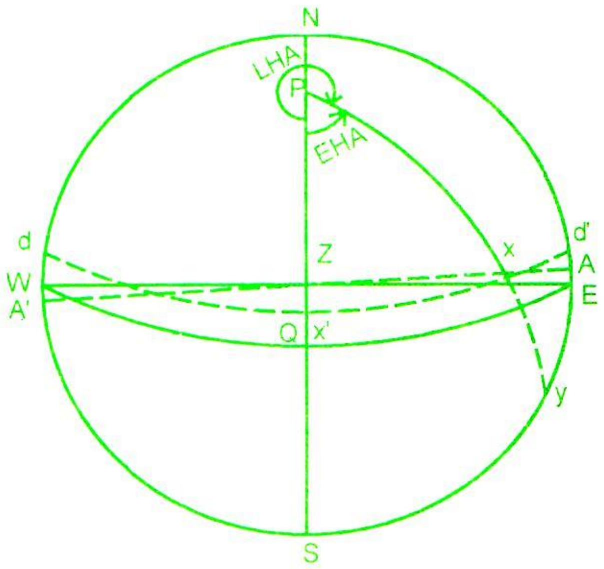
Position circles may also be transferred, using the same principle by transferring the centre of the position circle through the ship's run.

Problems on position fixing by crossed bearings, horizontal or vertical sextant angles, running fix, DF bearings etc. are available in any standard book on chart-work. Similarly problems on position fixing by doubling the angle on the bow, four point and beam bearings, 'special angles' etc. are also available in such text books.

11.4 POSITION LINES FROM CELESTIAL OBSERVATIONS

Figure drawing for astronomical calculations

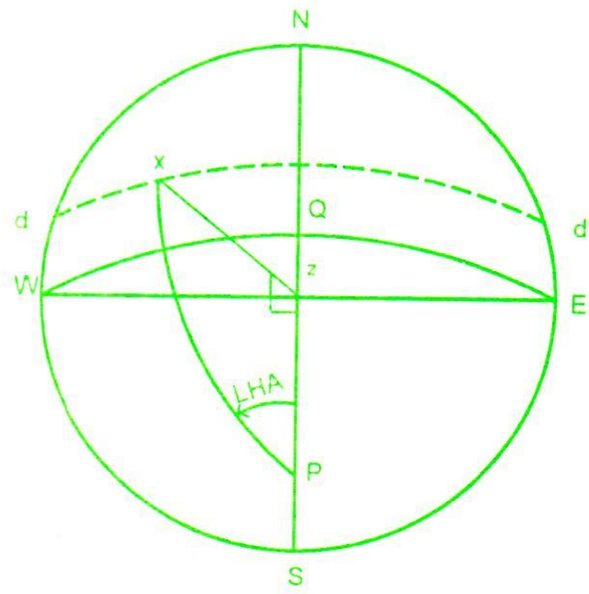
Figures drawn on the plane of the observer's rational horizon help in understanding astronomical calculations.



(FIG.11.4)

In Fig. 11.4 NESW, the outer circle represents the observer's rational horizon. Z is the observer's zenith, PZS, the observer's celestial meridian and WZE, the observer's prime vertical. WQE represents the Equinoctial, dd' the declination circle of the body and P the elevated celestial pole.

To simplify measurements, it would be convenient to draw the circle using a radius of 9 units to represent 90°, between Z and the rational horizon. ZQ is measured equal to the latitude and Q is marked to the South or North of the zenith, according to the observer's latitude being North or South respectively. In the above figure QZ represents the observer's North latitude. NP, the altitude of the elevated pole (North celestial pole in this case) is equal to ZQ, the latitude of the observer. QX' represents the North declination of the body X' which is on the observer's meridian. If the declination was South, QX' would have been measured Southward from Q. X represents the same body before reaching the meridian. PX represents the distance of the body from the pole (90° - dec) normally referred to as the polar distance of the body. PZ equals to (90°-lat), is referred to as the co-latitude. A'ZA represents the vertical circle through the body. PXY represents the celestial meridian through the body. Most astronomical calculations involve the solution of the spherical triangle PZX. In the figure, AX represents the true altitude of the body, ZX equal to (90°-true altitude), is the zenith distance of the body. PZ is the co-latitude and PX is the polar distance of the body. Angle Z is the azimuth of the body, the minor angle P represents the easterly hour angle, and the major angle P, the local hour angle of the body.

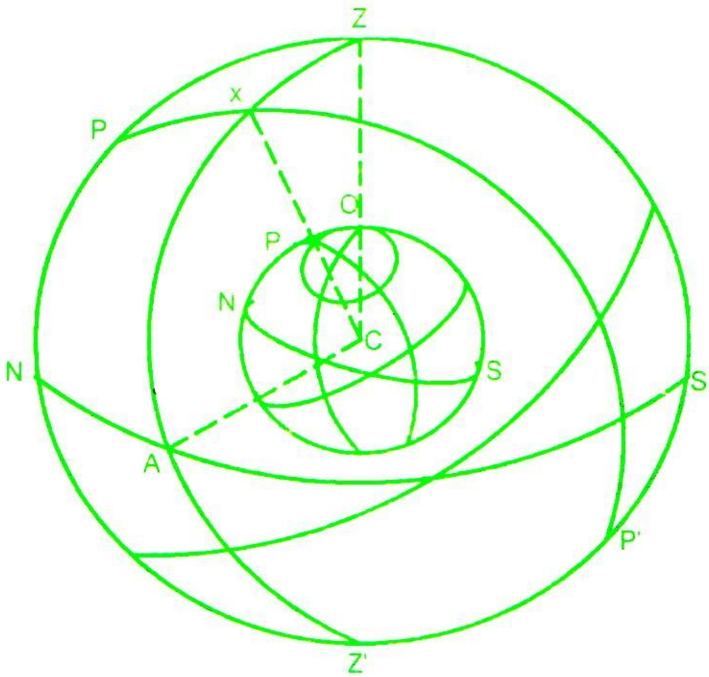


(FIG.11.5)

Fig. 11.5 shows the various elements for an observer in South latitude, the body's declination being North. It should be noted that LHA, which is a westward measurement, is measured counter-clock-wise from the observer's meridian, as the measurement is being made around the South celestial pole.

11.5 ASTRONOMICAL POSITION LINES

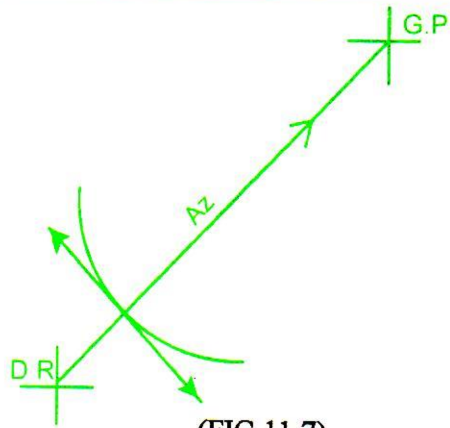
As stated earlier, the GHA and the declination of any heavenly body can be obtained from the nautical almanac for any second of time. It has also been explained that the geographical position of the heavenly body is the point on the Earth vertically below that body. The body's declination and its GHA corresponds to the latitude and longitude respectively of its geographical position. Further by subtracting the true altitude of the body from 90°, we obtain the body's zenith distance, which is the arc of a vertical circle or the angle at the centre of the Earth between the body and the observer's zenith. As shown in fig. 11.6,this angle is equal to the angle subtended at the Earth's centre by the geographical position of the body and the observer's position on the Earth.



(FIG.11.6)

Therefore, the zenith distance of a body expressed in minutes of arc is the distance in miles on the Earth, between the observer and the body's GP. Thus, if a circle is drawn on the Earth's surface with the body's GP as the centre, and zenith distance in minutes of arc as the radius in miles, we would obtain a circle of position on which the observer must be situated. The position circle so obtained is also known as a 'circle of equal altitude'. The intersection of two or more such position circles determines the position of a ship.

Unless the zenith distance is very small,plotting the GP of the body and thence the entire position circle, on the chart in use, is not practicable. The radius of the position circle in miles (equal to the zenith distance in minutes) would normally be hundreds of miles. Further, such a circle will not appear as a circle on a mercator chart. If the zenith distance is very small, a position circle can infact be plotted on the chart without appreciable loss of accuracy particularly in low latitudes. When the zenith distance is large, we are not interested in the entire position circle. Since the DR of the vessel is known, we are interested in only a small arc of the position circle near the ship's DR. This small arc of the very large position circle approximates to a straight line. We also know from geometry, that the radius of a circle meets the circumference at 90°. From any point on the circumference of the position circle, the radius represents the direction to the body's GP; that is the body's azimuth.



(FIG.11.7)

Thus from an observation of the altitude of a celestial body, we can obtain a position line, drawn as a straight line at right angles to the azimuth. It should be noted that position lines obtained by bearings of terrestrial objects are laid off in the direction of the bearing, while those obtained from the observation of celestial bodies are laid off as straight lines, perpendicular to the azimuth of the body.

From what has been stated above, it may appear that the intersection of the line of azimuth of a celestial body and the position circle obtained from the body's zenith distance would fix the ship's position. This is not possible because, the accuracy of such a fix depends on the accuracy with which the azimuth is calculated and laid off. A small inaccuracy in the calculated azimuth would put the ship miles away from the actual position, because the distance between the ship and the body's GP may be hundreds of miles.

To plot the part of the position circle we are interested in, we require to know :

(i) the position through which to draw it and
(ii) the direction in which to draw it.

With respect to the latter, it has already been explained that, position lines obtained from astronomical observations will lie at right angles to the azimuth of the body. Thus once the azimuth of the body is calculated, the direction of the position line is easily obtained. It should be noted that when the body bears exactly North or South, as in the case of a latitude by meridian altitude calculation, the position line will run exactly East - West, coinciding with the latitude of the ship. When the body bears exactly East or West, the PL will run North - South, coinciding with the longitude of the ship.

After having obtained the altitude of a celestial body at a known instant of time, various methods are used to obtain the direction of the PL and the position through which to draw it.

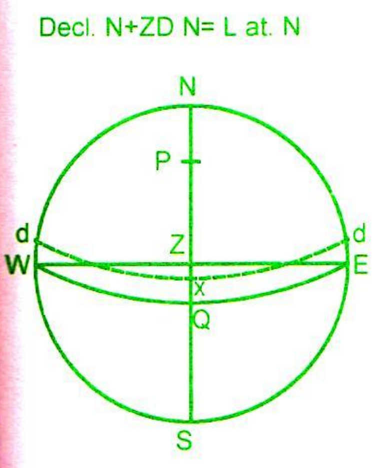
11.6 LATITUDE BY MERIDIAN ALTITUDE

A simple yet important method of obtaining the direction of the position line and the position through which to draw it, is by measuring the altitude of a celestial body when it is on the observer's meridian. For a stationary observer, the meridian altitude of a body will be the maximum altitude attained by the body. Since the body is on the observer's meridian then, its bearing will be exactly North or South and hence the PL will run exactly East-West. A position line running East-West, coincides with the latitude of that place. We can therefore calculate the observer's latitude from the altitude of celestial body when on the observer's celestial meridian.

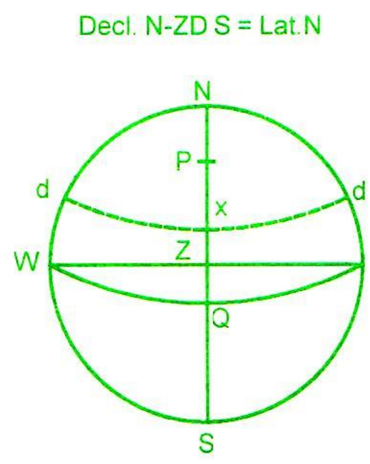
In brief this method of finding the latitude involves the following :

1. Using the DR longitude, find the GMT of meridian passage of the body, at the observer.
2. Convert the GMT to ship's time and observe the meridian altitude of the body then.
3. Correct the altitude and name it North or South according to the bearing of the body when on the meridian.
4. Subtract the true altitude from 90° to obtain the meridian zenith distance and name it opposite to the bearing.
5. From the almanac, obtain the body's declination for that GMT.
6. Apply the declination to the meridian zenith distance, using the rule "same names - 'ADD', different names - SUBTRACT" and name the latitude so obtained according to the greater of the two.

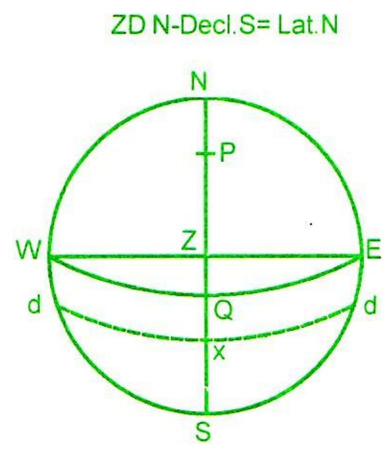
   The PL obtained will be East-West



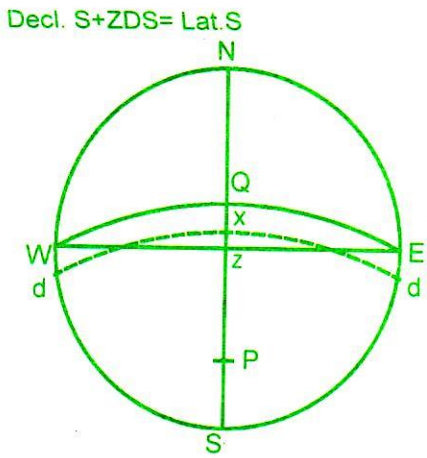
(FIG.11.8)



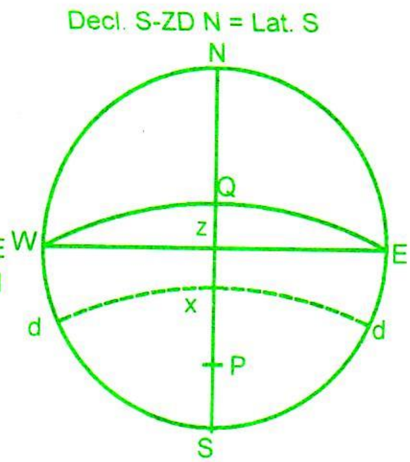
(FIG.11.9)



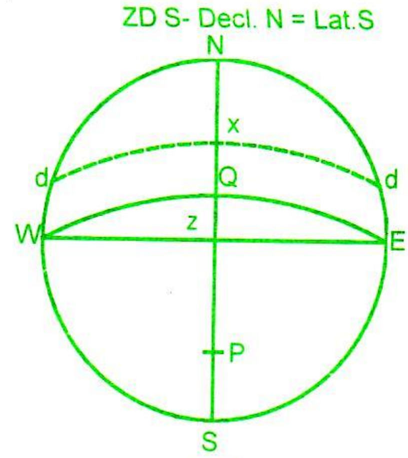
(FIG.11.10)



(FIG.11.11)



(FIG.11.12)



(FIG.11.13)

The above figures show six possible cases of meridian altitude problems, three for observers in North latitude and three for observers in South latitude. The reader should again refer to item '6' in the method indicated for solution of latitude by meridian altitude problems and understand that in each case, Lat = MZD ± Declination. He should also note, how the latitude obtained is named N or S in each case.

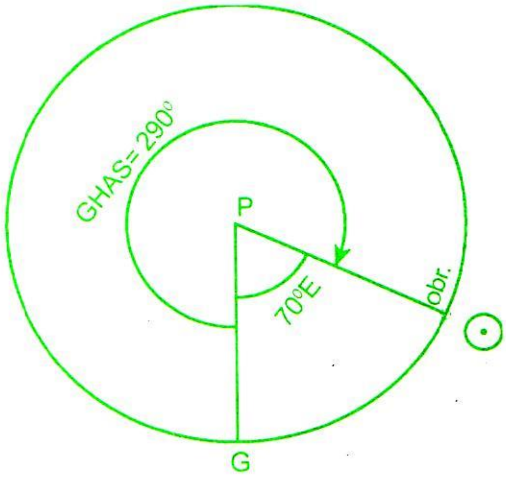
The reader will already be familiar with latitude by meridian altitude problems in his practical navigation work. Solution of such problems is therefore not included in this book. However accurate calculation of the time of meridian passage of various bodies and the principles used in obtaining the latitude by a meridian altitude observation are shown in the following pages.

To find the time of meridian passage of various heavenly bodies :

SUN

Examples

1. Find the LMT of meridian passage of the Sun, in longitude 70°E on 13th October, 1976.



(FIG.11.14)

LHA Sun = GHA Sun + E (-W) longitude

Since the Sun is on the observer's meridian LHA = 360°
Therefore 360° = GHA Sun + 70°
Therefore GHA Sun = 360° - 70° = 290°

To find the GMT, when GHA Sun is on 13th October:-		290° 00.0
13th October GHA at 07h GMT	=	288° 26.4'
		1° 33.6'

From the increment table for Sun, against the value of 1°33.6', we read 6m 14s

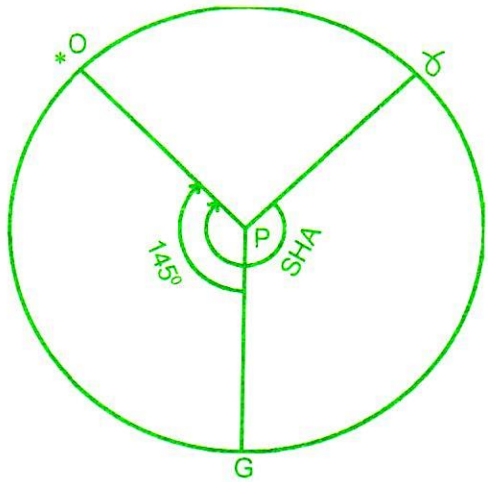
GMT meridian passage of Sun over 70°E	=	13d 07h 06m 14s
Longitude In Time (LIT) EAST	=	04h 40m 00s
Therefore LMT Meridian Passage	=	13d 11h 46m 14s

Note

As stated earlier, the GMT meridian passage of the Sun over Greenwich, tabulated at the foot of the daily pages gives the LMT meridian passage of the Sun over any longitude, to the nearest minute. For the 13th of October, the tabulated time is 11h 46m. For solving latitude by meridian altitude sight, accuracy to the nearest minute would suffice as the correct time is required only for obtaining the Sun's declination which would hardly change in a few seconds.

STAR

2. Find the LMT of meridian passage of star Betelgeuse, SHA = 271°31', in longitude 145° West on 14th October, 1976.



(FIG.11.15)

Since the star is on the meridian, LHA * = 360°

LHA *	=	GHA γ + SHA * + E (-W) longitude	
360°	=	GHA γ + 271°31' - 145°	
GHA γ	=	233°	29'.0
GHA γ at 1400 hrs. on 14th October =		233°	14.4'

		00°	14.6'

From the increment table for Aries, against the value of 00°14.6', we read 00m 58s

GMT meridian passage of * over 145°W	= 14d 14h 00m 58s
Longitude In Time (LIT) WEST	= -09h 40m 00s
Therefore LMT Meridian Passage	= 14d 04h 20m 58s

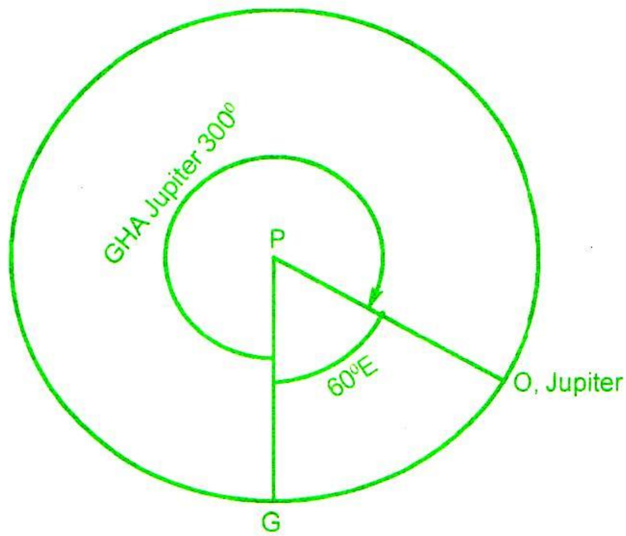
Note

A very approximate LMT of a star's meridian passage can be obtained as LMT meridian passage of Aries + Right Ascension of star.

PLANET

3. Find the LMT meridian passage of Jupiter in longitude 60°E, on 13th October, 1976.

Since Jupiter is on the meridian, its LHA is 360°.
LHA Jupiter = GHA Jupiter + E (-W) Longitude
360° = GHA Jupiter + 60°
∴ GHA Jupiter = 300°



(FIG.11.16)

By inspection of the almanac, for the 13th, we find that the GHA of Jupiter comes to this value between 22 and 23 hours GMT. When the LIT of 4h is added to this is to obtain the LMT, the date becomes 14th. Since the date at the ship, in the question is the 13th, it is necessary to obtain the GMT for the previous day (12th), so that the LMT would fall on the date at ship (13th). Conversely, if when inspecting the almanac, it was found that LMT would fall on the preceding day, we would have to obtain the GMT for the following day, so that the LMT obtained would be on the correct date at ship.

GHA Jupiter	=	300° 00.0'
GHA Jupiter at 2200h on 12th oct.	=	293° 11.6'
		06° 48.4'

(Planets attain this increment in about 27m)

For 'v' of +2.7', 'v' correction for 27m	=	-1.2'
		06° 47.2'

From the increment tables for planets against the value of 6° 47.2' we get 27m 09s

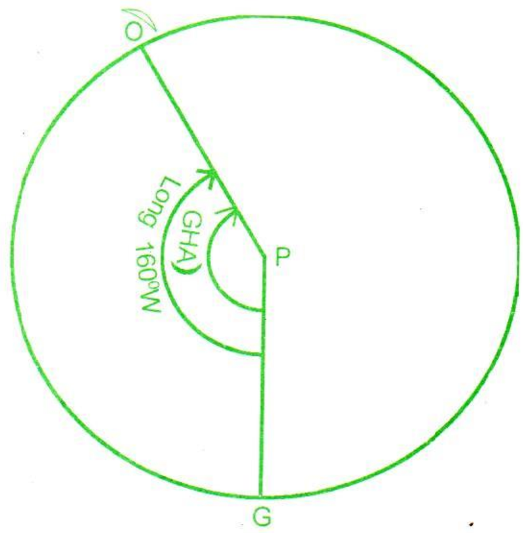
GMT meridian passage of Jupiter	=	12d	22h	27m	09s
LIT 60°E	=		04h	00m	00s
LMT Meridian Passage	=	13d	02h	27m	09s

Note

When finding the GHA for a given time, 'v' correction is applied as per its sign. Since, in the above problem, we are finding the GMT for a given GHA, this correction should be applied in the reverse manner. The exact amount of the 'v' correction should be taken off from the increment table for the minute where the increment in GHA is found by inspection.

MOON

4. Find the GMT of meridian passage of the Moon in longitude 160°W on 13th October, 1976.



(FIG.11.17)

Since the Moon is on the meridian, LHA Moon = 360°.
LHA Moon = GHA Moon + E (-W) Longitude
360° = GHA Moon - 160°
GHA Moon = 520° - 360° = 160°
By inspection of the almanac, for the 13th, we find that the GHA of Moon comes to this value between 14 and 15 hours GMT.

GHA Moon	= 160° 00.0'
On 13th October at 1400 hrs. GHA Moon	= 154° 11.2'
	05° 48.8'
Moon attains this increment in about 24m.	
For 'v' of + 11.7' 'v' correction for 24m =	- 4.8'
	05° 44.0'

From the increment tables for Moon, against the value of 05°44.0', we get 24m 02s.
GMT meridian passage of Moon over 160°W
= 13d 14h 24m 02s

Note

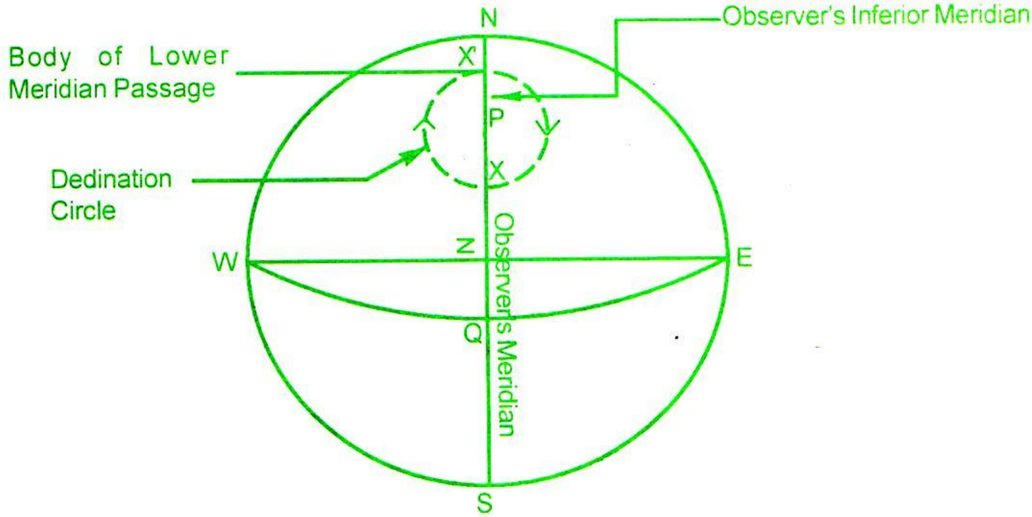
We have found the GMT for the 13th. It is essential to check that the LMT falls on the date in question, at ship. By applying the LIT of 10h 40m (160°W), we find that it does, in this question.

If it did not, we would have to proceed as shown in Example 3

It is suggested that, till one is very sure of one's working the calculated meridian passage time may be checked for correctness by working out the LHA of the body for that time. If correct, the LHA will be exactly 360°.

11.7 LOWER MERIDIAN PASSAGE

Lower meridian passage of a body occurs when it is on the observer's inferior or anti-meridian i.e. when the body is on the celestial meridian 180° away from the observer's celestial meridian. Obviously the LHA of the body then will be 180°. In Fig.11.18 the body is on the observer's inferior meridian at X'. At X, it is on the observer's meridian.



(FIG.11.18)

To find the time of lower meridian passage of a body, the working is similar to finding the time of meridian passage explained earlier, except that the LHA used should be 180° instead of 360°.

If the upper meridian passage time of the Sun is known, the lower meridian passage can be obtained by adding or subtracting 12h, as the rate of increase of Sun's GHA is 15°/hr.

In the case of stars, the lower meridian passage time can be obtained by adding or substracting 11h 58m 02s to its upper meridian passage time, as the star returns to the meridian every 23h 56m 04s.

Lower meridian passage times of planets and the Moon should be worked out independently using an LHA of 180°.

For a body to be visible at its lower meridian passage, (i) its declination should be of the same name as the observer's latitude and (ii) latitude + declination should be equal to or greater than 90°. When the above conditions are satisfied, the body remains above the horizon all the time. It never sets or rises. Such bodies are called **circumpolar bodies,** though theoretically all bodies are circumpolar, as all of them describe apparent paths along their declination circles, with the pole at the centre.

Examples

1. On 14th October, 1976, required the GMT at which Jupiter will be on the observer's meridian below the pole, the observer being in longitude 112°W.

LHA Jupiter	=	GHA Jupiter - W Long
GHA Jupiter	=	LHA Jupiter + W Long
LHA Jupiter	=	180°
Longitude (W)	=	112° 00
GHA Jupiter	=	292°00
On 14th Oct. at 2100hrs GMT, GHA Jupiter	=	280°16.9'
		11° 43.1'
(Planets attain this increment in about 47 minutes)		
For 'v' of + 2.7'	=	-2.1'
		11° 41.0'

From the increment tables for planets against the value of 11°41', we get 46m 44s

∴ GMT Meridian Passage of Jupiter over 112°W

= 14d 21h 46m 44s

2. On 14th October 1976, required the LMT of upper and lower transits of star Schedar for an observer in longitude 82°30'E.

Since the star is on the Meridian, LHA*	=	360°
LHA Schedar	=	360°
Longitude (E)	=	- 82° 30'
GHA Schedar	=	277° 30'
		+360°
		637° 30'
SHA Schedar	=	350° 11.4'
GHA γ	=	287° 18.6'
14d 17h	=	278° 21.2'
35m 41s	=	8° 56.8'

GMT upper meridian passage = 14d 17h 35m 41s
Longitude in Time (LIT) EAST = + 05h 30m 00s

LMT upper meridian passage	=	14d	23h	05m	41s
			- 11h	58m	02s
LMT lower meridian passage	=	14d	11h	07m	39s

11.8 LATITUDE BY LOWER MERIDIAN ALTITUDE

(On the meridian below the pole)

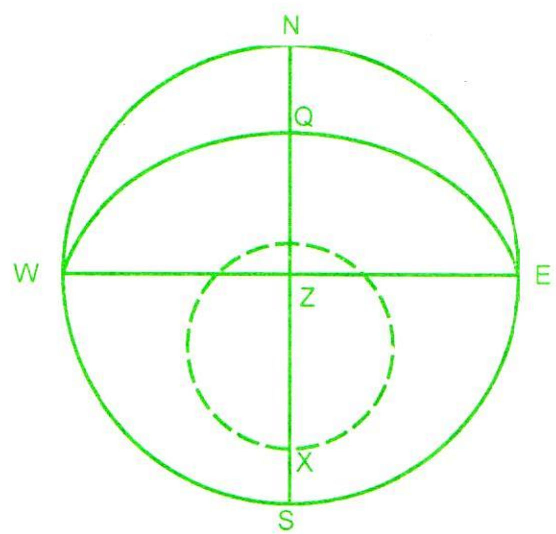
A body at its lower meridian passage is said to be on the meridian below the pole, as its altitude then is less than the altitude of the Celestial pole. A reference to Fig.11.18 will help in understanding this clearly. In the same figure, the latitude of the observer = QZ, and QZ = NP (the altitude of the Pole.)

NP = NX' + X'P. Therefore the observer's latitude is equal to the true altitude of the body at lower meridian passage + the polar distance of the body.

Example

Star Canopus had a true altitude of 17°15', when on the meridian below the Pole. Calculate the observer's latitude. (declination of Canopus = 52°40.8'S)

Polar distance of Canopus (PX)	=	37°19.2'
Lower meridian altitude (SX)	=	17°15.0'
Latitude (SP)	=	54°34.2'S



(FIG.11.19)

Note The latitude is named South, since the body will be visible at lower meridian passage, only if the observer's latitude and the body's declination are of the same name. At lower meridian passage, the bearing of the body will also be N or S according to the body's declination being N or S, respectively.

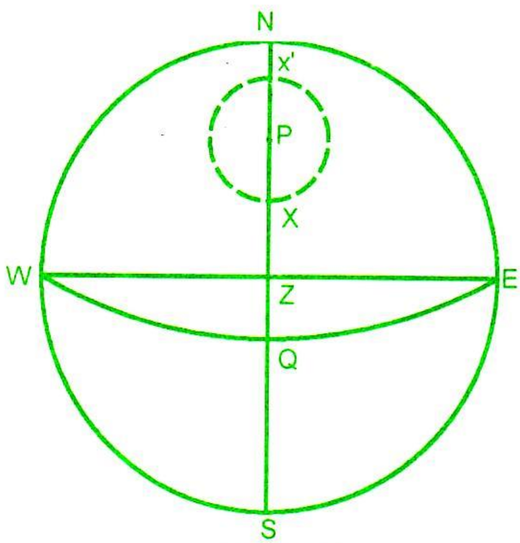
11.9 CIRCUMPOLAR BODIES

If the altitude of a circumpolar body is observed when on the observer's meridian and again when on the observer's inferior meridian, the latitude of the position as well as the body's declination can be calculated.

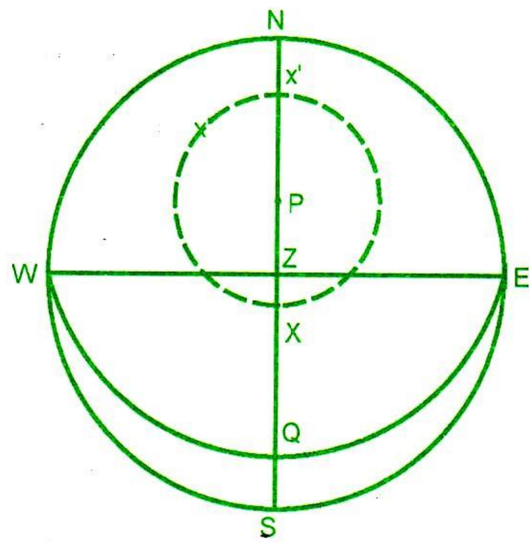
The following figures show two cases of circumpolar bodies for an observer in North latitude. The elevated pole (the pole above the horizon) is the North celestial pole.

Fig.11.20 shows a case where the body bears the same at both upper transit X and lower transit X'. Both X and X' are North from Z.

Fig. 11.21 shows a case, where X', the body at lower transit is North of the observer and X at upper transit is South of the observer.



(FIG.11.20)



(FIG.11.21)

Having observed, the altitude of the body at upper and lower meridian passage, the observer's latitude and the body's declination can be calculated as follows :

1. Draw an approximate figure as shown above, placing the body at upper and lower transit (X and X' respectively) using the bearing (N or S) and the altitude above the horizon on each occasion.

2. Place the elevated pole midway between the two positions and draw in the declination circle of the body, with the pole as the centre and the circle passing through both X and X'.

3. Draw WQE, the Equinoctial so that PQ = 90°.

4. Obtain the diameter of the declination circle as upper meridian altitude - lower meridian altitude [in case (i) where bearings are the same on both occasions] or 180° - (upper meridian altitude + lower meridian altitude), in case (ii) where bearings are different on the two occasions.

5. The diameter, divided by 2, gives the polar distance PX or PX'.

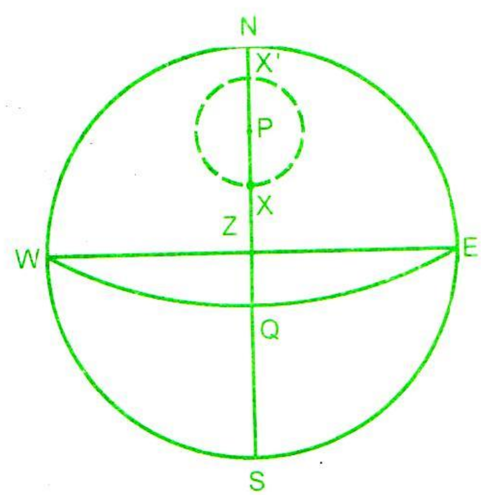
6. 90° - polar distance = the declination, which is named the same as the elevated pole, ie body's bearing at lower transit.

7. Polar distance + the lower meridian altitude = the altitude of the pole = latitude of the observer, which is also named the same as the elevated pole.

Examples

1. A star when on the meridian above the pole, bore North with a true altitude of 70°04', and when on the meridian, below the pole, bore North with true altitude 22°05'. Find the observer's latitude and the star's declination.

Upper meridian altitude NX	=	70°04'
Lower meridian altitude NX'	=	22°05'
XX'	=	47°59'
Polar distance 47°59' / 2	=	23°59.5'
decl. = 90° - polar dist.	=	90° - 23°59.5'
	=	66°00.5'N
Observer's latitude	=	NP = NX' + X'P
	=	22°05' + 23°59.5' = 46°04.5'N



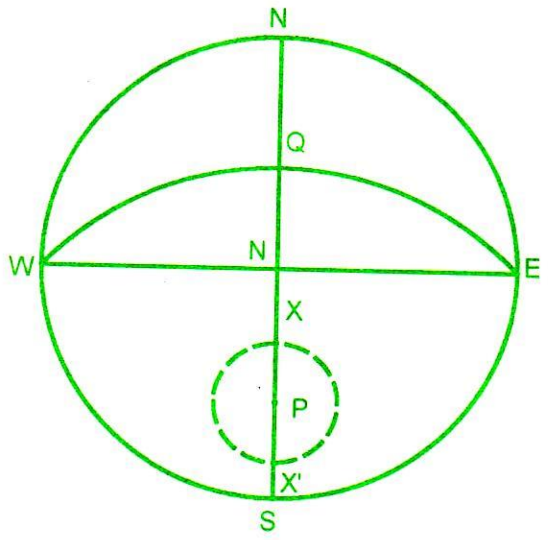
(FIG.11.22)

2. During the same night, a star bore South with true altitude 28°34' and again with a true altitude 76°46'. Calculate the star's declination and the latitude of the observer.

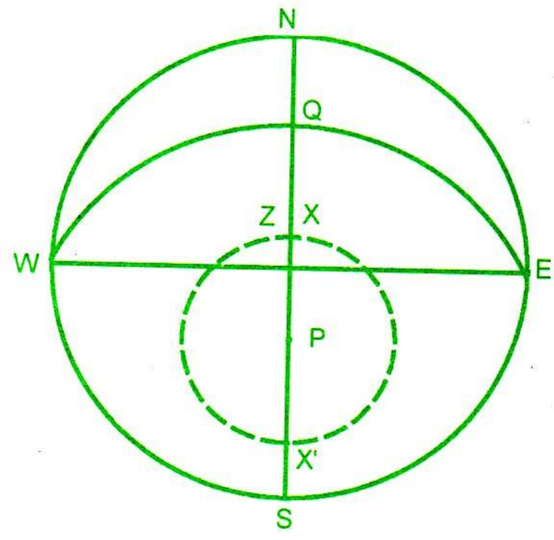
	SX	=	76°46'
	SX'	=	28°34'
	XX'	=	48°12'
Polar dist.	PX	=	48°12'/2 = 24°06'
	Decl.	=	90° - 24°06' = 65°54'S

Lat. = SP = SX' + X'P

= 28°34' + 24°06' = 52°40'S



(FIG.11.23)



(FIG.11.24)

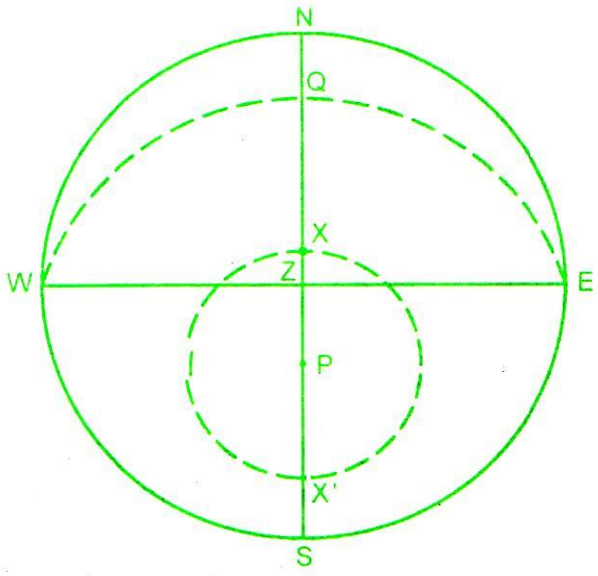
3. To a stationary observer, an unknown star bore 000°(T) with true altitude 78°12'. After about 12 hours, the same star bore 180°(T) with true altitude 18°54'. Calculate the observer's latitude and the declination of the star. (FIG. 11.24)

NX	=	78° 12'
SX'	=	18° 54'
		97° 06'
XX'	=	180°
		-97° 06'
	=	82° 54'
Polar dist.		= 82° 54' / 2 = 41° 27'
Declination of star		= 90° - 41°27' = 48°33'S
Observer's latitude		= SP = SX' + X'P
		= 18° 54' + 41° 27'
		= 60° 21'S

HARDER PROBLEMS

1. A star with declination 52°12' South had a true altitude of 24°15' at lower transit. Find the sextant altitude of the same star at upper transit. I.E. 1.5' off the arc. HE 10m.

Altitude at lower transit	=	SX'	=	24°15'
Polar dist.= 90° - 52°12'	=	X'P	=	37°48'
Again polar dist. =	PX=	37°48'		
	SX=	99°51'		
True alt. at upper transit =180° - 99°51'				
	80° . 09.0' N			
Refraction	+ 0.2'			
Dip	+ 5.6'			
Index error	- 1.5'			
Sext. alt. at upper transit	80° . 13.3'			



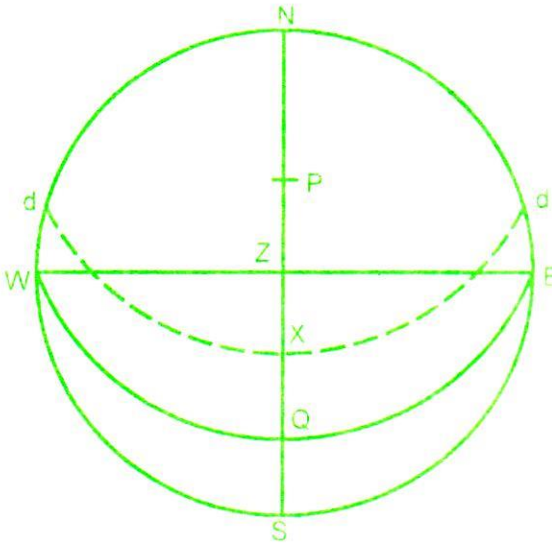
(FIG.11.25)

2. To an observer at the North Pole, the Moon had a true altitude of 20°12'. In what latitudes would the meridian altitude of the Moon be double this.

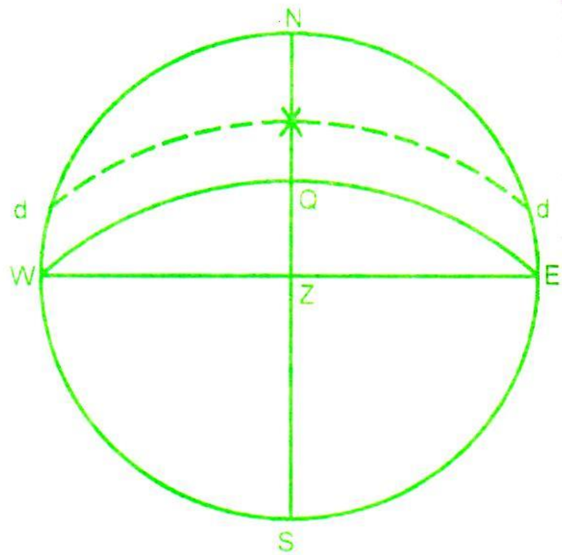
For an observer at the North pole, his zenith is coincident with the North celestial pole and therefore his rational horizon coincides with the Equinoctial. The altitude of the Moon above the rational horizon therefore corresponds to the angular distance of the Moon from the Equinoctial, that is, its declination. Therefore declination of the Moon is 20°12'N. For the Moon to have an altitude of 40°24', in two latitudes, when its declination is 20°12' N, the observer has to be in a North latitude in one case and a South latitude in the other. (Refer figs. 11.26 & 11.27)

(a) Alt. SX = 40°24'
dec. QX = 20°12'N
SQ = 20°12'
QZ = lat. = 90° - SQ = 90° - 20°12' = 69°48'N

(b) Alt = NX = 40°24'
dec QX = 20°12'
NQ = 60°36'
Lat. = QZ = 90° - 60°36' = 29°24'S



(FIG.11.26)



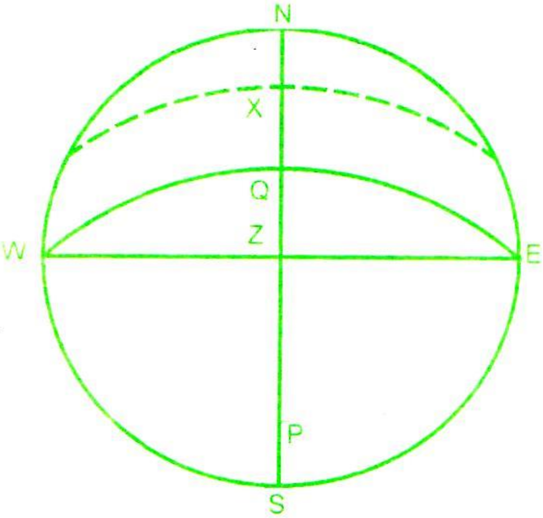
(FIG.11.27)

3. Find two latitudes in which a star having a declination of 68°46'N will bear North with a true altitude of 16°12'.

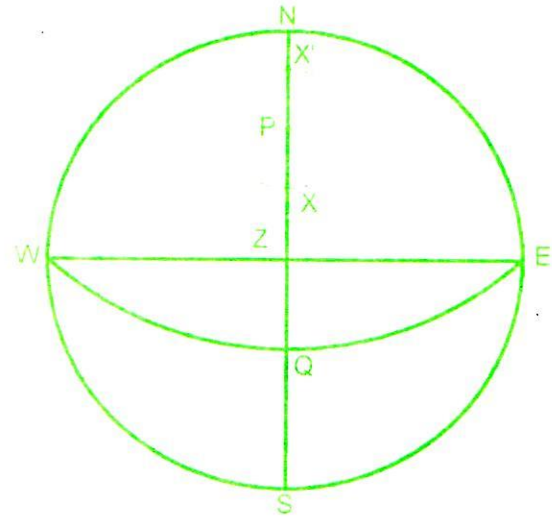
   For a star to bear the same, and have the same altitude when on the meridian, it would have to be above the pole in one case and below the pole in the other. We have to thus find two latitudes where the given conditions are satisfied. (Figs 11.28 & 11.29)

(a) Altitude	=	NX	=		16°12'
dec.	=	QX	=		68°46'N
		NQ	=		84°58'
Lat.	=	QZ	=	90° - 84°58'	05°02'S

(b) Altitude	=	NX'	=	16°12'
polar distance	=	PX'	=	21°14'
Lat. NP			=	37°26'N



(FIG.11.28)



(FIG.11.29)

4. A star when on the meridian above the pole had 4 times the altitude as it had when on the meridian below the pole. Calculate the observer's latitude and the star's declination in terms of the lower meridian altitude, if the star bore North on both occasions. (Refer fig. 11.29)

Let lower meridian altitude = k^0

Then upper meridian altitude = $4k^0$

NX = $4k^0$

NX' = k^0

XX' = $3k^0$

Polar dist. = $3k^0/2 = 1.5k^0$

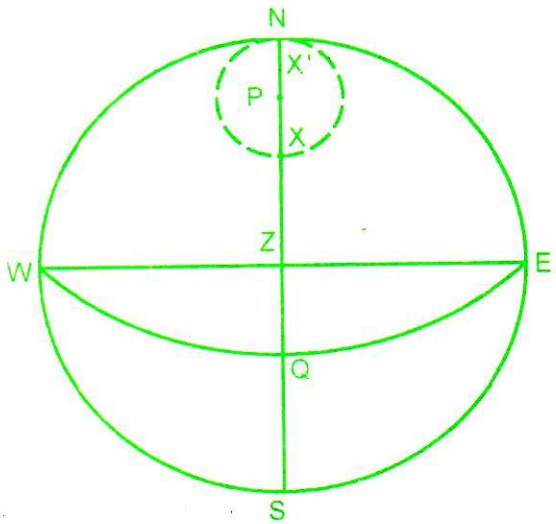
dec. = (90° - polar dist.)

= 90° - $1.5k^0$ (lower meridian alt.) N

Lat. = NP = NX' + X'P = $k^0 + 1.5k^0 = 2.5k^0$N

Lat. = 2.5 (lower meridian alt.)N

5. For a star to be circumpolar to an observer in a certain North Latitude, its altitude at upper transit should not exceed 47°16'. Find the observer's latitude and the star's declination.



(Fig.11.30)

As can be seen from the figure, in the limiting condition, the star is just circumpolar ie it grazes the rational horizon at lower transit. If the upper transit altitude was in excess of 47°16', the star would be below the observer's rational horizon at lower transit.

Latitude of the observer = NP = ½ NX = 47°16'/2 = 23°38'N
Polar dist. = XP = 23°38'N
decl. = 90° - 23°38' = 66°22'N

Note

As stated under the topic "Celestial Position Lines", PLs obtained from celestial observations are drawn as straight lines, perpendicular to the azimuth of the bodies. In the above problems involving calculations of the observer's latitude from the meridian altitude, lower meridian alt or from altitudes of a body when on the meridian above and below the pole, the bearing of the body in every case is exactly North or South. Therefore, the PL's run East-West in each case and thus coincides with the observer's latitude.

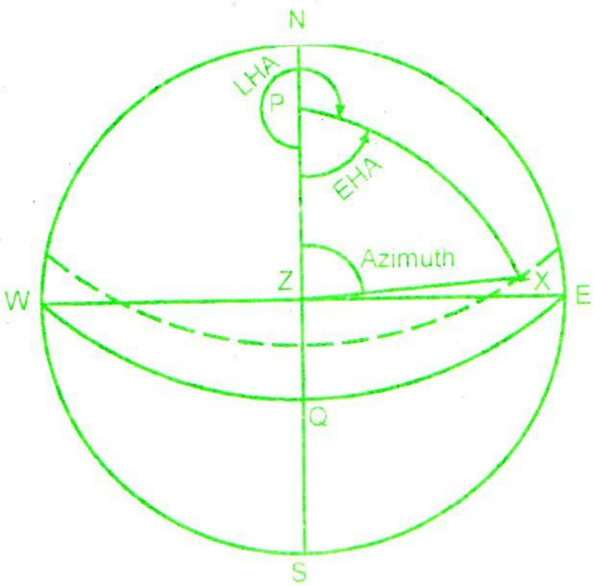
11.10 AZIMUTH

The azimuth of a heavenly body has already been defined as the angle at the observer's zenith or the arc of its rational horizon, contained between his celestial meridian and the vertical circle through the body. The azimuth of a body can be calculated by spherical trigonometry or by the use of A,B,C tables.

We calculate the azimuth of a body mainly -

(i) to find the error on the compass by comparing the true azimuth of the body with its compass azimuth, and
(ii) to obtain the bearing of the body and thus the direction of the PL which is always perpendicular to the bearing.

It should be noted that when the body is on the observer's meridian or inferior meridian i.e. its LHA is 360° or 180°, its azimuth will be 000° or 180° and when the body is on the observer's prime vertical, its azimuth will be 090° or 270°.



(FIG.11.31)

Since LHA is measured westwards from the observer's meridian, the azimuth of a body whose LHA is between 000° and 180° will be westerly and that of a body whose LHA is between 180° and 360° will be easterly.

From the above figure, it can be seen that angle Z can be calculated without any ambiguity using the haversine formula, provided we know the three sides viz. PX(90° ± decl), ZX(90°-alt) and PZ(90°-lat).

Hav Z = hav PX-hav(PZ ~ ZX) / sinZX . sin PZ

Knowing the LHA and declination of the body, as well as the observer's latitude, ABC tables can also be used to determine angle Z.

Calculation of azimuth forms part of the working of 'sights'. Problems on azimuth calculations are available in any text book on practical navigation. The calculation involves the following :-

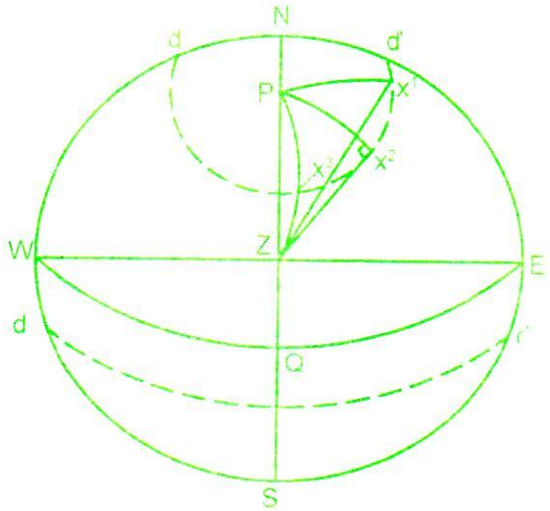
(i) Obtain the GMT, date and time

(ii) Obtain the declination of the body and determine its local hour angle at that time.

(iii) With the hour angle and DR Latitude as arguments, obtain the value of 'A'. With the hour angle and declination as arguments, obtain the value of 'B'. The algebraic sum of A and B gives 'C'. Table 'C' is now entered with the DR latitude and the value of 'C' to obtain the azimuth. The rules for naming A, B and C as well as the azimuth obtained, vary in the different nautical tables.

11.10.1. Maximum Azimuth From the figure, it can be seen that, a body whose declination (the declination circle dd in fig.) is opposite in name to that of the observer's latitude will have a maximum azimuth, when on the horizon.

If the declination of the body (declination circle d'd' in fig.) is of the same name as the observer's latitude and is also greater than the latitude, its azimuth will increase initially, reach a maximum value and thereafter decrease. In the figure the maximum azimuth of the body is angle NZX_2. At this time, the vertical circle through the body is at a tangent to the declination circle, and PX_2 the radius of the declination circle meets ZX_2 the vertical circle (and tangent) at 90°. When the body is at maximum azimuth, the angle at the body therefore is 90° and we can solve the PZX triangle using Napier's rules for right angle spherical triangles.



(FIG.11.32)

Examples

1. Find the maximum azimuth of a star of declination 66°47'S for an observer in latitude 43°39'S.

From Napiers' rule (fig.11.33)

sin PX = cos (90-PZ).cos (90-Z)

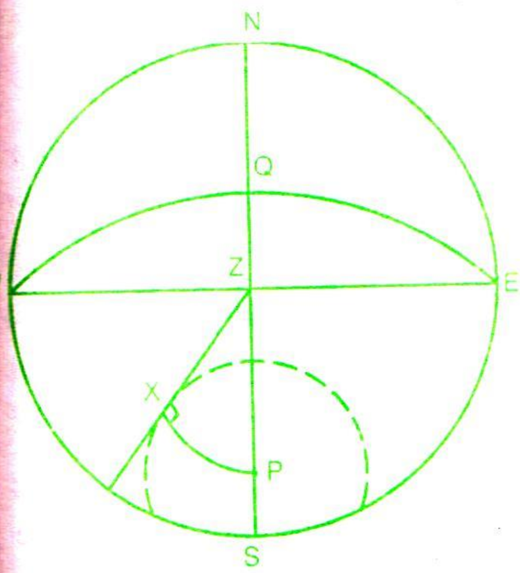
cos dec = cos lat. sin Z

Sin Z = cos dec. sec. lat

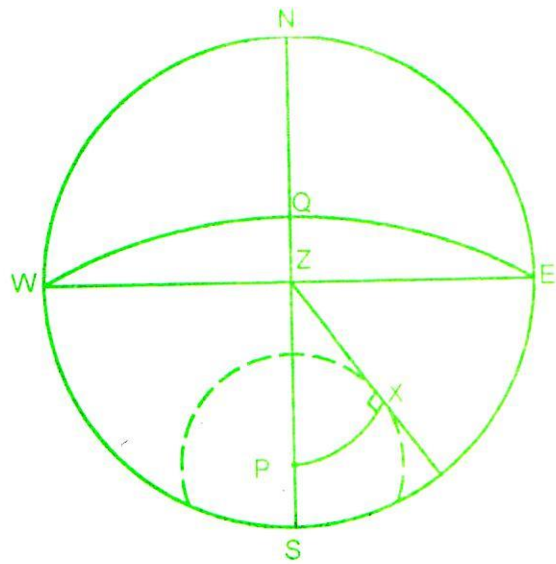
= cos 66°47'. sec. 43°39'

Angle Z = 33°00.7'

Maximum azimuth = S33°00.7'E or S33°00.7'W



(FIG.11.33)



(FIG.11.34)

2. To an observer, star Fomalhaut, dec.29°44.6'S bore 180°(T) when on the meridian. If its true altitude when at maximum azimuth was 26°03', find the observer's latitude.

From Napier's rule (fig.11.34)

sin (90-PZ)= cos PX cos ZX

sin lat = cos 60°15.4' cos 63°57'

Lat. = 12°35'S

The student may solve the following :-

1. Find the true altitude of a star, declination 19° 18.4' North, when at its maximum azimuth, in latitude 12°14'N.

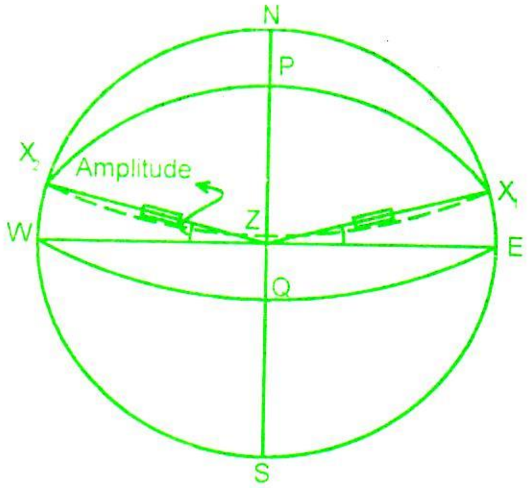
2. In latitude 20°S, a star had a maximum azimuth of S 70°°E, find its declination.

Answers

1. 39°51.5 2. 27°59.3'S

11.11 AMPLITUDE

The amplitude of a body is the angle at the observer's zenith or the arc of his rational horizon contained between the observer's prime vertical, and the vertical circle through the body, at theoretical rising or setting.



(FIG.11.35)

When observing the amplitude of a body, its centre should be on the rational horizon, that is, its true altitude should be exactly 0° which implies that its zenith distance will be exactly 90°.

If the true altitude of the Sun is 0°, the observed altitude of its lower limb, for an observer at sea level will be about 0°18' because of semidiameter correction and refraction. This was shown in the problems on altitude correction. It should be noted that stars and under normal conditions, planets also cannot be observed for amplitude as they are not visible at rising or setting, due to the horizon haze.

It is important to understand that amplitude is measured from the observer's prime vertical, as shown in the figure, and not from his meridian. Amplitude is therefore named from East towards N or S when rising and from West towards N or S when setting. The angle so obtained can thereafter be converted to 360° notation to obtain the bearing. For a body with northerly declination, the amplitude will be northward of E or W and for a body with a southerly declination, the amplitude will be southward of E or W.

Using Napiers rule on the quadrantal spherical triangle PZX indicated in the figure, the reader should prove that : **sin amplitude = sin declination x sec. latitude**

Since the calculation involved is very simple, it is suggested that amplitude should always be calculated using the above expression rather than using the various amplitude tables, as the results obtained by calculation are definitely more accurate. The calculation involves :

(i) Obtaining the GMT, date and time (refer chapter XII on rising, setting, twilight.)

(ii) Obtaining the body's declination for that time

(iii) Calculation of the amplitude

(iv) Conversion of the amplitude to three figure notation.

Note When rising, the amplitude is named E ......° N or S (according to the name of declination) and when setting it is named W ......° N or S (according to the name of the declination).

11.12 OBSERVATION OF CELESTIAL BODIES, OFF THE MERIDIAN

There are two methods of obtaining the position line from an observation of the altitude of a celestial body when it is not on or near the meridian. They are :

(i) The Marc St. Hilaire (Intercept) Method, and

(ii) The longitude by chronometer method.

We have already seen that the direction of the position line is at right angles to the azimuth. It is however necessary to obtain a position through which to draw that position line.

The two methods named above give positions through which the position line can be drawn.

11.12.1 Intercept Method

In the intercept method, the DR latitude and longitude of the vessel at the time of the sight are used in the calculation. From the DR latitude, we obtain PZ. Using the DR longitude and the GHA of the body at the time of the sight, we obtain angle P. From the declination of the body, we obtain PX.

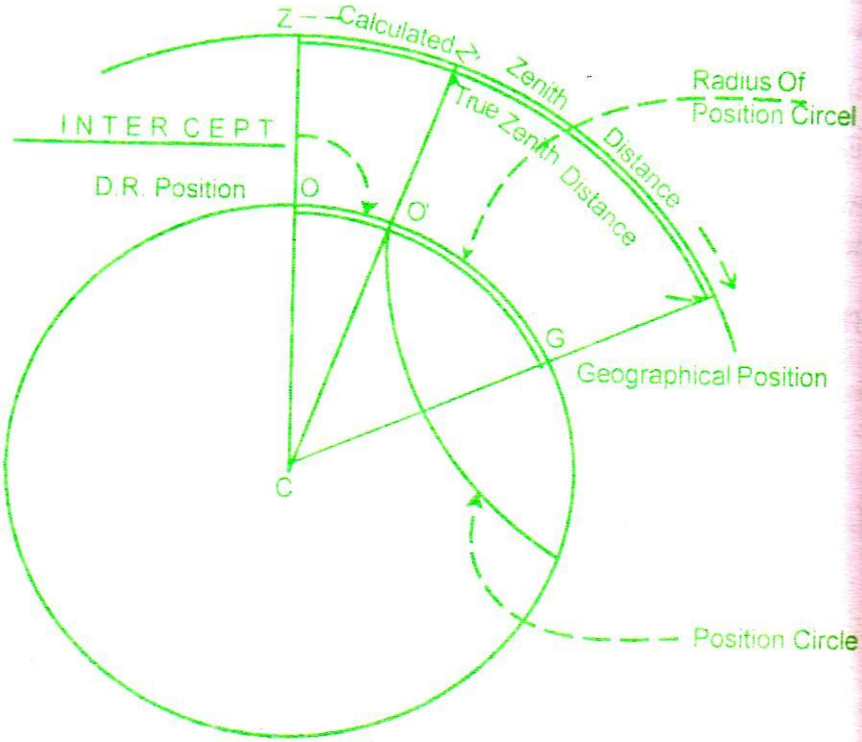
With the above, using the Haversine formula, we solve the spherical triangle PZX for side ZX, the zenith distance.

Hav ZX = (hav P sin PZ sin PX) + hav (PZ ~ PX)

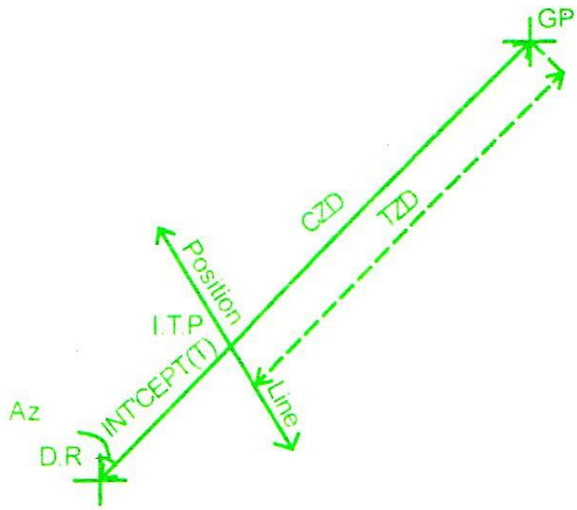
i.e. Hav zenith dist. = Hav hour angle x cos lat x cos dec. + hav (lat ~ dec)

The zenith distance, so obtained is its value, for an observer at the DR used in the calculation. The true zenith distance of the body is also found by correcting the sextant altitude and subtracting the true altitude from 90°. The difference between the calculated zenith distance, and the true zenith distance gives the intercept.

The true zenith distance in minutes of arc is the distance in miles between the observer and the GP of the body. Similarly, the calculated zenith distance in minutes of arc is the distance in miles between the DR used and the GP of the body. The intercept, which is the difference in minutes of arc between the calculated ZD and the true ZD is therefore the distance in miles from the DR, by which the observer is closer to or further away from the GP of the body. If TZD is lesser than the CZD, the observer is obviously closer to the body, and if the TZD is larger, he is further away from the body than the DR. We can therefore plot the intercept from the DR position in the direction of the azimuth or away from that direction as the case may be, and draw the PL at right angles to the azimuth through the Intercept terminal point (ITP) obtained.



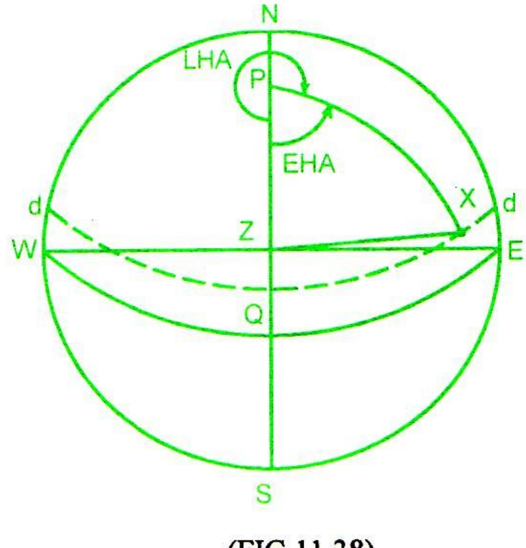
(FIG . 11.36)



(FIG.11.37)

Obtaining an intercept from a celestial observation involves :-

1. Observing the sextant altitude and GMT at that instant.
2. Obtaining the GHA and declination of the body for that time.
3. Determining the hour angle of the body, using GHA and DR longitude.
4. Calculating the CZD using the haversine formula.
5. Obtaining the TZD from the sextant altitude.
6. Comparing the TZD and CZD to obtain the intercept, which will be named **Away** if TZD is **greater** and **Towards** if TZD is **lesser.**
7. Obtaining the azimuth and from it, the direction of the PL.



(FIG.11.38)

11.12.2 Longitude by Chronometer

In the longitude by chronometer calculation, the ship's DR latitude is used. This gives us the co-latitude, PZ in the spherical triangle PZX. From the observed altitude, we obtain the true zenith distance, ZX in the triangle. The declination of the body for the time of the observation is obtained from the almanac to give PX, the third side of the triangle. Using the haversine formula :

Hav P = havZX - hav(PZ ~ PX) / sin PZ x sin PX

i.e. Hav P = hav zenith dist. - hav (L ~ D). sec lat x sec dec

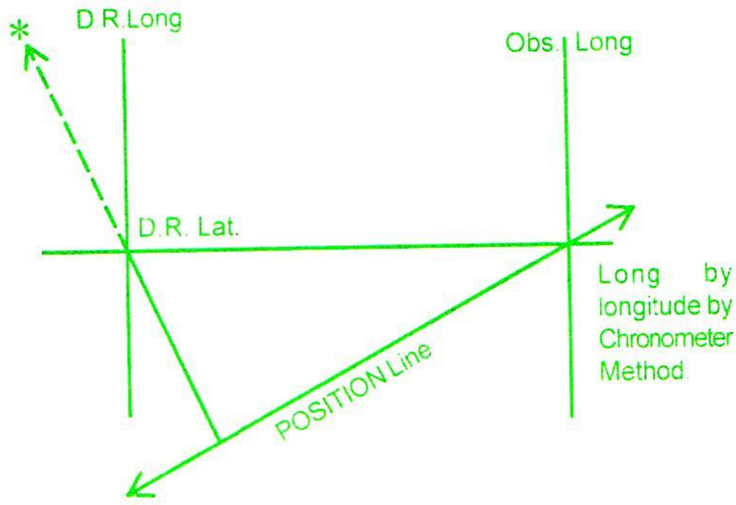
We obtain LHA of the body from the calculated angle P. The GHA of the body for the time of the observation is obtained from the almanac. The longitude is then obtained as the difference between GHA and LHA of the body.

It should be clearly understood that the longitude so obtained is the longitude in which the PL crosses the DR latitude used in working the sight. It **does not** give the longitude of the ship. If a different DR latitude was used in the calculation, the longitude obtained would also be different. The PL can therefore be drawn through the DR latitude and calculated longitude. If the bearing of the body was exactly True East or West, the PL would run exactly North-South and therefore the longitude obtained by this method would be the same irrespective of the DR latitude used. This would be the actual longitude of the ship, as the PL coincides with the meridian.

The longitude by chronometer method of obtaining a position through which the PL passes, is not very accurate if the body is too close to the observer's meridian, as the rate of change of azimuth with respect to hour angle is then fairly large. The Marc St.Hillaire method does not suffer from this limitation.

Obtaining the longitude in which the PL crosses the DR latitude involves

1. Observing the sextant altitude and GMT at that instant.
2. Obtaining the GHA and declination of the body for that time.
3. Obtaining the true zenith distance from the measured altitude.
4. Determining angle P (using the haversine formula) and from it, the LHA.
5. LHA ~ GHA gives the longitude. As explained earlier, the longitude will be west if the GHA is larger and East is the GHA is smaller than the LHA.
6. Calculating the azimuth and thence the direction of the Position Line.



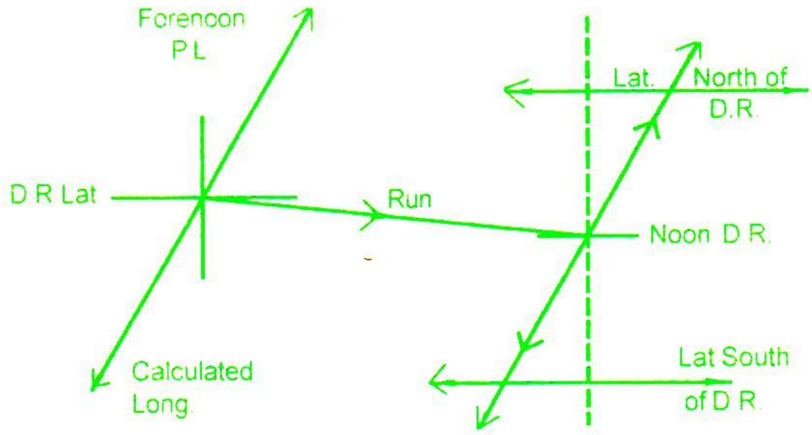
(FIG.11.39)

As shown in the figure, it should be understood that the intercept from the DR position, obtained by the Marc St. Hilaire method and the longitude obtained by the Longitude by chronometer method gives the same PL.

11.13 NOON POSITION

During morning and evening twilights several stars are available for 'sights' and an accurate, celestial fix can be obtained by crossing the various position lines. During the day, when the Sun may be the only body available for sight, we can obtain only one position line. This does not fix the position of the ship. A common method employed to obtain the noon position of the ship, at sea, is to transfer a position line obtained from a Sun sight in the morning upto the time of the Sun's meridian passage. At that time, the latitude of the ship is obtained from meridian altitude of the Sun. This position line will run East-West. The position of the ship at the time of the Sun's meridian passage, that is, at **apparent noon** is where the transferred PL intersects the latitude obtained by meridian altitude. It is not the ship's position at 1200 hrs. by the ship's clock.

The above method would give the true position of the ship at apparent noon, only if the run allowed between sights is the exact course and distance made good by the ship during the interval between the sights.



(FIG.11.40)

If the DR latitude used for working the morning sight was correct, the longitude obtained then would be correct and therefore the longitude at apparent noon, calculated by applying the run to the morning position would also be the correct longitude of the ship then. If the morning DR latitude was in error the noon latitude obtained by meridian altitude will differ from the latitude obtained by running up the morning DR latitude to apparent noon. As the morning longitude calculated depends on the DR latitude used (which was in error) the longitude at apparent noon, worked by running up the morning observed longitude, would also be in error. The amount of the error in longitude can be obtained by, multiplying the value of 'C' obtained for the morning sight, from ABC tables by the difference between the DR and observed latitudes at Noon.

Longitude correction = (DR Latitude ~ obs. Lat) x 'C'.

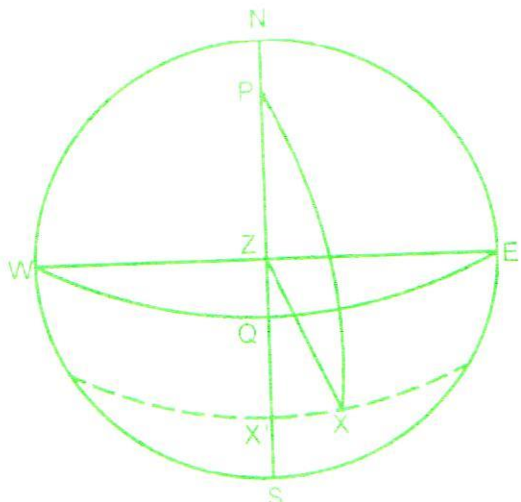
The proof of this relationship is beyond the scope of this book. Whether the true longitude at noon lies to the east or west of the noon, can be easily determined by drawing a rough sketch as shown above.

11.14 EX-MERIDIAN SIGHTS

It may not always be possible to obtain the altitude of a body, when on the meridian. If the body is observed when near the meridian that is, when its hour angle is small, the latitude in which the PL intersects the DR longitude used in the working, can be calculated. It is important to note that the latitude calculated by Ex-Meridian method is not the latitude of ship then. The ship will be on the position line obtained which would be nearly East-West as the body is then near the meridian, and will bear nearly North or South. The latitude obtained by this method would be the correct latitude of the ship only if the DR longitude used in the working is the true longitude of the ship. It should also be noted that the PL obtained and the position through which to draw it (calculated latitude and DR longitude used in the working) are for the **time of the observation** and **not** for the **time of meridian passage.**

The limits of hour angles within which an observation may be worked by Ex-Meridian method depends on Lat ~ Dec (L ~ D). When latitude and declination are of the same name, the rate of change of altitude will be larger and therefore the Ex-Meridian limits are smaller than when latitude and declination are of opposite names.

Ex-Meridian table IV, both in Norie's and Burton's tables, gives the limits of hour angle within which observations may be obtained and worked, Ex-Meridian method, without appreciable error. These tables are tabulated as a function of the observer's DR latitude and the body's declination, both for same names and for opposite names. As a rough rule, the hour angle in minutes of time should be less than the approximate meridian zenith distance of the body in degrees



(FIG 11.41)

The zenith distance of the body is least, when it is on the meridian. When the body is near the meridian (before or after meridian passage), the zenith distance is slightly larger.

With reference to the above figure, the method used and the approximations which are employed, in working a sight Ex-Meridian method are discussed below.

In the figure, X is a body close to the meridian, and X' the same body when on the meridian. Assuming that the declination remains unchanged between positions X and X', PX'=PX.

$$ZX' = PX' \sim PZ$$
$$\therefore MZD = PX' \sim PZ = PX \sim PZ$$

By the Haversine formula applied to the triangle PZX
hav (PX ~ PZ) = hav ZX - hav P. sin PZ. sin PX = hav zenith dist. - hav P . cos lat . cos dec

The latitude is then obtained by applying the declination of the body to the calculated PZ ~ PX, (L ~ D) as for a meridian altitude sight. The calculation involves -

1. Working out the EX-Meridian limits for the body
2. Obtaining the altitude of the body within the Ex-Meridian limits and the GMT date and time then.
3. Obtaining the declination and GHA of the body from the almanac and thence the LHA, using the DR longitude.
4. Correcting sextant altitude to find the true zenith distance ZX.
5. Calculating PZ ~ PX (L ~ D), by the Haversine formula, using PZ as obtained from the DR latitude.
6. Obtaining the latitude by applying the declination to the calculated (PZ ~ PX) as in a meridian altitude calculation.
7. Obtaining the azimuth by ABC tables and thence the direction of the PL.
8. The PL is drawn through the DR longitude and the calculated latitude.

The latitude obtained is that at which the PL intersects the DR longitude at the time of observation and **not** at the time of meridian passage, because the ZX used in the calculation is that for the time of observation and not for the time of meridian passage.

In working sights by this method, one of the parameters we use is the latitude, and it is the latitude that we are trying to calculate. Provided, the DR latitude used is reasonably accurate, no appreciable error will be caused since, in the formula, cos latitude is being multiplied by hav P. As angle P is small, any error due to the use of an inaccurate latitude is reduced considerably.

An approximation we use in the calculation is that the declination remains unchanged between the time of observation and of meridian passage.

Ex-Meridian method should not be used when the approximate MZD is less than about 4°, as the errors involved and approximations used would not, then, be within acceptable limits.

11.14.1 Ex-Meridian Tables

Ex-Meridian sights may be worked faster, by using the Ex-Meridian tables provided in Norie's and Burton's tables. From Ex-Meridian table I, in Nories or Burtons using DR latitude and the body's declination, factor 'A' or factor 'F' respectively is obtained. Table II, is entered with A or F, as the case may be, and the LHA of the body, to obtain the first correction.

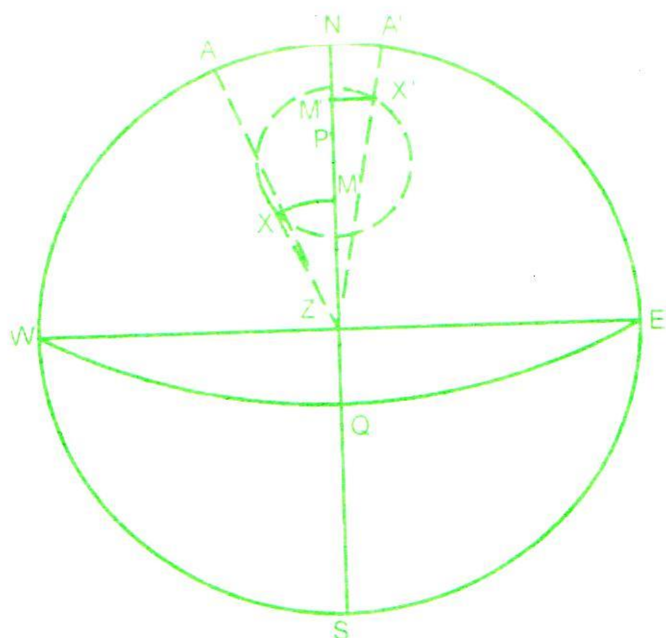
By entering table III, with the first correction and true altitude we obtain the second correction. The second correction is subtracted from the first correction to obtain the 'reduction'. The 'reduction' is subtracted from the TZD to obtain the Meridian ZD. The declination is applied to the MZD to obtain the latitude as explained earlier. The azimuth is then obtained in the normal manner. Full explanation on the use of Ex-Meridian tables is provided in Norie's and Burton's nautical tables.

It should be noted that Ex-meridian observations may be worked accurately by the Marc St. Hillaire or Intercept method as no approximations are involved in that method.

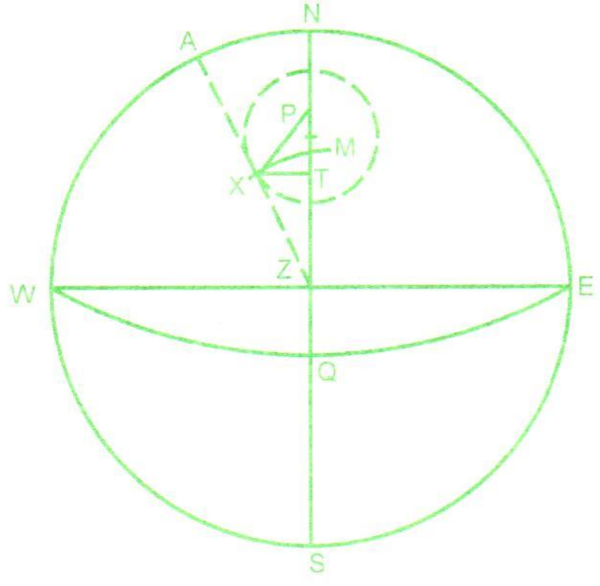
For Practical Navigation at sea therefore, the Ex-Meridian method is obsolete and this method is used only in academic work where the candidate may be required to obtain the latitude in which the PL intersects a given DR longitude.

.15 POLARIS SIGHTS

know that the altitude of the celestial pole is equal to the latitude of the observer. Fortunately there is [illegible]rly bright star (Polaris), situated very close to the celestial North Pole. There is no such star near the [illegible]stial South Pole. Since the declination of Polaris is about 89°10'N, it is situated less than 1° from the [illegible]th celestial pole. An observer can therefore determine his latitude from the true altitude of Polaris, by [illegible]lying small correction to it.



(FIG.11.42)



(FIG.11.43)

In the above figures, the declination circle of Polaris has been deliberately exaggerated for the sake of clarity.

When Polaris, is at some position such as 'X' in figure 11.42, its altitude is equal to AX. If with Z as centre and radius equal to ZX, an arc is drawn to meet the meridian at M, Altitude AX = NM. Latitude NP can then be obtained by subtracting the small correction PM from the true altitude NM. When Polaris is at some position such as X', in fig. 11.42 the latitude NP, can similarly be obtained by adding the small correction PM' to the true altitude A'X' i.e. NM'.

In figure 11.43 if XT is a perpendicular from X to NZ, PM the correction is approximately equal to PT and if the small triangle PXT is considered a plane triangle, PT equals PX. cos P i.e. polar distance x cosine LHA. A further correction is necessary to allow for the fact that triangle PXT is spherical and not a plane triangle,and also to account for the small difference between PM and PT. In the 1976 almanac, these two corrections are computed for a standard latitude of 50°N, using mean values of SHA and declination of Polaris as 327°39.0' and 89°09.5'N respectively.

The above corrections are worked for all LHAs of Polaris from 0° to 359°, at one degree intervals. As shown in the figure this correction may be +ve or -ve, depending on the LHA of Polaris. Its value can never exceed the polar distance of Polaris. To enable easy computation of latitude, a constant 58.8' is added to the actual corrections so that the values tabulated in the almanac are always positive. These adjusted values are tabulated in the almanac as a function of LHAγ, and is the first correction 'a_0' of the Polaris tables. It is tabulated as a function of LHAγ and not as a function of LHA Polaris so as to avoid the necessity of calculating LHA Polaris by adding to LHAγ, the SHA of Polaris, the value of which changes fairly rapidly due to precession of the equinoxes. Since LHA Polaris relates to LHA Aries by the SHA of Polaris, 'a_0' and LHAγ can be co-related.

The second correction, 'a_1' in the Pole star tables of the almanac, is a correction for variations in a_0 due to the variation in the observer's latitude from 50°N, which was assumed for calculation of 'a_0', This correction may also be positive or negative depending on whether the observers lat was greater than or less than 50°N. For easy computation of latitude, a constant 0.6' is added to the actual correction to make it always positive and the adjusted values are tabulated in the almanac as a function of LHAγ and the observer's latitude.

The third correction a_2 is for variation in the values of SHA and declination of Polaris from the mean values used in working the a_0 corrections. a_2 corrections are also, increased by a constant 0.6' to make them always positive, before tabulation in the almanac as a function of LHAγ and the month.

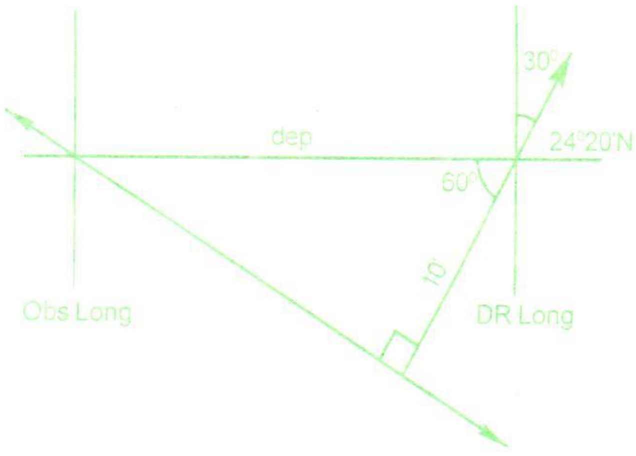
Since a total amount of 58.8' + 0.6' + 0.6' = 60' = 1° is arbitrarily added to the three corrections, the latitude is obtained as, true altitude of Polaris + a_0 + a_1 + a_2 - 1°. An example showing the use of these tables to obtain the latitude is provided below the Pole star tables in the almanac. The almanac also provides an azimuth table for Polaris as a function of LHAγ and the latitude. Theoretically the observer is on the PL drawn through the DR longitude and the latitude calculated. In practice, except in high latitudes, the azimuth being very small, may be disregarded and the PL assumed to coincide with the parallel of latitude.

The principles involved in the various methods of obtaining from celestial observations, the direction of the position line and the position through which it passes, have been indicated above. The method of work has also been shown in each case.

Since the reader will be simultaneously working problems of the different types in his Practical Navigation curriculum, such problems are not included in this book. It is assumed that in his study of Practical Navigation, the reader is familiar with position fixing by crossing position lines obtained from simultaneous celestial observations, as well as transferred position lines obtained from celestial observations at different times. In his Chart Work, he would also have dealt with position fixing by simultaneous, and transferred terrestrial position lines and position circles. It is important that he knows these aspects of position fixing before proceeding with the problems that follow in this chapter.

The type of problems that follow, involve celestial as well as terrestrial position lines and are not generally available in Practical Navigation or Chart Work text books. Solution of these problems involve the principle of position lines and therefore form part of Principles of Navigation. These problems may be solved graphically or mathematically. Both methods of work have been shown in the solved examples so that the reader achieves proficiency in both methods of approach.

1. A vessel in DR 24°20'N, 040°30'W, obtains an intercept of 10 miles away from an observation of a celestial body bearing 030°(T). Find, what longitude would have been obtained if this sight had been worked by longitude by chronometer method.

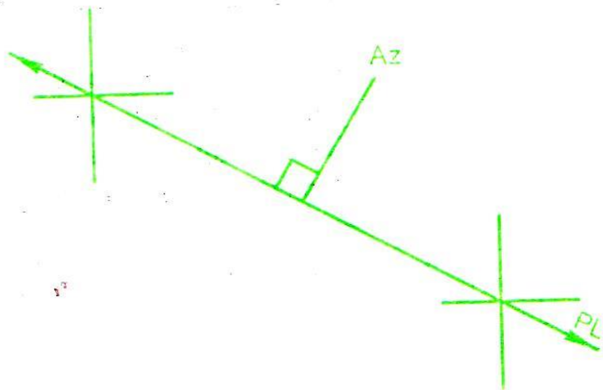


(FIG.11.44)

dep = 10/cos 60 = 20 miles
d'long for dep 20 miles in latitude 24°20' = 22'
∴ Obs.long. = 40°30'W + 22' = 40°52'W.

2. Two ships, one in position 20°10'N 12°15'E and the other in position 20°16'N 12°07'E, observed the Sun simultaneously and obtained the same true altitude. The Sun bore 047°(C) from the first ship and 042°(C) from the second ship. If the variation was 6°E, find the deviation of the compass at each ship.

Since the true altitude and therefore the true zenith dist. is the same from both positions, the same position circle passes through the two positions. The two positions being close to each other, the arc of the position circle between them may be considered a straight line i.e. as a position line. The azimuth will therefore be at 90° to the direction between the two positions. (FIG. 11.45)



(FIG.11.45)

	Lat	Long	m.p.
1st observer	20°10'N	12°15'E	1227.72
2nd observer	20°16'N	12°07'E	1234.08
	6'N	8'W	6.36

tan co = d'long / DMP

Direction between the two positions N51°31'W = 308°29'
∴ True azimuth = 038°29'

Note

The reverse direction may be ruled out as the compass bearings are 047° and 042°.

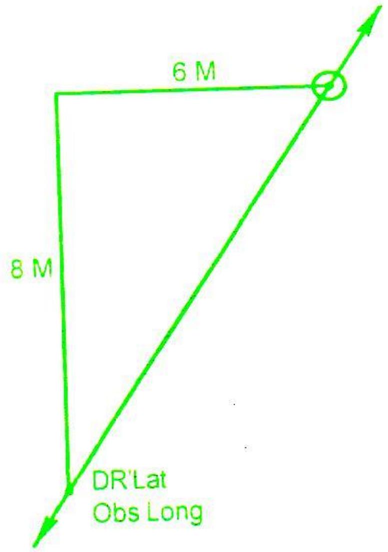
True Az.	38°29'		38°29'
Compass Az	47°		42°
Error	08°31'W		03°31'W
Var.	06°00'E		06°00'E
Dev.	14°31'W	Dev	09°31'W

Example

3. A vessel in DR latitude 24°S, worked a morning Sun sight longitude by chronometer method. At the same time a fix by radar put the vessel 8 miles to the north and 6 miles to the east of the position through which the PL was drawn. Find the Sun's true bearing. (FIG. 11.46)

The vessel is somewhere on the PL obtained from the sight. The fix obtained by radar must also therefore be on the PL. The PL therefore passes through the DR latitude and observed longitude, as well as through the fix 8 miles to the north and 6 miles to the east.

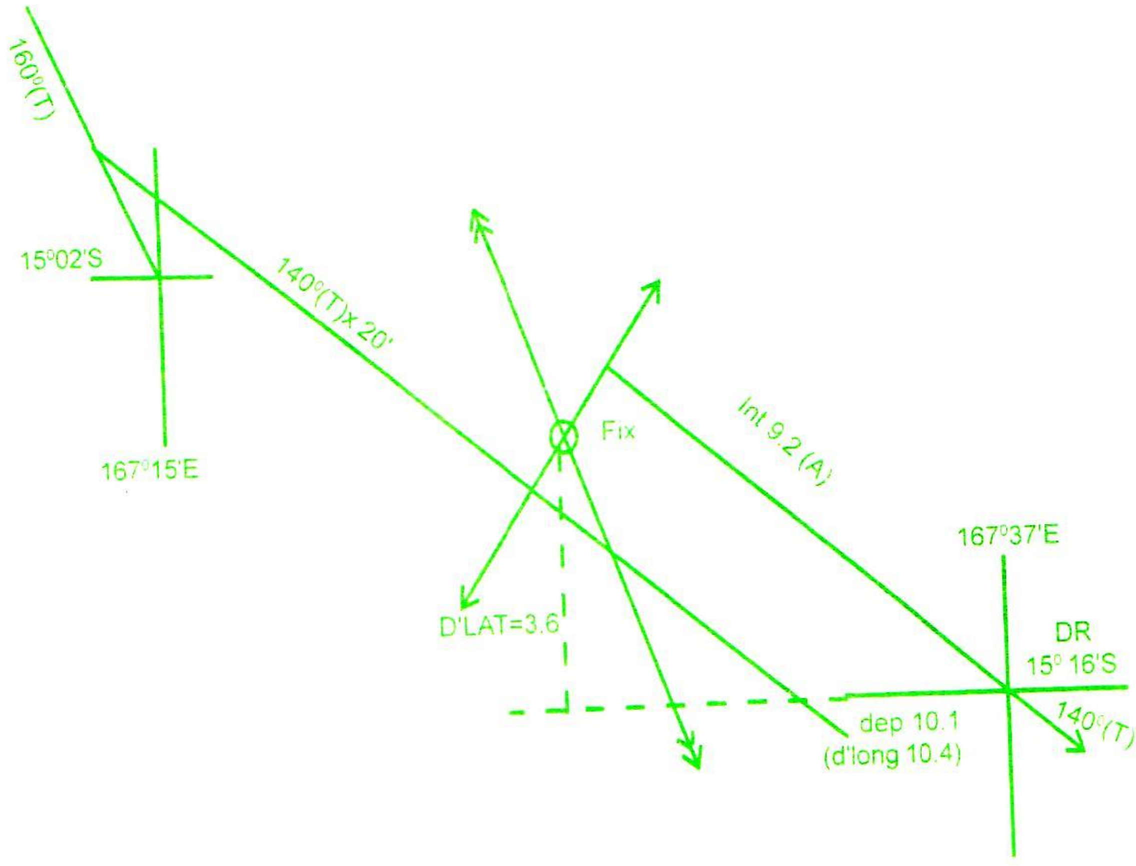
dep / d'lat = 6 / 8 = tan course
Course 36°52'. PL runs 036°52' ←→ 216°52'
The azimuth, which is 90° to the PL could be 126°52' or 306°52'. However, as it is a morning sight of the Sun, the azimuth must be easterly.
∴ True azimuth = 126° 52'(T)



(FIG.11.46)

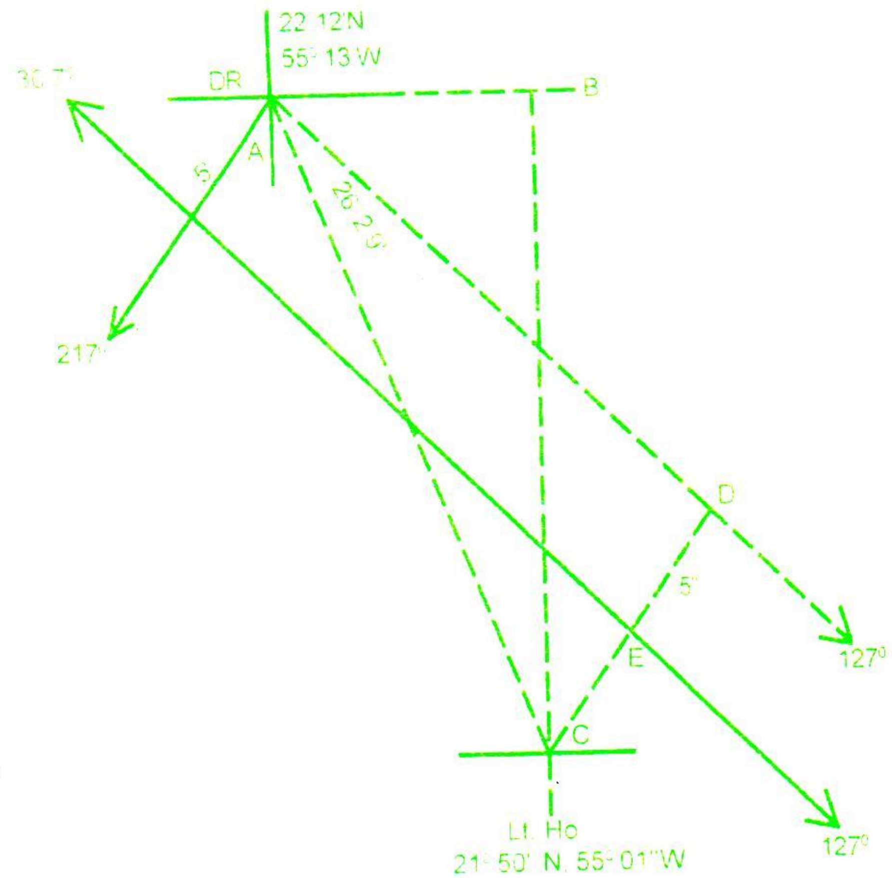
4. A vessel steering 140°(T) sights an island in position 15°02'S 167°15'E, 20° on her starboard bow. She then steams 20 miles when a celestial observation of a body worked using D.R. position 15°16'S 167°37'E gave bearing 140°(T) and intercept 9.2 miles away. Find the ship's position at the time of the celestial observation. (FIG. 11.47)

	15°02'S	167°15'E		
	15°16'S	167°37'E		
	d'lat 14'	d'long 22'	dep = 21.25	
DR		15°16.0'S		167° 37.0'E
	d'lat	3.6'N d'long	10.4'W	
		15° 12.4'S		167° 26.6'E



(FIG.11.47)

5. A star sight worked using DR 22°12'N, 055°13'W, gave azimuth 217°(T), intercept 5 miles towards. The ship then steered 127°(T). Find how far she would pass a light house in position 21°50'N 055°01'W. (FIG. 11.48)



(FIG.11.48)

DR used	:	22°12'N	055°13'W	1357.8	mp
Light house	:	21°50'N	055°01'W	1334.2	mp

d'lat 22'S d'long 12'E DMP 23.6

Converting d'long to dep., dep. = 11.13

tan co, A to C = d'long / DMP = 12 / 23.6 Course = 26°57.

Distance AC = d'lat.sec Co = 22 sec 26°57.1' = 24.68'

CD = AC sin 26°02.9'

= 10.84 M

ED = 5.00 M

Dist CE = 5.84 M

Distance off, she would pass the light house = 5.84 M.

6. A star (GHA 253°12', dec. 02°04'N), was observed to have a true altitude of 89°52'. At the same instant, a light house with a maximum range of 12 M, situated in latitude 2°12'N, long 106°54'E bore 012°(T). Find, by plotting, the ship's position. (FIG. 11.49)

Star's GP	02°04'N	106°48'E
Posn of Lt.house	02°12'N	106°54'E

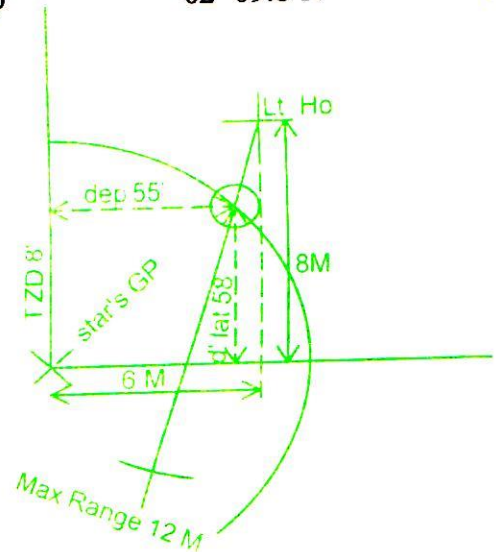
d'lat 8'N d'long 6'E

Converting d'long to dep.; dep. = 6 miles East.

d'lat & dep. of fix from GP of star

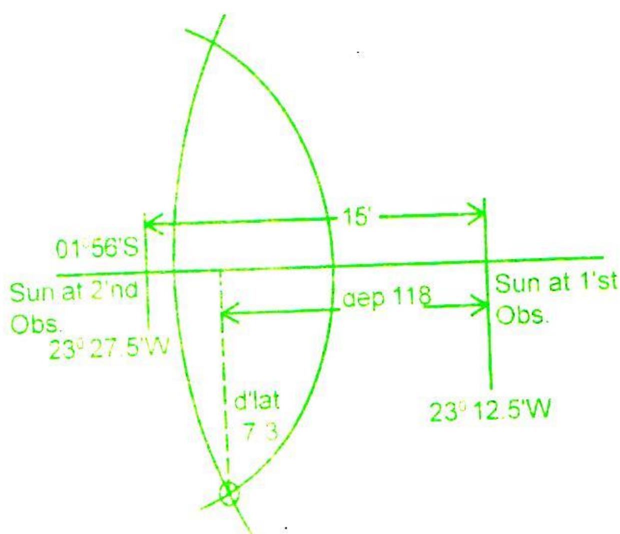
d'lat = 5.8'N; dep. = 5.5'E
converting dep. to d'long,
d'long = 5.5'E

GP of star :		02° 04.0'N		106° 48.0'E
	d'lat	05.8'N	d'long	05.5'E
Position of ship		02° 09.8'N		106° 53.5'E



(FIG.11.49)

7. An observer on an anchored vessel, obtained the true altitude of the Sun as 89°46' when the Sun's GHA was 23°12.5' and declination was 01°56'S. The true altitude of the Sun obtained exactly 1 minute later was 89°52'. If during the interval, the Sun crossed the observer's meridian to his north, find the ship's position by plotting.



(FIG.11.50)

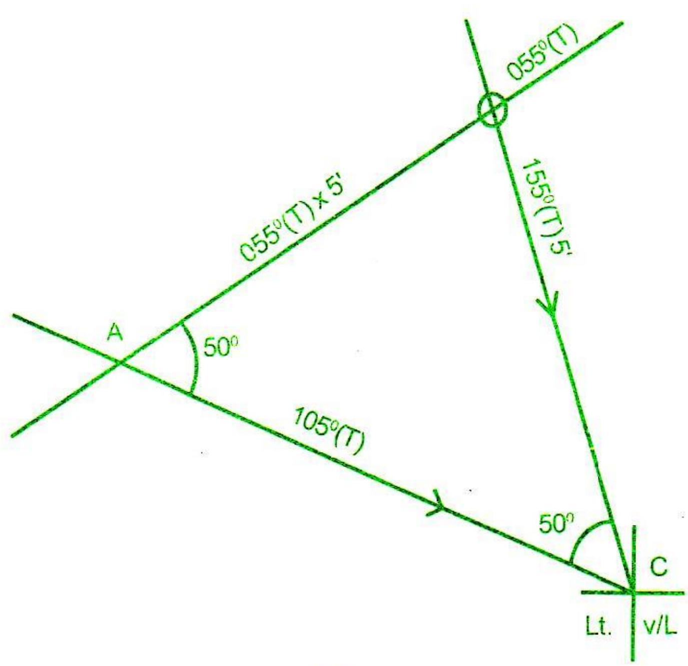
Position circles are plotted as in the previous question with the geographical positions of the Sun as centre and radii equal to the ZDs. (Fig. 11.50) The GP of the Sun at the time of 1st observation is known. Since the second observation is exactly one minute later, the Sun's GHA would have increased by 15' i.e. its long of GP would be 15' to the West. The declination hardly alters in one minute. The latitude of the Sun's G.P. is therefore unaltered.

To plot Sun's GPs
d'long of 15' = dep. of 15'

Of the two points at which the position circles intersect, the one to the south is the true fix as the Sun crossed the observer's meridian to his north.

GP of Sun at first observation :	01°56.0'S	023°	12.5'W
In Lat 01°56'; dep. 11.8'	d'log		11.8'W
d'lat 7.3'S			
Position of Ship :	02°03.3'S		023°
24.3'W			

8. A ship was steering a compass course of 060°, Var. 12°W, Dev. 7°E. A light vessel in position 45°31.5'S 15°20'W bore 110°(C) and after steaming for 5 miles it bore 167°(M). Find the ship's position at the time of the second bearing. (FIG. 11.51)



(FIG.11.51)

Variation	12° W	
Deviation	7° E	
Error	5° W	
Compass course	060°(C)	
Error	5° W	
True course	055°(T)	
1st brg.	110°(C)	2nd brg. 167°(M)
Error	05° W	Variation 12° W
True brg.	105°(T)	True brg. 155°(T)

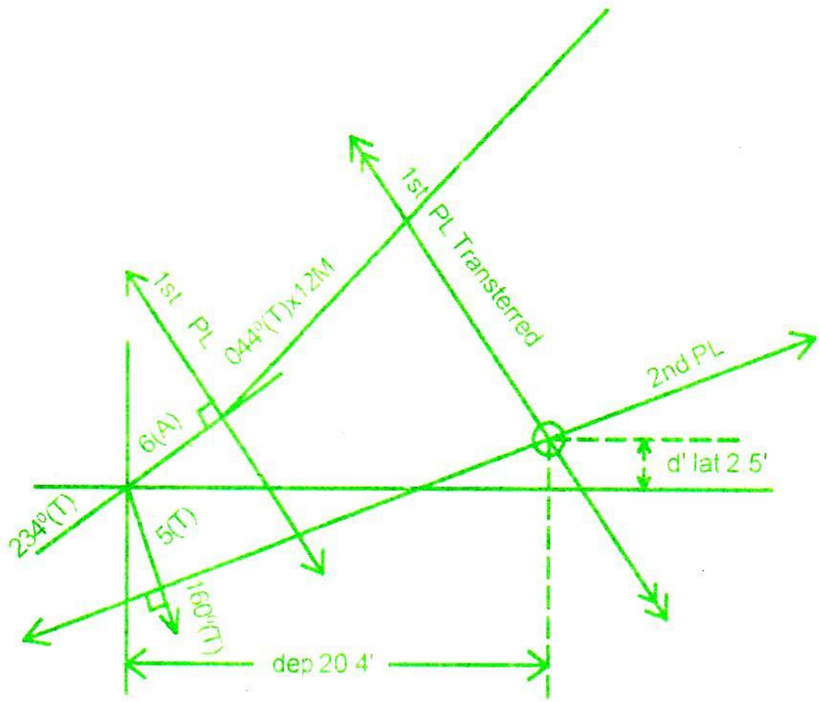
Angle between True course and 1st brg. = 105° - 55° = 50°
Angle between 1st and 2nd bearings = 155° - 105° = 50°

The triangle ABC is isosceles and side AB = side BC = 5M
To find position B, with reverse bearing 335° as course & distance 5 M d'lat = 4.53'N, dep. = 2.11'W; d'long = 3.02'W

Position of Lt. vessel	45°31.50'S	15°20.00'W
	d'lat 4.53'N	d'long 3.02'W
Position of ship	45°26.97'S	15°23.02'W

9. In DR 26°10'N, 060°04'W, a star sight gave azimuth 234°(T), intercept 6 miles away. The vessel then steamed 044°(T) for 12 miles, when a sight of the Moon was obtained and worked with the original DR. This gave an azimuth of 160°(T) and an intercept of 5 miles towards. Find by plotting, the ship's position, at the second observation. (FIG. 11.52)

To obtain the fix at the time of the second observation, the first PL should be transferred for the vessel's run between the observations.

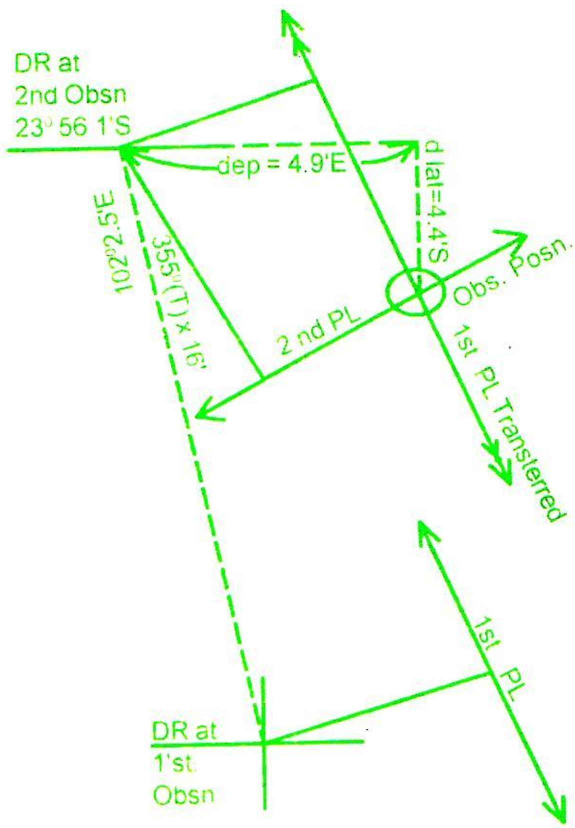


(FIG.11.52)

DR lat. 26°10.0'N long. 060°04.0'W		
D'lat	0°02.5'N d'long 0°22.7'E	
	(dep. = 20.4'E)	
Obs.lat.	26°12.5'N	long. 059°41.3'W

10. A ship in DR 24°12'S, 102°04'E, obtained a sight which gave azimuth 074°(T), intercept 3.5 miles towards. She then sailed 355°(T), 16 miles when another sight worked with the DR position carried forward gave azimuth 335°(T), intercept 6 miles away. Find by plotting, the ship's position at the second observation.

The accompanying diagram (fig. 11.53) shows the original DR, the run and the final plotting from which the observed position can be obtained. Alternatively, the 2nd DR can be worked mathematically using traverse table and the plotting carried out using that position, to a larger scales to obtain the same observed position. The latter method is particularly recommended when the distance run is fairly large as otherwise the scale and consequently the accuracy is reduced.

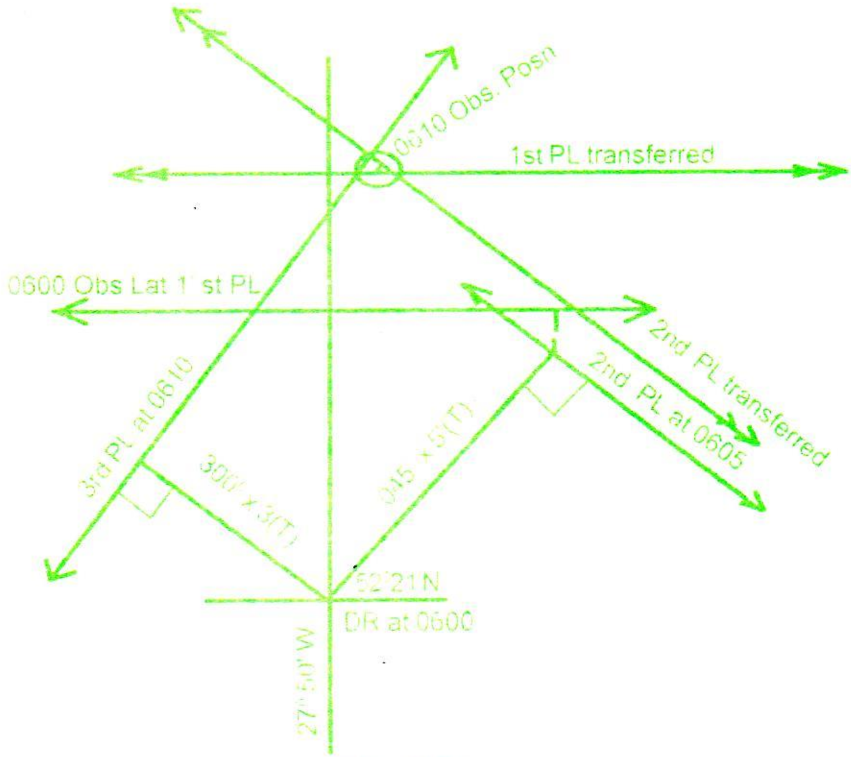


(FIG11.53)

DR position : Lat 24° 12' S Long 102° 04 ' E

355° x 16' d'lat		0°15.9 N d'long		0° 01.5'W (dep = 01.4'W)	
2nd DR	lat	23° 56.1' S	long	102° 02.5' E	
From plot, d'lat		4.4' S	d'long	5.3'E (dep= 4.9 'E)	
obs.posn.	lat	24° 00.5' S	long	102° 07.8' E	

11. At 0600 hrs by ship's clock on a vessel in DR position 52° 21'N, 27°50'W, an observation of star A on the meridian gave latitude 52° 26'N. A second sight, obtained at 0605 hrs of star B and a third sight, obtained at 0610 hrs of star 'C' when worked using the original DR gave azimuth 045° (T) int. 5 M towards, and azimuth 300° (T) int. 3 M towards respectively. The ship was steering 000° (T) at 12 knots. Find by plotting the ship's position at 0610 and at 0600 hours. (FIG. 11.54)



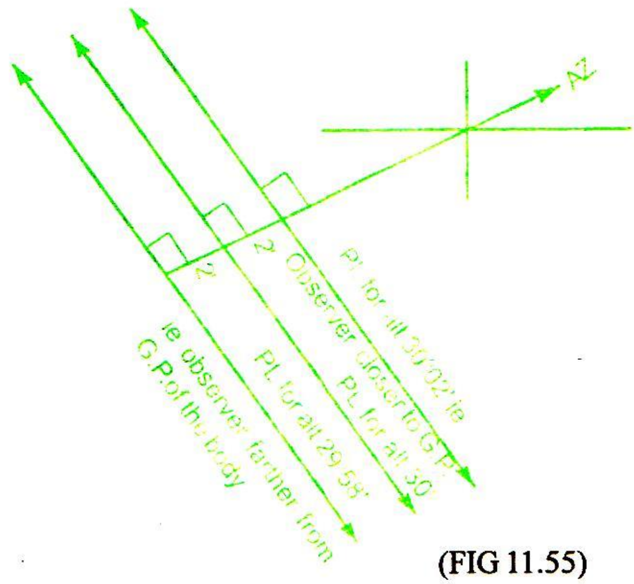
(FIG. 11.54)

0600 Obs	lat	52° 26.0' N	DR long	27° 50' W
	d'lat	0° 02.2N	d'long	0°01.2 E
				(dep = 0.7'E)
0610 Obs	lat	52° 28.2'N	Obs long	27°48.8' W
	d'lat	0° 02.0'S	d'long	NIL (dep Nil)
00600 posn.	lat	52° 26.2'N	long	27°48.8' W

11.16 ERRORS N POSITION LINES

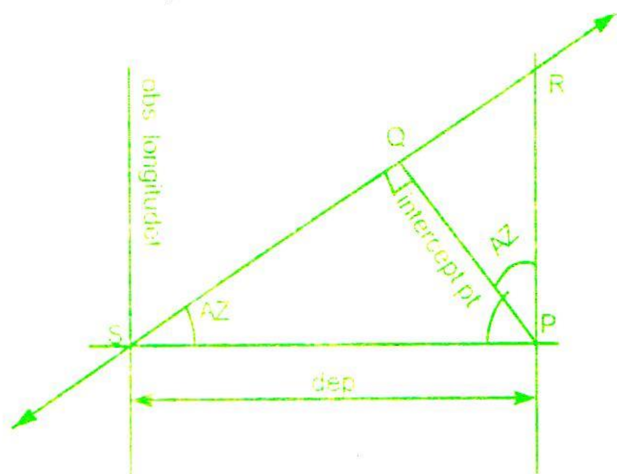
11.16.1 Error in Intercept due to error in altitude

Any error in the altitude will affect the position through which the PL passes. As already explained earlier, the TZD in minutes is the distance in miles on the Earth's surface between the observer and the GP of the body. If the true altitude is greater, the true zenith distance is lesser by the same amount. The PL will therefore be displaced towards the direction of the celestial body observed, by a number of miles equal to the number of minutes the altitude was in error. Conversely if the true altitude is lesser, the true zenith distance is larger and the PL is displaced in a direction away from the observed celestial body. Thus an error in the altitude reflects as an error in the intercept equal to an error in minutes in the altitude. The illustration below is an example of how the PL is displaced, because of an error in the altitude.



(FIG 11.55)

11.16.2 Error in long due to error in altitude



(FIG. 11.56)

In triangle PQS, PS = PQ cosec azimuth
Error in dep. = error in alt. cosec azimuth
Error in d'long . cos lat = error in alt. cosec azimuth
(dep = d'long cos lat)
Error in long. = error in alt. cosec azimuth . sec lat.

11.16.3 Error in long. due to error in time

For every second of time, the GHA of a heavenly body increases by 0.25'. Thus for every four seconds of time, the GHA increases by one minute of arc. When working a sight, longitude by chronometer method, we obtain the longitude as GHA - LHA = Long (W)

LHA - GHA = Long (E)
If the time is greater the GHA increases. The LHA which was calculated using the measured altitude is unchanged.

Therefore an increase in GHA causes a corresponding increase in the (West), longitude obtained and a corresponding decrease in the (East) longitude obtained. It can therefore be seen, that for an error in time of 1 second, an error in the longitude of 0.25' will be caused. If the actual time is greater, the GHA will be larger and therefore the west longitude obtained will be larger and the east longitude obtained will be lesser. In other words if the actual time is greater, the longitude shifts to the West by 0.25' per second of error in time. Conversely if the actual time is lesser, the longitude shifts eastwards by 0.25' for every second of error in time.

If the sight had been worked intercept method, the error in intercept due to an error in the time may be obtained by first finding the error in longitude as explained above and then using the expression. Error in intercept = Error in long . sin azimuth . cos lat, which relationships the reader can develop using the figure 11.56.

11.16.4 Error in longitude due to error in latitude

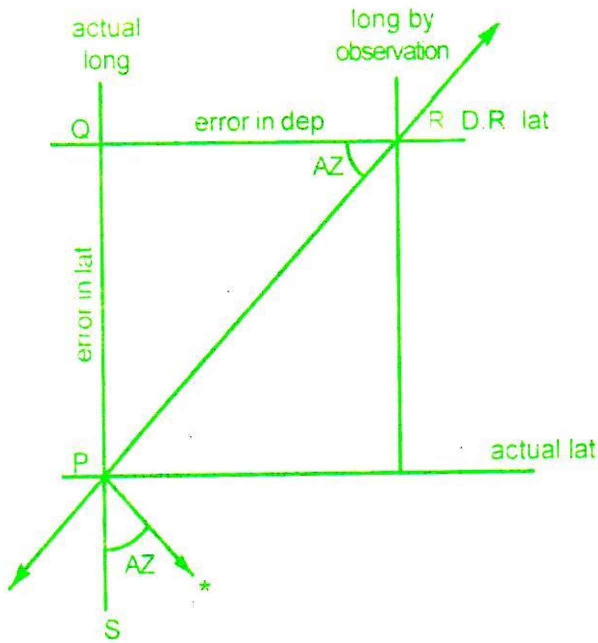


FIG.11.57)

In triangle PQR

QR / PQ= cot azimuth

QR = PQ Cot Az

Also QR = d'long cos . lat. (dep = d'long cos lat.)

QR = d'long cos lat = PQ cot Az

d'long = PQ cot Az sec lat.

d'long = error in lat. cot Az sec lat.

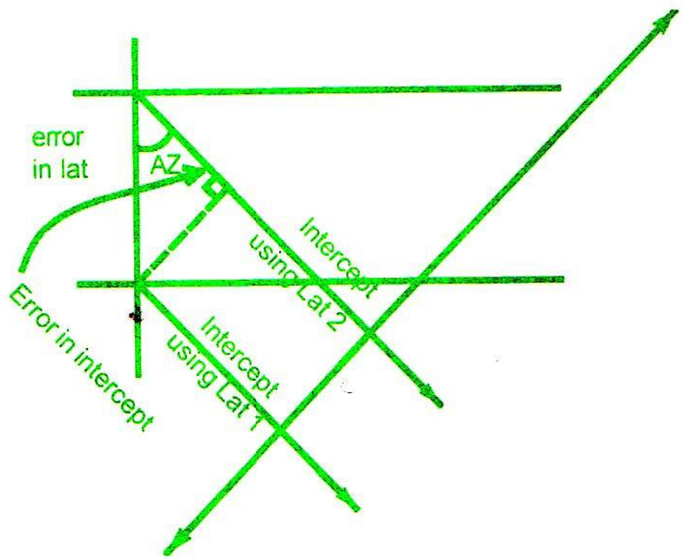
d'long = error in long. = error in lat . cot Az . sec lat.

In the ABC tables, the value of C = cot Az.sec lat

∴ Error in long=error in lat × C

11.16.5 Error in intercept due to error in latitude

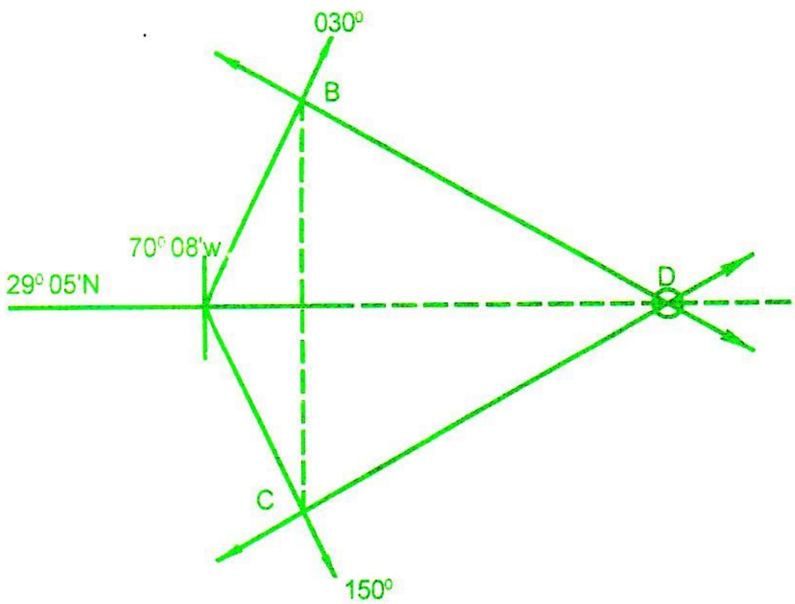
From the figure it can be seen that Error in intercept = error in lat . cos azimuth.



(FIG.11.58)

Example

1. A position 29° 05' N, 70°08' W was obtained by plotting position lines obtained from two celestial observations, the bodies bearing 030° (T) and 150° (T). It was later found that the index error of the sextant, which was 2.5' off the arc had not been applied. Find the true position of the ship.



(FIG. 11.59)

Join AD

In right angled triangles ABD and ABC, AD is common and

AB = AC

triangles are congruent

i.e. Angle BAD = Angle CAD = 120° / 2= 60°

AD = AB sec 60° = 2.5 sec 60° = 5.0

since angle BAD = 60°, D is East (T), 5M from A

Converting dep. of 5 M to d'long, using traverse tables.

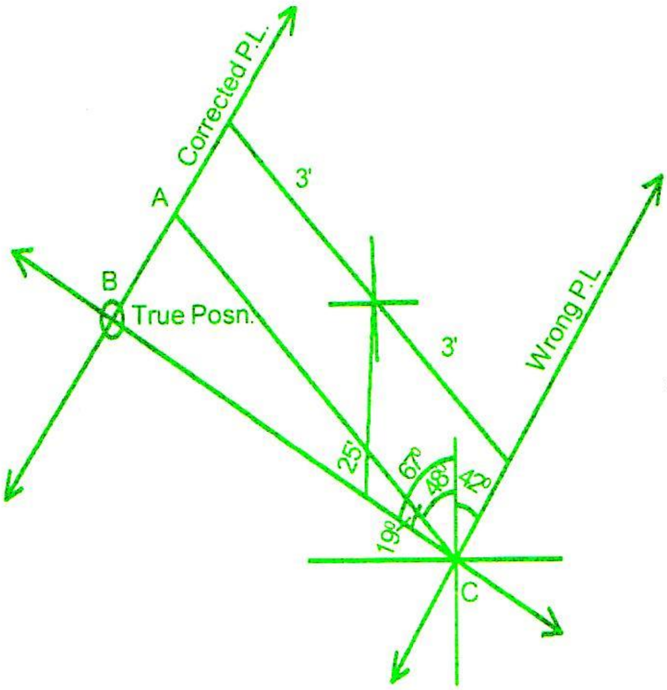
d'long = 5.7' E

Position of A	29° 05' N		70° 08' W	
d'lat	00' N	d'long	05.7' E	
True Position	29° 05' N		70° 02.3 W	

Note : This problem can also be solved graphically.

Example

2. A position 20°10'S, 112°04' W was obtained by observing two stars one bearing 132° (T), intercept 3 miles away and the other bearing 203° (T), intercept 2.5 miles towards. It was then discovered that the first intercept had been wrongly laid off, as towards instead of away. Find the true position of the ship.



(FIG. 11.60)

AC = 6'

Angle ACB = 19°

AC / BC=Cos 19⁰	BC	=	AC sec 19°
		=	6 sec 19°
		=	6 x 1.0576 = 6.3456

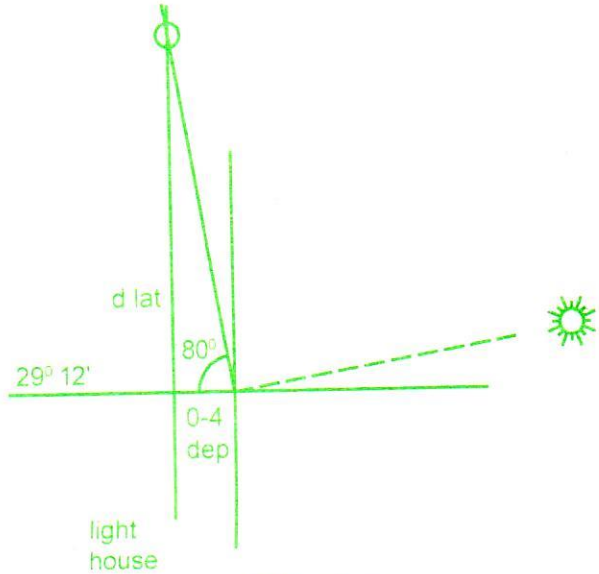
By traverse tables, with course N 67°°W and distance 6.346 miles.

d'lat = 2.48' N and dep =			5.84' W
		d'long	6.2 W
Lat	20°10.00' S	Long	112⁰ 04.0' W
d'lat	2.48 N	d'long	06.2' W
lat	20° 7.52 S	Long	112° 10.2 W
True position	Lat		20° 7.52' S
		Long	112° 10.2' W

Example

3. A vessel in DR latitude 29° 12' S, obtained a Sun sight which when worked longitude by chronometer method using the DR latitude gave a certain longitude, the Sun bearing 080° (T). At the same instant, a light house bearing 180° (T) put the vessel 0.4 miles to the west. Find the error in the latitude. (Fig. 11.61)

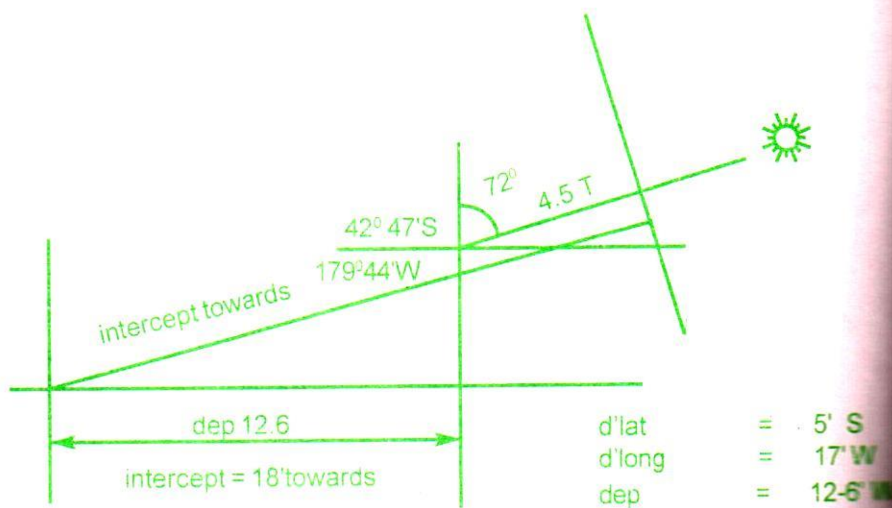
d'lat	=	dep. tan 80°
	=	.4 tan 80°
	=	2.268'
Error in latitude	=	2.268'



(FIG. 11.61)

Example

4. In DR 42° 47' S, 179° 44' W, an observation of the Sun bearing 072° (T) gave intercept 4.5 miles towards. By plotting, find the intercept which would have been obtained if the DR position used was 42° 52' S, 179° 59' E.



(FIG. 11.62)

d'lat = 5' S
d'long = 17' W
dep = 12.6' W
Intercept 18M (Towards)

Example

5. In correcting the altitude, when working a celestial observation, in... cept method, an index error of 2.5' on the arc was allowed as 2.5'...

the arc by mistake. If the calculated intercept was 3 miles towards, find the true intercept.

Since the 2.5' on the arc error was allowed as 2.5' off the arc, the total error in the true zenith distance and therefore the intercept is 5 miles. As the actual error was on the arc, the true altitude will be lesser and therefore the true zenith distance greater. The PL should therefore be drawn through a point 5 miles further away from the direction to the body. The true intercept will therefore be 2' away.True intercept = 2 miles away.

Example

6. A celestial observation when worked longitude by chronometer method, using DR latitude 31° 12'N, gave longitude 62°05' W, azimuth 235° (T). It was then found that the chronometer error of 28 seconds slow had not been applied. Find :-

   (a) The longitude which would have been obtained if the error had been correctly applied.

   (b) If the observation had been worked intercept method, what would be the amount of correction to be applied to the worked out intercept and in what direction is this to be applied ?

   a) Since the chronometer error was 28 seconds slow, the actual time 28 seconds larger. The GHA is therefore 28 x .25 = 7' larger.

   The longitude should be 7' westward of the worked out longitude.

   True longitude = 62° 05' W + 7' W = 62° 12' W

   b) Error in intercept = error in long sin az. cos lat.
   = 7' sin 55° cos 31° 12'
   = 4.9'

   Since the true longitude is westward, the PL has shifted westwards. As the bearing of the body is also west wards, the PL has shifted towards the body.
   Corrn. to intercept = 4.9 miles towards

Example

7. An observer in DR 24°12' N, 65°15' E, obtained an intercept of 3 miles towards, with an azimuth of 142° (T), it was then found that the

index error of 2' on the arc had been allowed as 2' off the arc and a chronometer error of 18 seconds fast was also applied the wrong way. Find the true intercept. (FIG. 11.63)

Allowing for the correct application of index error alone, the PL would shift 4 miles away from the direction to the body.

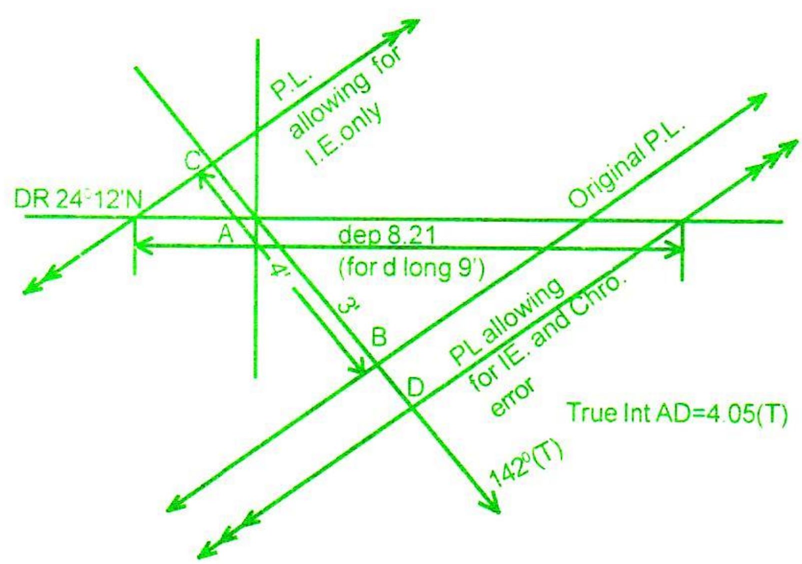
The chronometer error was taken as 18 seconds slow and was therefore added instead of being subtracted. The actual time is therefore 36 seconds lesser, causing an error in longitude of
36 x .25 = 9' to the eastward.

This will cause an error in the intercept = 9' x sin azimuth cos lat
= 9 sin 38° cos 24°12'
= 5.05 miles eastwards i.e. towards the body

True intercept = 3 miles towards (-) 4 miles away
(+) 5.05 towards

True intercept = 4.05 towards

Such problems can also be solved graphically as shown in fig. 11.63



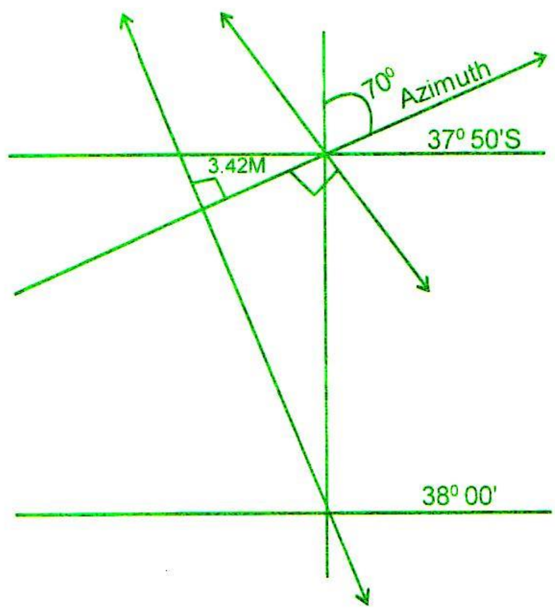
(FIG. 11.63)

Example

8. In DR 37°50'S, 76°00' W, a sight of the Sun bearing 070° (T), gave intercept 2.5 miles away. It was then found that a chronometer error

of 30 seconds fast had not been applied. Find the true intercept if the error had been correctly applied and the sight was worked using DR latitude 38°°S instead of 37° 50' S.

Allowing for chronometer error having been correctly applied the actual time would be 30 seconds lesser causing the GHA to be 30 x .25 = 7.5 lesser and therefore the longitude to be 7.5' to the east wards. This would cause an error in the intercept of
7.5' . sin 70° . cos 37°50'
= 5.57' eastwards i.e. towards the body
Error in intercept due to error in lat.
= error in lat. cos az
= 10' . cos 70°
= 3.42' (Towards)
True intercept = 2.5 (A) - 5.57' (T) - 3.42 (T)
True intercept = 6.49 (T)



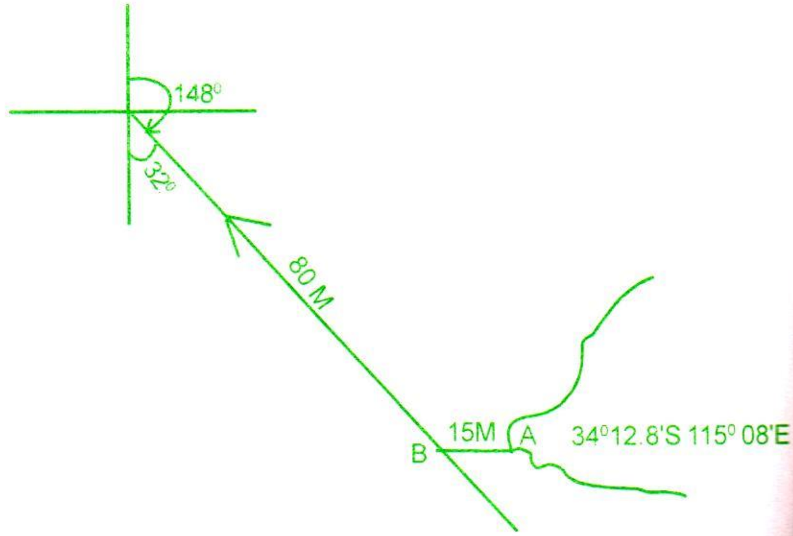
(FIG. 11.64)

Note

It is suggested that the student may also solve the above problem graphically to gain practice in graphical solution of such problems.

Example

9. In DR latitude 33° 05' S, a Sun sight worked longitude by chronometer method gave longitude 113°53' E, the chronometer error used was 13m slow. The vessel then steered 148° (T), 80 miles, when a point of land in latitude 34° 12.8'S, 115° 08' E bore 090° (T), 15 miles off. Find the actual error of the chronometer. (Refer fig. 11.65)



(FIG. 11.65)

Since the chronometer error of 13 m slow was used and actual chronometer error has been asked, the error used must not have been quite correct.

As the bearing and distance of the point of land, and further course and dist. run is given, we can work backwards from the point of land and obtain the correct longitude when the sight was taken.

Course	dist	d'lat	dep
270°	15	0.0	15'W
328°	80	67.8' N	42.4'W
		67.8' N	57.4'W
		= 1°7.8'N	

For dep 57.4'W d'long in mean lat .33° 38.9' = 1°09'W
lat when sight was taken = 34°12.8'S-1°7.8'
= 33° 05'.S.

Since the DR latitude used and latitude now obtained is same, there is no error in latitude.

To find correct longitude at the time of sight
Long of pt of land 115° 08' E
d'long 1° 09' W
Correct long at observation
113° 59' E
Long obtained using wrong chronometer 113° 53'E
diff 6' of arc = 24 seconds of time.
Since LHAS - GHAS gives longitude East and as the longitude ob-

tained by this relation was 113° 53', instead of 113° 59' the value of GHA used was 6' too large. This in turn means that the value of GMT used was 24 seconds too large.

Chronometer error used was 13 m slow. By adding 13m 00s to the chronometer time, we obtained a GMT which was 24s too large. The error added should have been lesser by 24 seconds.

Error used	13m	00s	(slow)
difference		24s	
Actual chronometer error	12m	36s	(slow)

EXERCISE XI

1. A Sun sight when worked with DR lat 41° 52' N, gave longitude 10°15' W and when worked with DR latitude 42°02' N, gave longitude 9° 55' W, find the true bearing of the Sun.

2. A vessel in DR latitude 25° 34' N, obtained a Sun's sight which was worked longitude by chronometer method, giving azimuth 113° (T) longitude 115° 15' E. The vessel then steered 220° (T) at 16 knots for 3 hours, when a latitude of 25°02' N was obtained by a meridian altitude of the Sun. Find by plotting the position of the Ship at apparent noon.

3. By plotting the PL's obtained from observations of two celestial bodies, one bearing 250° (T), and the other bearing 140° (T), the position obtained was 30°50' S, 45° 07' E. It was then found that the sextant had an Index error of 2' on the arc. Find the true position of the ship.

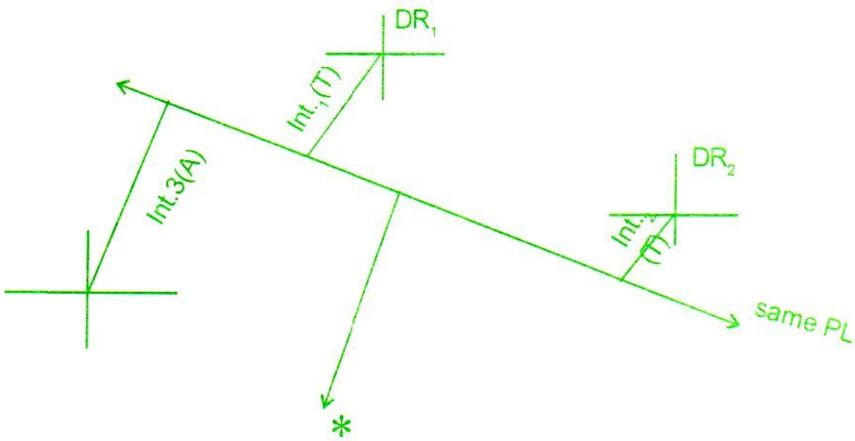
4. A vessel in DR latitude 23°54' N, worked a Sun sight using longitude by chronometer method and obtained a longitude of 74°12'E. The chronometer error was taken as 11m 07s fast. The ship then steered 070° (T), 35 miles, when a light vessel in latitude 24° 01.5' N longitude 73°24' E bore 138° (T), 6 miles off. Find the actual error on the chronometer.

5. A vessel in DR latitude 45°N, obtained a certain longitude from a Sun sight worked longitude by chronometer method. The azimuth of the Sun was 105° (T). At the same time a point of land bearing 180° (T) put the vessel 3.5 miles further to the west. Find the error in the latitude.

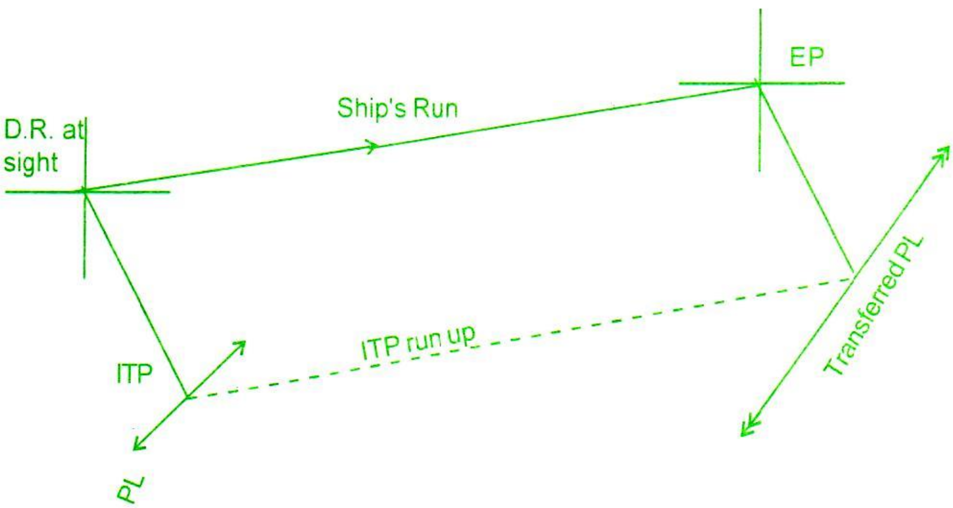
6. A sight of a star bearing 142° (T), worked using DR 50° N, 08°W gave an intercept of 2 miles away. The vessel then steered 052° (T). How far will she pass a light house in latitude 50° 16.5' N, longitude 7° 11' W.

Position lines

In working out the intercept, from a celestial sight, it should be understood that the position line obtained would be in the same geographical location irrespective of the D.R. used for working the sight.Obviously , the DR used will be in the vicinity of the position line.



The PL obtained is valid for the time of observation. If the PL is required for another time, it may be transferred by allowing the run of the ship for the interval between the time of observation and the time at which the PL is required. If the PL is required for a time later than the time of observation, The PL will be run up in the direction of the ships course. If the PL is required for a time earlier than the time of observation, the PL would shift backwards by the amount of the ships run. If the run is large, it would be adviseable to calculate D'lat and D'long for the run because accuracy would be lost in plotting the run as the scale used will be very small. If the runs are not large, the entire work may be done on a plotting sheet. The PL may be shifted by allowing the run from the I.T.P. It may also be shifted by allowing the run from the D.R. in which case, the intercept must be plotted from the EP at that time.

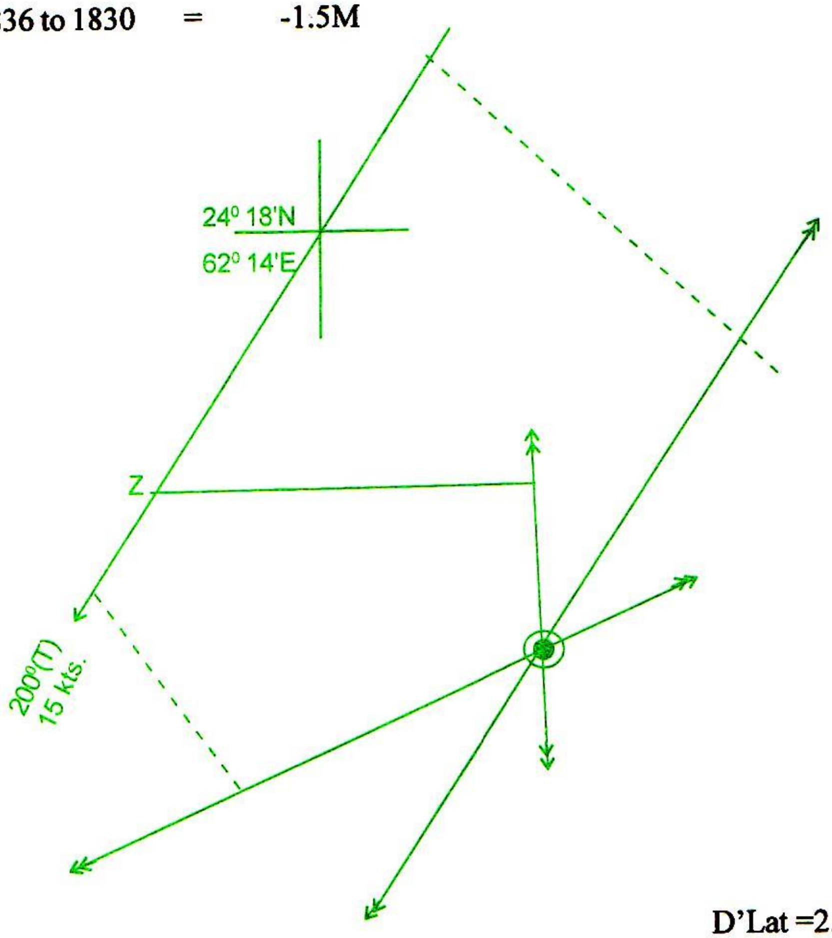


Ex. 1. A ship steering 200° (T) at 15 kts. obtained the following intercepts :-

	AZ.	Intercept
1820	340°	1.2' Away.
1824	086°	1.8' Towards.
1836	133°	3.0' Towards

All sights were worked using DR 24°18N, 62°14'E
Fidd the observed position at 1830 hrs.

Run from 1820 to 1830 = 2.5 M
from 1824 to 1830 = 1.5 M
from 1836 to 1830 = -1.5M



D'Lat =2.4'S
Dep = 1.3 M E
D'Long = 1.4'E

D.R: 24°18'N 62014'E
<u>2.4'S 1.4'E</u>
Obs. Posn: 24'15.6'N 62o15.4'E

Cocked Hat

Terrestrial Bearings :- If unknown errors of different signs on magnitudes exists on the bearings obtained, there is no means of obtaining a correct fix. However, if the unknown error is of the same sign and magnitude on two bearings, the ship's position will lie on a circle passing through the two objects and the ship's fix. We may therefore obtain a position circle using the horizontal angle between the two objects, since the horizontal angle between them is unaffected by the constant error on both bearings. If two such position circles are obtained by the horizontal angles between three objects, we obtain the ships fix as point-of intersection between the two position circles.

Celestial position lines:- When three position lines are plotted, they may form a cocked hat. If there are unknown errors of different signs or magnitudes on the three position lines the correct fix can not be obtained. If however the errors on all three position lines are equal and of the same sign (e.g. an index error on the sextant), the fix may be obtained by construction.

i) If the bearings of the three bodies are not contained within 180°, draw the bisectors of the three internal angles of the cocked hat formed by the erroneous position lines. The bisectors will meet at a point within the cocked hat. This point is the ship's fix. It will be seen that the fix is displaced equally from all three erroneous position lines Fig a.

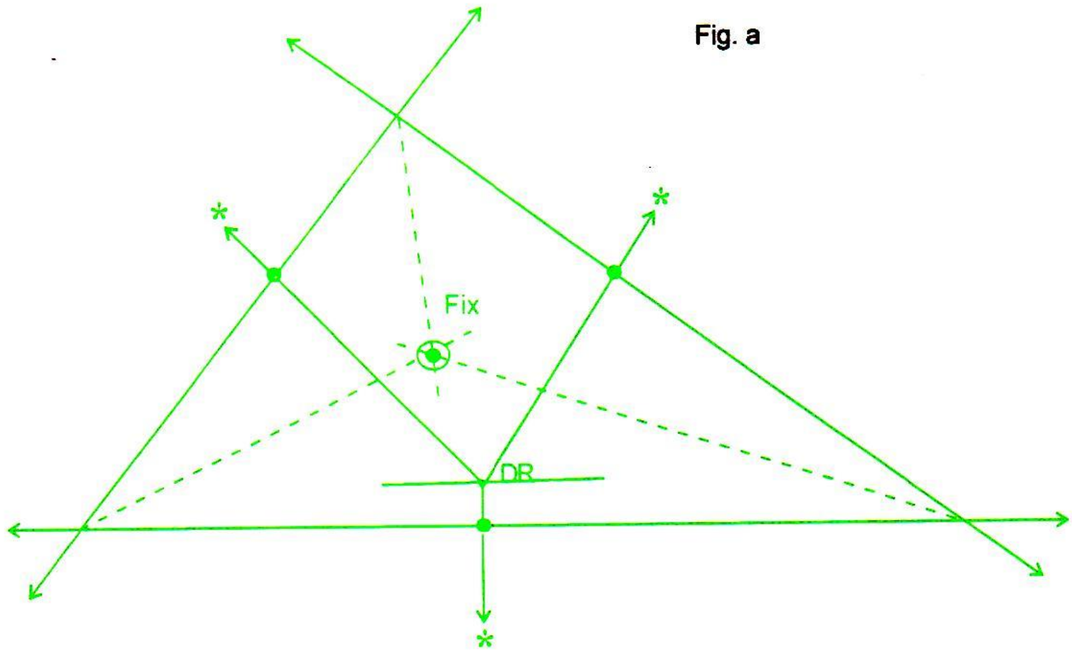


Fig. a

ii) If the bearings of the three stars observed are contained within 180°, The construction to obtain the fix is slightly different. In this case, the two external angles associated with the PL of the star with the middle bearing are bisected and the internal angle between the PLs of the stars with the outer bearings is bisected. The fix lies outside the cocked hat formed by the erroneous position lines. Again it will be seen that the fix is displaced equally from all three erroneous position lines, and in the same sense – away in all cases –Fig b.

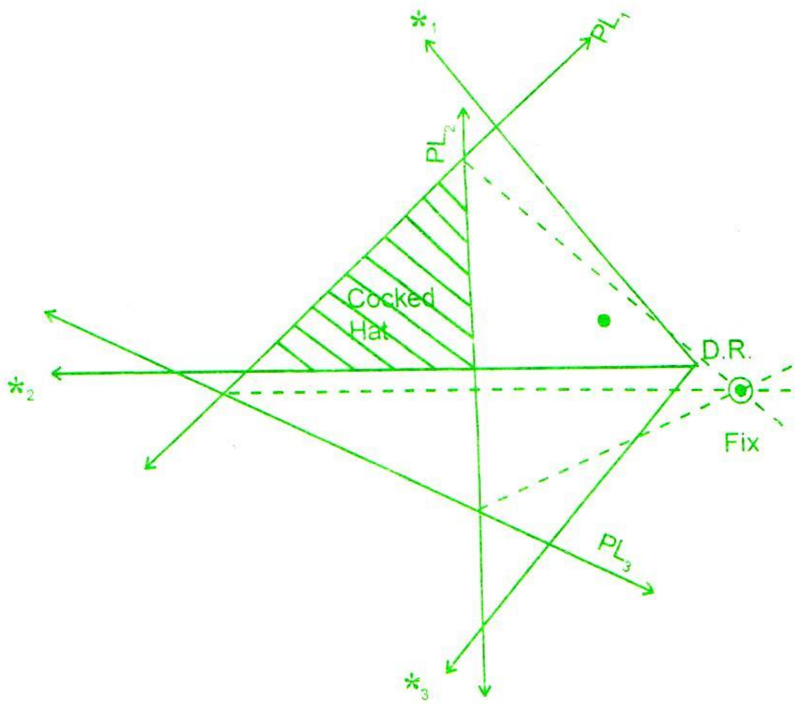


Fig b

Exercise

1. In D.R. lat 21° 01'S 179°00'W, a sun sight gave an intercept of 7M towards, Azimuth 038° (T). The ship steered 252° (T), 38 M. when the latitude obtained by meridian attitude was 21° 04'S. Find her noon position.

Ans : 21° 04'S 179° 38.5'W

2. A ship steering 132° at 25 kts. Obtained the following intercepts, all of them worked with D.R. 47° 38'N 30°17'W.

Time	1838	Az. 258°	Int. 6.7 M	Towards
	1843	Az. 141°	Int.2.4 M	Away
	1855	Az.027°	Int. 4.9 M	Away

Find her position at1850.

Ans. 47° 34.8'N 30° 21.8'W

3. The following results were obtained from simultaneous celestial observations worked using D.R. 15°45'S 64°10'E. Fuid the ships position and the index error of the sextant.

i)	Az: 023° (T)	Int.	6.4 M	Towards
ii)	147° (T)		3.0 M	Away
iii)	244° (T)		1.4 M	Towards

Ans: 15° 39.8'S 64°08.3'E; IE 2.2' on the arc.

4. Using D.R. 53° 40'N, 28°05'W; the following results were obtained from celestial observations.

i)	Az. 120° (T)	Int. 7.3' Towards
ii)	Az. 168° (T)	Int. 3.7' Towards
iii)	Az.212° (T)	Int. 1.7' Towards

If the dip correction was not applied to any of the observations, find the position of the slup and the approx. HE of the observer.

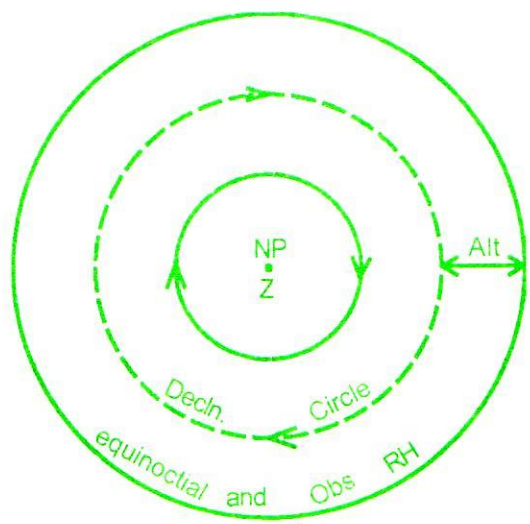
Ans : 53° 42.7'N 27° 59.5'W

HE 10.3 to 10.6 m

12

RISING-SETTING OF CELESTIAL BODIES AND TWILIGHT

As the Earth rotates on its axis from west to east, all heavenly bodies appear to describe an east to west motion around the Earth each day. They appear to move along circular paths, around the Celestial poles. Thus a heav
enly body appears to rise in the east, move westwards, gaining in altitude until it is on the observer's meridian. It is then said to culminate or transit the meridian. After culmination, it continues to move westwards decreasing in altitude till it sets over the western horizon. For a stationary observer, the interval between rising and culmination of a body will be equal to the interval between its culmination and setting, provided its declination remains unchanged. Also under the same circumstances, its amplitude at rising will be equal to that at setting.



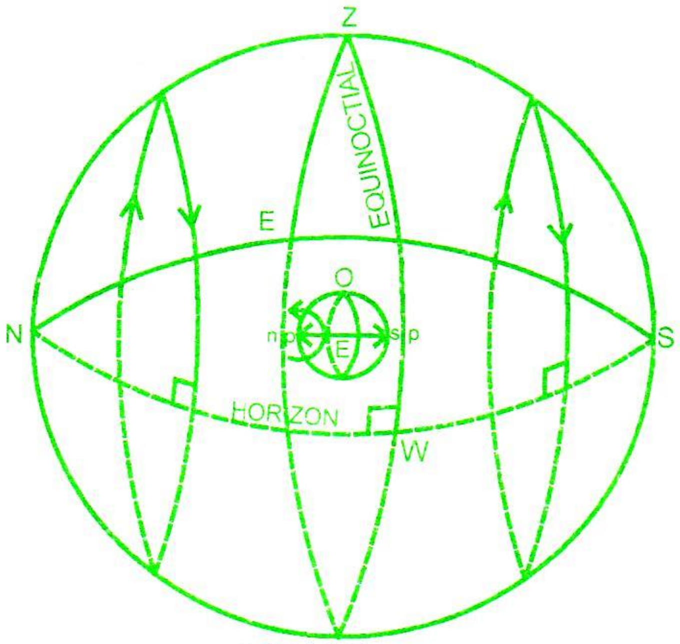
(FIG.12.1)

Consider an observer at the North Pole. His zenith would be coincident with the celestial North Pole, and his rational horizon would coincide with the Equinoctial. A celestial body with zero declination would appear to the observer to move along his rational horizon completing a circle in exactly the same period as the Earth completes a rotation of 360° i.e. 23h 56m 04s of Mean solar time. Celestial bodies with north declination would also appear to move along a circle maintaining constant altitudes equal to their declinations. They would remain above the horizon at all times. Bodies with south declinations would always remain below the horizon and would not therefore be visible.

As the Earth rotates from W to E the celestial bodies appear to move E to W with constant altitudes.

To an observer on the Equator, the rational horizon would be in the plane of the Earth's axis. The Equinoctial and all declination circles will be bisected at right angles, by his rational horizon. All celestial bodies whether having northerly, southerly or zero declination will therefore remain above the horizon for exactly half the day and below the horizon for the remaining half.

All bodies will rise and set perpendicular to the horizon.



(FIG.12.2)

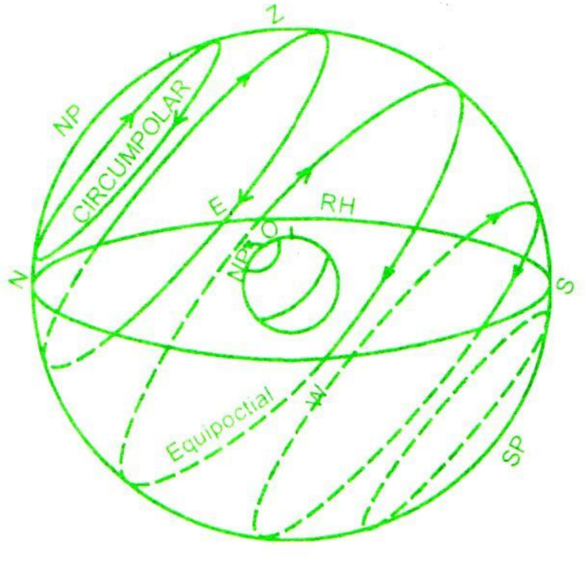
For an observer in an intermediate north latitude, the north celestial pole would be between his zenith and his rational horizon. The rational horizon will bisect the Equinoctial at his east and west points. A celestial body with zero declination would therefore be above the horizon for exactly half the day and below the horizon for the other half.

The altitude of the celestial pole is equal to the latitude of the observer. As the observer's latitude increases, the elevated pole therefore approaches his zenith. As will be appreciated from fig. 12.3, the angle at which the Equinoctial intersects his rational horizon will then reduce.

Therefore a major part of the declination circles of bodies with northerly declinations (of the same name as the observer's latitude) would lie above the horizon and a smaller arc below it. Bodies with northerly declinations would therefore remain above the horizon for a greater part of the day. They would rise and set bearing northwards of his east and west points respectively.

If the northerly declination of the body is large enough, its declination circle would lie entirely above the horizon. Such bodies would not therefore rise or set, but would remain above the horizon throughout the day. They are then said to be circumpolar. Explanations and problems on circumpolar bodies have been already provided earlier in this book.

Declination circles of bodies with a southerly declination (of the opposite name to the observer's latitude) will lie with a major arc of the circle below the horizon and a minor arc above. Such bodies would therefore remain above the horizon, for a smaller part of the day only. They would appear to rise and set bearing southwards of the observer's east and west points respectively. If the southerly declination was large enough, the declination circle would lie entirely below the horizon and the body would then not be visible during any part of the day.



(FIG.12.3)

As the observer's latitude increases, his celestial horizon approaches the Equinoctial. Declination circles being parallel to the Equinoctial, a greater number of declination circles then lie entirely above the horizon, that is, more bodies become circumpolar.

The arcs of declination circles, lying above the horizon will increase as the declination increases in the case of bodies having declination of the same name as the observers latitude. Such bodies then remain above the horizon for larger periods. In the case of bodies with declination of the opposite name to the observers latitude, the arcs of the declination circles above the horizon reduces as their declination increases, causing these bodies to remain above the horizon for reduced periods of time. Thus, the period of time, a body remains above the horizon during a day, depends on the observer's latitude, as well as the body's declination.

Let us now consider the Sun. When the Sun is above the horizon, we have 'Day' and when it is below the horizon, we have 'Night'. The declination of the Sun varies from 23½°N to 23½°S. As has been explained above, when the Sun has a northerly declination, it will remain above the horizon for more than 12 hours for observers in north latitudes, and for less than 12 hours for observers in south latitudes. Thus the Northern hemisphere would have longer days and shorter nights, while the soultern hemisphere will have shorter days and longer nights. As explained earlier, if the observer's latitude and the Sun's declination are of the same name and if the latitude + Sun's decln ≥ 90, the Sun would be circumpolar. Observers in such latitudes would have continuous day light and no night, as the Sun would never set. When the Sun is at its maximum declination north, observers in latitudes above 66½°N would experience this phenomenon known as the 'Midnight Sun'. At this time, observer in latitudes above 66½°S would have continuous night and no day, as the Sun would always remain below the observer's horizon.

Converse would be the case when the Sun has its maximum southerly declination. When the Sun's declination is zero degree its apparent diurnal path is along the Equinoctial. Since the rational horizon of an observer in any latitude, bisects the Equinoctial, the Sun would then remain above the horizon for 12 hours and below the horizon also for 12 hours, for observers all over the Earth. Thus the Sun would rise at 6 a.m. and set at 6 p.m. local apparent times all over the Earth.

12.1 TWILIGHT

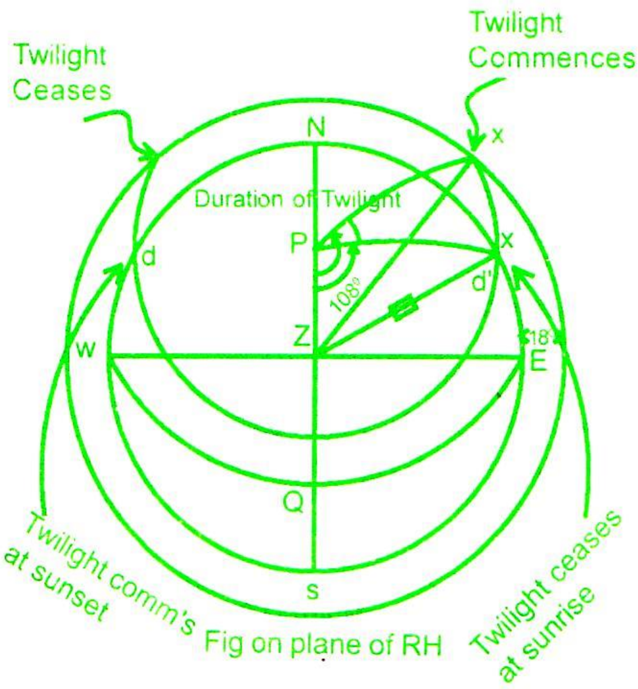
Twilight is the light received from the Sun, when the Sun is below the horizon, that is before sunrise in the morning and after sunset in the evening. Though the Sun is below the horizon, it illuminates the upper layers of the atmosphere. A part of this light is reflected and scattered in various directions. This scattered light illuminates the Earth's surface for some time, before sunrise and after sunset. Twilight completely ceases in the evening, when the Sun is 18° vertically below the horizon. After that there is total darkness. In the mornings, twilight commences when the Sun is 18° vertically below the horizon and ceases at sunrise. The entire period of twilight is divided into three stages, Civil, Nautical and Astronomical.

In the mornings, Astronomical twilight commences when the Sun's centre is 18° below the rational horizon, Nautical twilight commences when it is 12° below the rational horizon, and Civil twilight commences when it is 6° below the rational horizon.

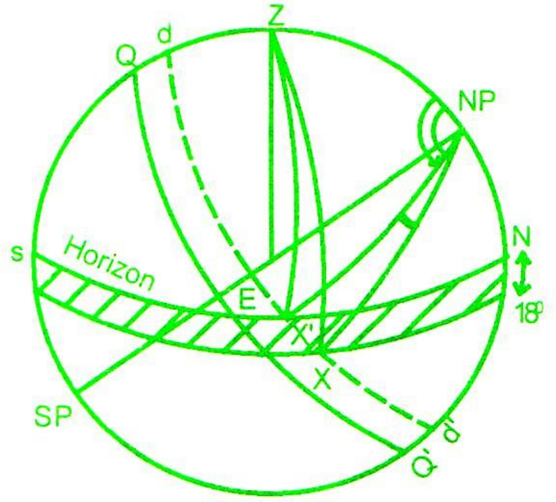
Each of them lasts until visible sunrise i.e. when the Sun's upper limb appears over the visible horizon.

In the evening, they all commence at visible sunset i.e. when the Sun's upper limb disappears over the visible horizon. Civil twilight, continues till the Sun's centre is 6° below the rational horizon, Nautical twilight till it is 12° below the rational horizon and Astronomical twilight till it is 18° below the rational horizon. During the period of Civil twilight, the horizon is very clearly visible and the sky is fairly bright. Therefore stars are not visible for stellar observation. When the Sun is between 6° and 12° below the horizon, the sky is dark enough for the bright stars to be seen and the horizon is clear enough for stellar observations. Star sights are therefore best obtained during this period. When the Sun is between 12° and 18° below the horizon, most stars are visible but the horizon is too dark for celestial observations.

Tables are provided in the Nautical almanac, listing the times of commencement of Nautical and Civil twilights in the mornings as well as the end of the Civil and Nautical twilights in the evening for various latitudes.

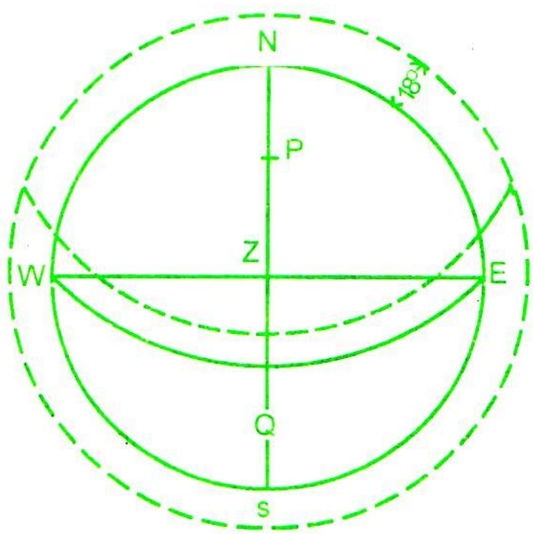


(FIG.12.4)

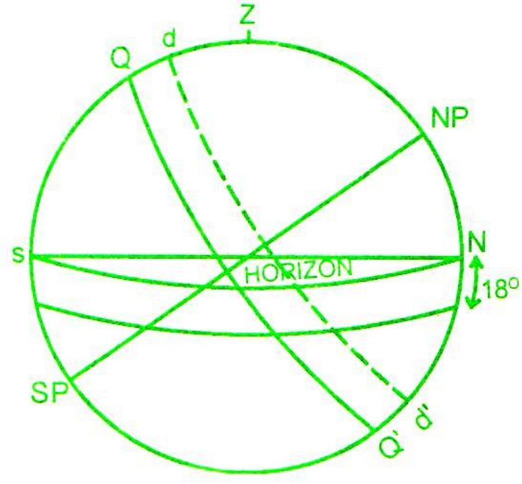


(FIG.12.5)

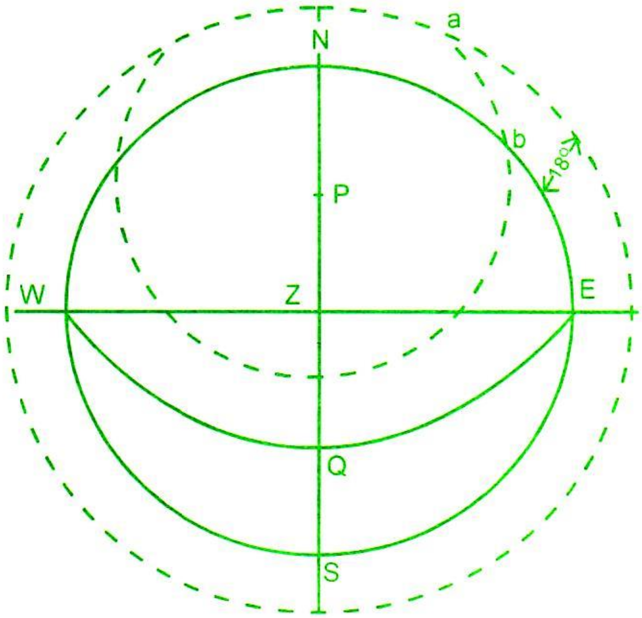
While on his sea voyages, the reader would have noticed that in lower latitudes the duration of twilight is shorter than in higher latitudes.



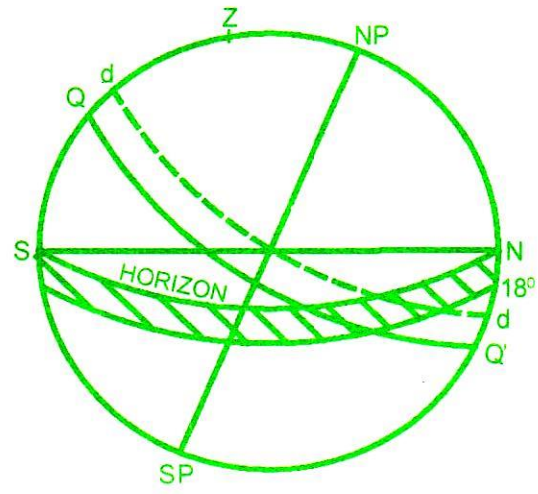
(FIG.12.6)



(FIG.12.7)



(FIG.12.8)



(FIG.12.9)

As is evident from the above figures, when the observer is in a low latitude (figs.12.6 and 12.7), the Sun rises and sets almost perpendicular to the horizon, covering the 18° twilight belt in a rather short arc and therefore in a rather short period of time. When the observer is in a high latitude however (figs.12.8 and 12.9) the Sun rises and sets at a much more oblique angle to the horizon, thus covering the 18° twilight belt over a much larger arc and therefore over a much larger period of time.

This explains why twilights lasts longer in higher latitudes, than in lower latitudes.

Twilights is possible only when the Sun is below the horizon. For an observer to have a twilight therefore, he must have some night. If not, the Sun would be continually above the horizon and he would have continuous day light and no twilight.

As explained earlier, an observer would have night for some part of the 24 hours, either
(1) if the observers latitude and the Sun's declination are of opposite names or
(2) if, they are of the same names and the sum of the latitude and declination is less than 90°.

For an observer to have continuous twilight throughout the night, the Sun must set. Also it must never go below the 18° twilight belt. This can only happen if the observer's latitude and the Sun's declination are of the same name and only, provided the sum of the latitude, declination and 18° is equal to or greater than 90°.

Thus for continuous twilight, throughout the night, the observer's latitude and the Sun's declination should be of the same name and the limiting latitudes are obtained as :-

(i) lat. + dec $\leq 90°$ - so that the Sun will set
(ii) lat. + decl. + $18° \geq 90°$ - so that the Sun will not go below the twilight belt.

12.2 THEORETICAL SUNRISE AND SUNSET

Theoretical sunrise and sunset occurs when the True Sun's centre is on the observers rational horizon. The true altitude of the Sun is then 0° and the true zenith distance 90°. The times of theoretical sunrise or sunset, can be obtained by solving the PZX triangle in which ZX is 90°.

It should be appreciated that, at visible sunrise and sunset the true altitude is not 0°, because of corrections for refraction, semi-diameter, dip. etc. This aspect has been illustrated in the problems on 'altitude corrections' which may, if necessary, be referred to.

Assuming the observer to be at the sea level, the true altitude of the Sun at visible sunrise and sunset is about 0°50', the true zenith distance then is therefore 90°50'. Because of this visible sunrise occurs before theoretical sunrise, and visible sun set after theoretical sunset. The nautical almanac lists the times of visible sunrise and sunset for various latitudes. Interpolation is necessary for latitude of the ship. Though the times given are strictly Greenwich, Mean time of the occurances on the Greenwich meridian for the middle day, they may be taken as the LMT of the occurance in any longitude for any of the three days on the page without appreciable error, particularly in low latitudes.

To find precise times of these phenomenon, interpolation for longitude and for the day (other than for the middle day on the page) would also be required.

Example : Find the approximate GMT of sunrise in latitude 47°12'S longitude 56° E on 14th Oct 1976.

In latitude	45°S	LMT sunrise	05h	10m
In latitude	50°S	LMT sunrise	05h	03m
In latitude	47°12'S	LMT sunrise	05h	07m
LIT(E)			03h	44m
Approx GMT of sunrise			01h	23m

12.3 MOONRISE AND MOONSET

At visible moonrise and moonset, the true altitude of the Moon is approximately 0°07' for an observer at sea level allowing 34' for refraction, 16' for semi-diameter and 57' for parallax (-34' - 16' + 57' = 07'). Thus in the case of the Moon, visible and theoretical rising occur at about the same time. So also the visible and theoretical moonset.

The GMT of moonrise and moonset on the Greenwich meridian is tabulated for each day in the nautical almanac, for various latitudes. The times of these phenomenon for the first day on the following page is also tabulated to help in interpolation. When moonrise or moonset does not occur on a particular date (about once every month), the time of the occurrence on the next day is tabulated with 24 hours added e.g. on 14th October, 1976 there is no moonrise in latitude 45°S, however the moonrise time is tabulated as 24h 10m which really indicates 15th October 00h 10m, as tabulated for the 15th.

To obtain LMT moonrise or moonset at any position, apart from interpolation for latitude, a correction for the observer's longitude has also to be applied to the tabulated times because of the large change in the times of moonrise or moonset on successive days.

To find precise times of moonrise or moonset, first interpolate for latitude for the day in question and also for the preceeding day, if in East longitude, and for the following day if in west longitude. The difference between the two times so obtained multiplied by the observer's longitude and divided by 360°, gives the correction for longitude to be **applied to the interpolated time for the day in question.** The correction is to be applied so that the resulting time obtained **lies between the two times used.**

Generally the longitude correction is to be subtracted for East longitudes and added for West longitudes. This rule may not hold good particularly in high latitudes and near spring and autumnal equinoxes, when moonrise and moonset times on succeeding days may become earlier.

Interpolation for latitude and the correction for longitude can also be obtained from table I and II, provided for the purpose, in the nautical almanac.

Examples

1. Required the time of moonrise in latitude 24°N, longitude 70°E on 13th October, 1976.

	12th	13th
30°N	20h 24m	21h 11m
20°N	20h 42m	21h 29m

Interpolating for latitude

LMT moonrise on 12th	:	20h 34.8m
LMT moonrise on 13th	:	21h 21.8m
Difference	:	47.0m

Correction = Difference in times x long. / 360^0
= 47 x 70 / 360° = 9.1m

The required LMT must be between 20h 34.8m and 21h 21.8m and the correction is to be applied to the time of the day in question.

21h	21.8m
	9.1m
21h	12.7m

∴ LMT moonrise in lat. 24°N, long. 70°E = 13d 21h 12.7m

2. Required the time of moonrise on 13th October, 1976 in latitude 32°S, longitude 110°W.

	13th	14th
30°S	22h 47m	23h 35m
35°S	22h 58m	23h 45m

Interpolating for latitude,

LMT	moonrise on 13th	22	51.4
LMT	moonrise on 14th	23	39.0

Correction = Difference in times x long. / 360
= 47.6 x 110 / 360° = 14.5m

LMT moonrise = 13th 23h 05.9m

3. Required the time of moon-rise on 13th October, 1976 in latitude 57°S, longitude 115°E.

	12th	13th	14th
56°S	2318	2408	0008
58°S	2329	2420	0020

It will be noticed that the tabulated times of moon-rise on 13th are 2408 and 2420 respectively which indicates that on the 13th, moon-rise does not occur over Greenwich meridian. The time tabulated as 2408 and 2420 on the 13th really indicates 0008 and 0020 on the 14th. We have to therefore interpolate between 12th and 14th.

Interpolating for latitudes
LMT moon-rise 12th 23h 23.5m
LMT moon-rise 14th 00h 14.0m

$$\text{Correction} = \frac{\text{Diff. in time . long.}}{360^0} = \frac{50.5 \times 115}{360^0} = 16.1\text{m}$$

Date in question 13th	: 13d	24h	14.0m
	(-)		16.1m
LMT moon-rise	13d	23h	57.9m

Problems on moon-set are also worked in exactly similar manner, as the moon-rise problem shown above.

4. On 18th March, 1976, find the time of moonset in latitude 70°N, longitude 160°E, given the moonset times tabulated in the almanac as follows :-

For 70°N 17th 053h, 18th 0532

Corrn.	= time diff. x long. / 360 = 5 x 160 / 360° = 2.2m		
Date in question		18th	05 32
Correction			2.2
			05 34.2

LMT moonset = 18d 05h 34.3m

Note

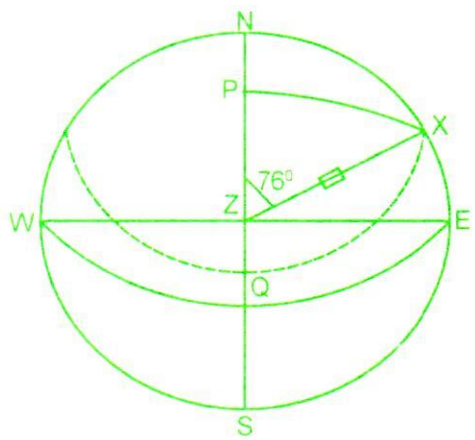
The correction is additive even though the longitude is East, because moonset has occurred earlier on the 18th than on the 17th.

The reader may solve more problems on this topic from any standard text book on Practical Navigation.

Problems on Rising, Setting and Twilight.

1. To an observer in a certain latitude, the Sun (Declination 12°14'N), bore 076° (T) at theoretical rising. Required the observer's latitude.

decl. 12°14' N
PX 77°46'
sin (90-PX) = cos Z.cos (90-PZ)
cos PX = cos Z.sin PZ
sin PZ = cos PX.sec Z
= cos 77°46' x sec 76°
PZ = 61°08.9'
Latitude = 28°51.1' N or S

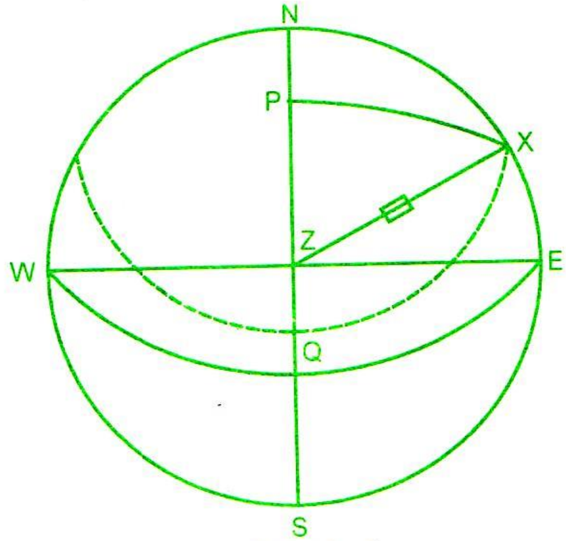


(FIG. 12.10)

2. In latitude 37°38'N, at theoretical sunrise, the Sun had a declination of 22°01'N and GHA 112°13'. Required the observer's longitude.

- sin (90 - P) = tan (90 - Pz) . tan (90 - PX)
- cos P = tan lat. x tan decl.

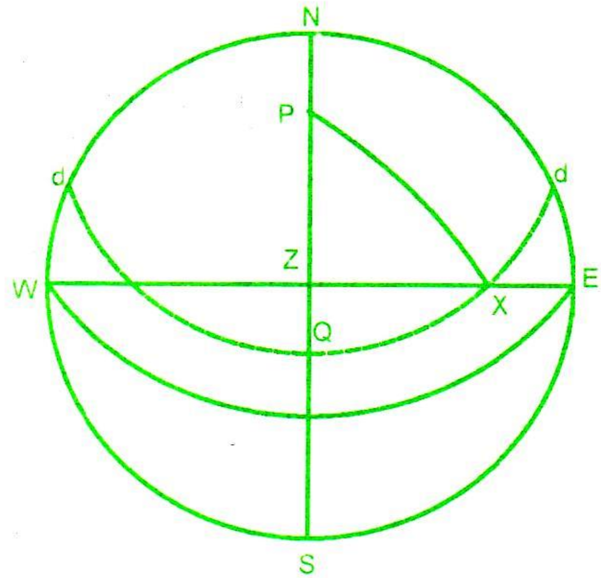
cos (180 - P)	=	tan L.tan D
	=	tan 37°38' x tan 22°01'
180 - P	=	71°50'
∴ P	=	108°10' (Easterly Hour Angle)
LHA Sun	=	251°50'
GHA Sun	=	112°13'
Long.	=	139°37' E



(FIG.12.11)

3. If the Sun's amplitude at Summer solstice was E31°N, to a stationary observer, find its altitude when on the prime vertical.

sin amplitude	=	sin decl. x sec lat.
sec lat.	=	sin amplitude x cosec decl.
	=	sin 31° x cosec 23°26.7'
lat.	=	39°25.2'
sin (90 - PX)	=	cos PZ.cos ZX
cos PX	=	cos PZ.cos ZX
cos ZX	=	cos PX.sec PZ
	=	cos 66°33.3' sec 50°34.8'
ZX	=	51°12.2'
T alt	=	90°- 51°12.2' = 38°47.8'



(FIG.12.12)

4. To an observer in the Northern hemisphere, in May of a certain year, the Sun bore 059°(T) at theoretical rising, Sun's declination 20°10'N. The vessel then steered 050°(T), 140 miles, till sunset, during which period the Sun's declination altered by 5'. Calculate the bearing of the Sun at theoretical sunset.

59° (T)	=	E31°N = ampl.
sin ampl.	=	sin decl. x sec lat.
sec lat.	=	sin ampl. x cosec decl.
sec lat.	=	sin 31° x cosec 20°10'
lat.	=	47°58.85'N
050° (T) x 140M		
d'lat	=	1°30'N
lat. at sunset	=	49°28.85'N
Decl. at sunrise	=	20°10'N
change in decl.	=	5'N (as in May, the Sun's Decl. is increasing northwards).
decl. at sunset	=	20°15'N
sin ampl.	=	sin decl. x sec lat. = sin 20°15' x sec 49°28.85'
amplitude	=	W32°11.4'N
bearing	=	302°11.4' (T)

5. In what latitude will the longest day be three times the shortest night ? (Refer fig. 12.13)

On the longest day the Sun's decl. is maximum and of the same name as observer's latitude. Also on the longest day, the observer has the shortest night. Therefore on that day, day : night : 3:1 i.e. 18 hours of day and 6 hours of night. The angle at P between X and X' = 18 hours

$\therefore$ ½ that angle (XPZ) = 9 hours = 135°

Solving PZX for PZ, the co-lat,

-sin (90 - P) = tan (90 - PZ) . tan (90 - PX)

-cos P = cot PZ cot PX

cot PZ = -cos P . tan PX

= -cos 135°. tan 66°33.3'

But -cos 135° = -[cos (180° - 45°)] = - [-cos45°]

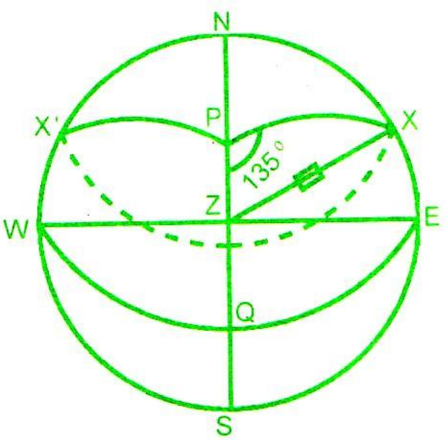
= cos45°

cot PZ = cos 45°.tan 66°33.3'

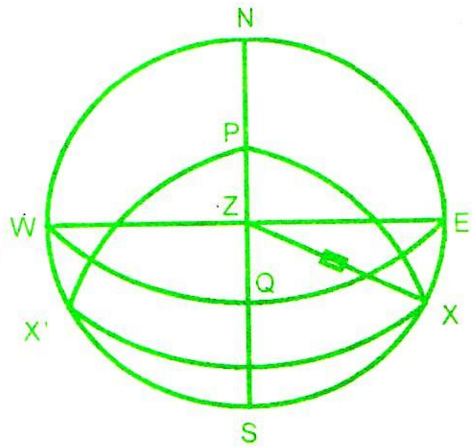
PZ = 31°31.2'

lat. = (90° - 31°31.2')

= 58°28.8'N or S



(FIG.12.13)



(FIG.12.14)

6. Required the latitude in which the period of darkness will be twice the period of day light, when the Sun's declination is 22°40'S.

Darkness : daylight : 2:1 Refer (FIG. 12.14)

i.e. 16 hrs. of darkness & 8 hrs.of day light in Δ PZX,

P = 4 Hrs. = 60°

-sin (90 - P) = tan (90 - PZ) . tan (90 - PX)

-cos P = cot PZ. cot PX

-cot PZ = cos P . tan PX = cos 60 x tan (90° + 22°40')

-cot PZ = cos 60° x -cot 22°40'

PZ = 39°52.2'

lat. = (90° - 39°52.2')

= 50°07. 8'N

7. To an observer in latitude 42°10'N a star of declination 20°17'N was on the observer's meridian at 02h 15m 00s LAT. At what LAT. will the star set ?

Refer (FIG. 12.15)

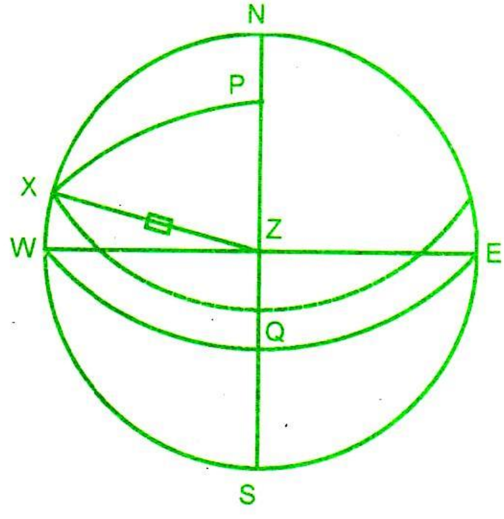
In the quadrantal Δ PZX
-cos P = cot PZ . cot PX
P = 109°33.4'

The first point of Aries and stars increase their GHA at the rate of 15°02.5' per hour, while in the case of the Sun, it is 15° per hour. Therefore the star's hour angle at setting cannot be converted to solar time interval by the normal method of dividing by 15°.

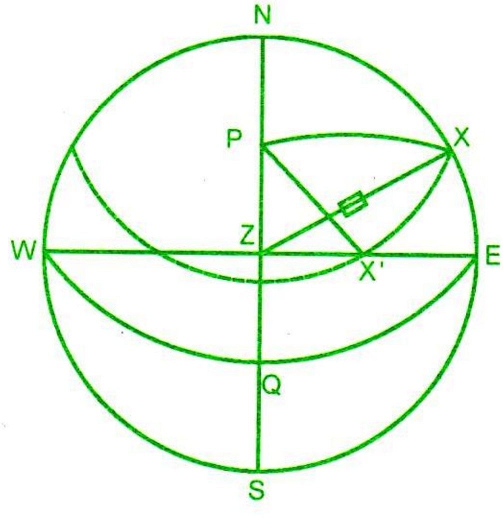
It should be done by dividing the hour angle by 15°02.5'

$\therefore$ time interval = 109°33.4' / 15°02.5' = 6573.4'/902.5'
= 7.2835 hrs.

	=	07h 17m 01s
Time of MP	=	02h 15m 00s
$\therefore$ Time of setting	=	9h 32m 01s



(FIG.12.15)



(FIG.12.16)

8. A star bore 065°(T) when rising. Its true altitude when bearing 090°(T) was 42°. Required the observer's latitude. (FIG. 12.16)

In the quadrantal PZX

cos PX = cos Z . sin PZ .... (i)

In the right angle PZX'

cos PX' = cos ZX'. cos PZ .... (ii)

Since PX = PX', therefore

(i) = (ii)

cos Z . sin PZ = cos ZX' . cos PZ

tan PZ = cos ZX'. sec Z

tan PZ = cos 48°x sec 65°

PZ = 57°43.4'

lat. = 32°16.6'N

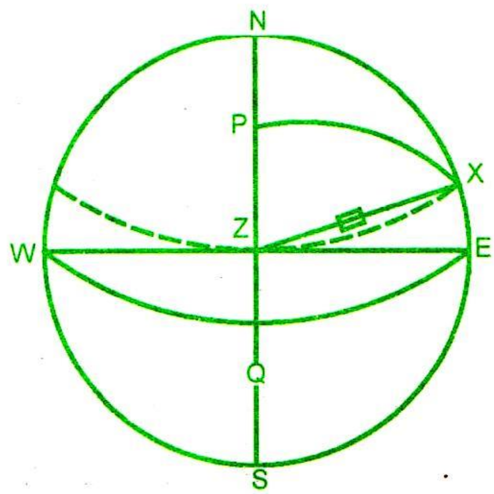
9. To a stationary observer, the Sun was at his zenith, 'h' hrs. after theoretical rising. Prove that -cos h = $\tan^2$. decln. (FIG. 12.17)

Since the Sun reached the observer's zenith QZ, the observer's latitude also equals the Sun's declination. In the quadrantal PZX sin (90 - P) = tan (90 - PZ) . tan (90 - PX)

-cos P = cot PZ. cot PX

-cos h = tan lat. tan decl.

-cos h = $\tan^2$ decl. (since lat. = decl.)



(FIG.12.17)

10. A vessel moored between two buoys found the compass bearing of the Sun at sunrise to be 104°(C), and that at sunset, 243°(C). If the variation at the place was 7°W, find the deviation of the compass.

Assuming the decl. of the Sun remained unchanged between rising and setting, the true Amplitude at rising should equal the true Amplitude at setting. Therefore the sum of the true rising bearing and true setting bearing is always equal to 360°. The mean of the two will therefore always be equal to 180°. The difference between 180° and the mean of the two compass brgs. will therefore give the error.

104°(C)	Error	6.5°	E
243°(C)	Var.	7.0°	W
347°(C)	Dev.	13.5°	E

Mean	173.5°(C)
True	180.0° (T)
Error	6.5° (E)

11. The Sun's declination being 20°S, calculate the latitude above which (a) there will be continuous day light (b) there will be continuous night.

For continuous day light, lat. and decl. are of same name and lat. + decl. ≥ 90°
∴ Lat. ≥ 90°- decl. = 90°- 20° = 70°S
For continuous night, lat. and decl. are of opposite names and lat. + decl. ≥ 90°
∴ Lat. ≥ 90°- decl. = 90°- 20° = 70°N

(a) Continuous day light in lat. 70°S or more
(b) Continuous night in lat. 70°N or more.

12. Find the latitudes within which an observer would have twilight throughout the night, when the Sun's decl. is 15°N.

To have night, (with lat. & decl. same name)
lat.+ decl. < 90° or lat. < 90° - decl. = 90°-15° = 75°N
Below 75°N and in all South latitudes, there will be night.
For twilight to last all night, lat. & decl. are of same name and lat. + decl. + 18° ≥ 90°
lat. ≥ 90°- decl.- 18° = 90°- 15°- 18° = 57°N
∴ In latitudes above 57°N, there will be twilight all night.
Therefore in all latitudes between 57°N and 75°N, twilight will be present throughout the night.

13. On 22nd December, find the latitudes within which twilight will last all night.

On 22nd December the Sun has maximum S'ly decl. of 23°30'.
For night, lat. ≤ 90°- 23°30' = 66°30'S
For continuous twilight, lat. ≥ 90°- 18°- 23°30' = 48°30'S
In all latitudes between 48°30'S and 66°30'S, twilight will last all night.

14. Calculate the limiting latitudes within which an observer would have nautical twilight throughout the night, when the Sun had a declination of 17°N.

For night, lat. < 90°- 17° = 73°N
For continuous nautical twilight,
lat. ≥ 90°- decl.- 12° = 90°- 17°- 12° = 61°N

Therefore in all latitudes between 61°N and 73°N nautical twilight will be present throughout the night.

15. If on the longest day the Sun's centre just touches the observer's rational horizon when on the meridian below the pole, find the observer's latitude.

On the longest day the Sun's declination is maximum i.e. 23°30'N or S.
If the Sun just touches the observer's rational horizon, it is the limiting cond-ition for continuous daylight, that is, lat. and decl. are of the same name and lat. + decl. exactly equals 90°.
Lat. = 90°- 23°30' = 66°30'N or S.

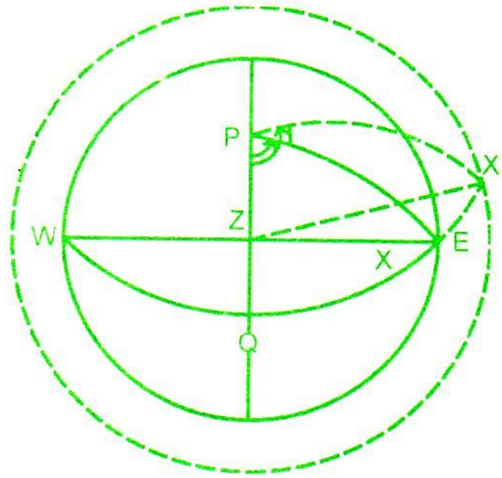
16. Calculate the duration of astronomical twilight in latitude 35°N on the day of spring equinox, assuming twilight ends in the morning and commences in the evening at theoretical sunrise and theoretical sunset respectively. (FIG. 12.18)

Astronomical twilight commences when the Sun's centre is 18° below the horizon i.e. when ZX' = 108°.

In the Δ PZX', ZX'	=	108°; PZ = 55°
PX' = polar distance	=	90°
sin (90 - ZX')	=	cos PX . cos (90 - PZ)
cos ZX'	=	cos P. sin PZ
cos 108°	=	cos P. sin 55°
cos P	=	cos 108° ÷ sin 55°
P	=	112°09.7'

In the Δ PZX by the sine rule, angle P will be 90° since ZX = 90°, PX = 90° and Z = 90°.

The duration of astronomical twilight is the angle XPX' i.e. 112°09.7'- 90°= 22°09.7', converted to time =1h 28m 39s



(FIG.12.18)

Note

If the problem was to be worked for a day when the Sun's declination is not zero, the triangle PZX could still be worked by Napiers rule, while PZX' would have to be worked using the haversine formula. If in addition, visible sunrise or visible sunset was used, instead of theoretical sunrise as in the above solution, both the triangles would be oblique and haversine formula would have to be used in both the cases.

EXERCISE XII

1. In what latitude would the longest day be 5 hours more than the shortest day ?
2. Required the declination of the Sun, if at theoretical rising it bore 080°(T) in latitude 12°N.
3. Required the LAT at the end of civil twilight in the evening, in latitude 20°S. Declination of the Sun 20°S.
4. At what LAT will astronomical twilight cease in the evening, in latitude 15°10'N, when the Sun's declination is 07°05'N.

5. If the Sun's declination is 15°S, in what latitudes will there be :

    (a) the phenomenon of the Midnight Sun
    (b) Twilight all night
    (c) Continuous night.

THEORY QUESTIONS

1. Explain the causes of variation in the length of day and night with change of latitude / Sun's declination.
2. Explain the difference between the theoretical and visible sunrise. When would you take an observation for an amplitude of the Sun ?
3. Define twilight. Explain clearly the cause of twilight and the reason why twilight lasts longer in higher latitudes.
4. Define the terms, civil, nautical & astronomical twilight.
5. Which is the best time for stellar observations ?
6. What conditions must be satisfied for twilight to last all night ?
7. When does the Moon set bearing 270° (T) ? What is the approximate true altitude of the Moon then, for an observer at sea level ?
8. If at theoretical sunrise, the LAT is 6 hours, what is the amplitude of the Sun ?
9. For an observer at sea level, would visible moonrise occur before or after theoretical moonrise ?

13

GREAT CIRCLE SAILING

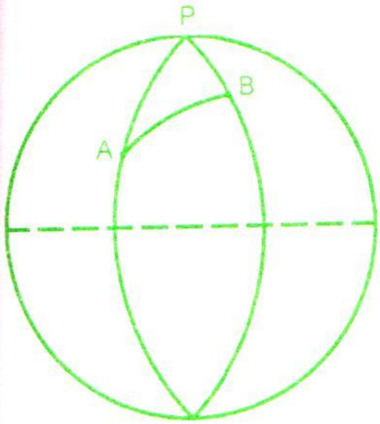
The shortest distance between any two points is the distance along a straight line between them. If a straight line between the two points is not possible, the shortest distance between them would be the arc of a circle passing through the two points and having the greatest possible radius. As the radius reduces, the distance along the arc between the two points will increase.

A straight line track is not possible between two points on the surface of the Earth (which can be considered spherical). Therefore the shortest distance between any two points on the Earth's surface would be the shorter arc of a circle passing through the two points and having the largest radius. A great circle by definition has a radius equal to the radius of the sphere itself, and therefore has the largest possible radius for that sphere. Thus the shortest distance between any two points on the Earth's surface is the shorter arc of the great circle passing through those points.

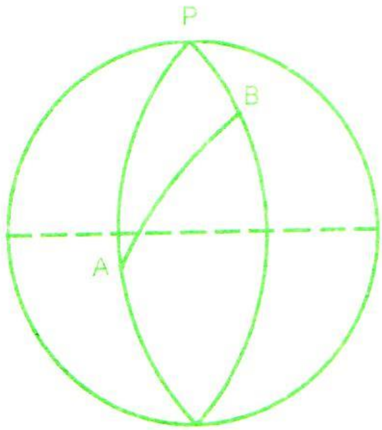
By sailing along a great circle track, a considerable saving in distance is obtained, as compared to sailing between the same two positions on a rhumb line track. The saving is greatest, when the positions are east and west of each other and least when they are north and south of each other. This is so because, on north-south courses, the rhumb line track and the great circle track are exactly the same i.e. along a meridian, while on east-west courses there is maximum separation between the two tracks. Whatever the course, the saving in distance would be greater in higher latitudes and least (nil) at the Equator. Thus the maximum saving in distance is achieved when sailing east-west in high latitudes. In practice, sailing exactly along a great circle track is impossible, because great circles intersect each meridian at different angles and so the vessel would have to continuously change her course at every point along the track. A vessel may however sail along a series of short rhumb lines between successive points on the great circle track, thus making good a track closely approximating to the great circle track, while doing rhumb line sailing. For detailed explanation and for explanation on the use of gnomonic charts for great circle sailing, the reader may refer to the earlier section on Gnomonic Charts.

13.1 SOLUTION OF GREAT CIRCLE SAILING PROBLEMS

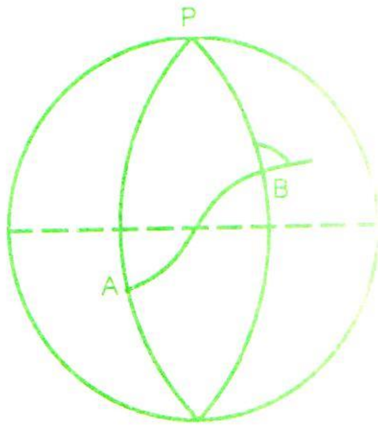
In solving great circle sailing problems, the spherical triangle to be solved is formed with one side as the great circle track between the two points and the other two sides, as the meridians through the two points. Refer Fig. 13.1.



(FIG.13.1)



(FIG.13.2)



(FIG.13.3)

The great circle track between two positions in the same hemisphere will curve **towards** the pole of that hemisphere. The great circle track from a position in one hemisphere to a position in the other hemisphere will curve towards the pole of the hemisphere in which the position with the higher of the two latitudes lies (Fig.13.2).

When sailing between two positions on equal latitudes in different hemispheres, the initial course will be equal to the final course. In other words, the inner angle at the departure position will be the supplement of the inner angle at the destination (fig. 13.3).

VERTEX

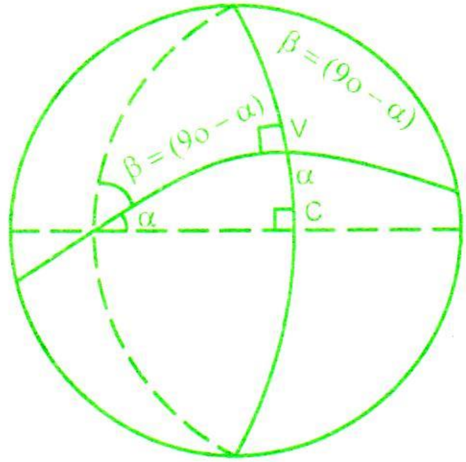
The vertex of a great circle is the point at which,the great circle is nearest to the geographic pole, that is the point at which it reaches the maximum latitude. Every great circle has two vertices (one in each hemisphere). In the solution of great circle sailing problems, we are only concerned with the vertex which is closest to the arc forming the great circle track.

It should be noted that -

(i) At the vertices,the great circle track will be exactly east- west and therefore the meridian of the vertex intersects the great circle track at an angle of 90°.

(ii) Every great circle will intersect the Equator at two points, 180° apart. The longitude of these points will be 90° away from the longitude of the vertices.

(iii) The angle at which the great circle intersects the Equator will be equal to the latitude of the vertices. In other words, the course at which a great circle track crosses the Equator will be equal to the co-lat of the vertices.



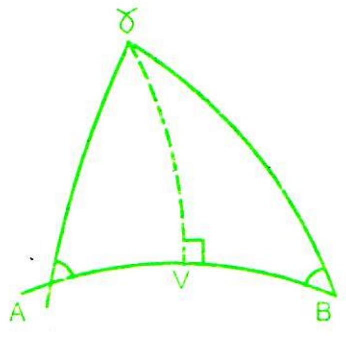
(FIG.13.4)

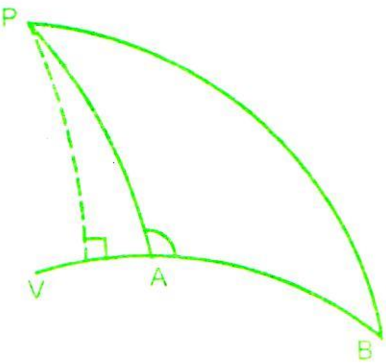
The purpose of determining the position of the vertex is to facilitate calculation of intermediate positions along the great circle track, between which short rhumb line courses could be steered. As stated earlier, the great circle track intersects the meridian of the vertex at 90°. Napier's rules can therefore be used to solve successive right angled triangles, to obtain the intermediate positions.

In using the position of the vertex for determining the intermediate points along the track, it is important to know, whether the vertex falls **inside** or **outside** the spherical triangle formed by the pole and the departure arrival positions. The following rule may be conveniently used for this purpose :

(a) In triangle PAB, if both angles A and B are acute, the vertex lies **within** the triangle.

(b) If one angle is acute and the other obtuse, the vertex lies **outside** the triangle on the side of the obtuse angle.





(FIG.13.5) (FIG.13.6)

It would be worth noting that in sailing between two positions, on the same latitude, the triangle involved will be isosceles, the angles at A and B would be equal and the vertex will be exactly midway between A and B.

In practice, the departure and arrival positions are known. It would be necessary to calculate the **great circle distance** between them, the **initial course,** the **final course, the position of the vertex and intermediate positions** along the great circle track.

The order of working the problem, is as follows:-

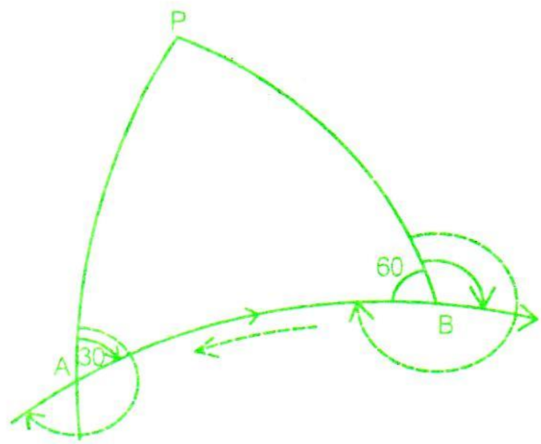
(i) Find the great circle distance using the haversine formula.

(ii) Find the initial and final courses either by the haversine formula or by the sine formula. If the sine rule is used, two possible answers will result in the case of each course. This may be resolved by the use of the ABC tables, provided the reader is familiar with that method, shown further on.

Where only the initial and final courses are required, they may be found with reasonable accuracy, by the use of ABC tables alone. However, where the position of the vertex and the positions of intermediate points along the great circle track are also required to be calculated, the error which may result from using ABC tables for obtaining the initial and final courses, would be cumulative and therefore that method of finding the course would be unacceptable.

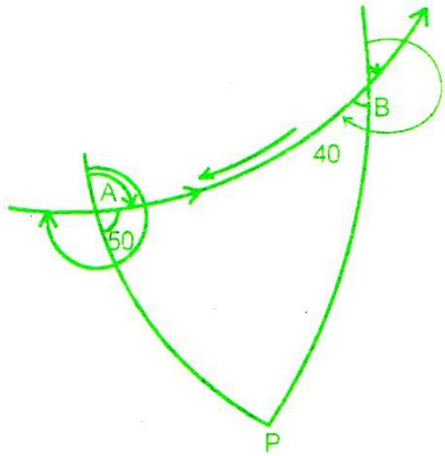
After calculating the values of angles A and B, one often experiences difficulty in naming the course and expressing it in the three figure notation. However this is easily done, since we know that, in three figure notations, the course is measured clockwise from the North meridian to the ship's head. In the following examples given the internal angles A and B, the reader should see for himself, how the ship course is expressed in the three figure notation.

(1) If proceeding from A to B, initial course = 030° and final course 120°. If proceeding from B to A, initial course 300° and final course 210°.



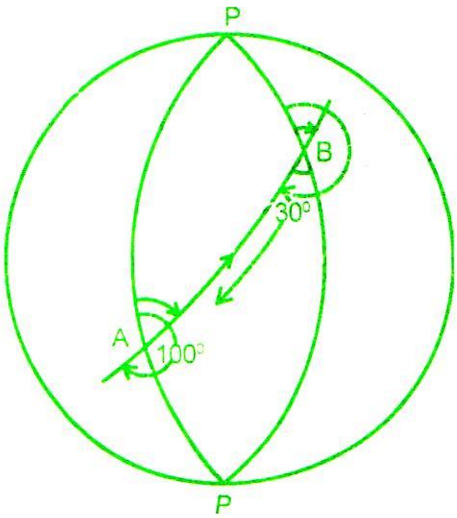
(FIG.13.7)

(2) If proceeding from A to B, initial course = 130°, final course = 040°. If proceeding from B to A, initial course = 220° and final course = 310°.



(FIG.13.8)

(3) If proceeding from A to B, initial course = 080°, final course = 030°. If proceeding from B to A, initial course = 210° and final course = 260°.



(FIG.13.9)

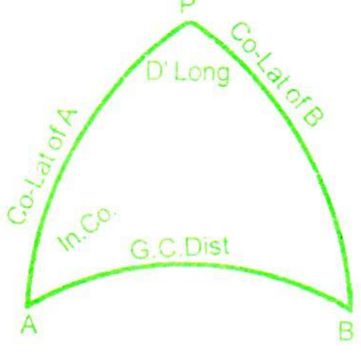
(3) **To find the position of the vertex -** As explained earlier, the meridian of the vertex will meet the great circle track at 90°, resulting in right angle spherical triangles in which one side and an angle (calculated earlier) are known. Napier's rule may then be used to find the d'long between the meridian of the vertex and that of either departure or arrival position, from which the longitude of the vertex is obtained. The co-lat of the vertex may also be calculated using Napier's rule, to give the latitude of the vertex.

(4) **To find the intermediate positions along the great circle track-** After having found the latitude and the longitude of the vertex, the latitudes or longitudes of the intermediate positions are decided upon. The longitude or latitude respectively of those positions, can then be calculated, by using Napier's rule on the spherical triangle right angled at the vertex.

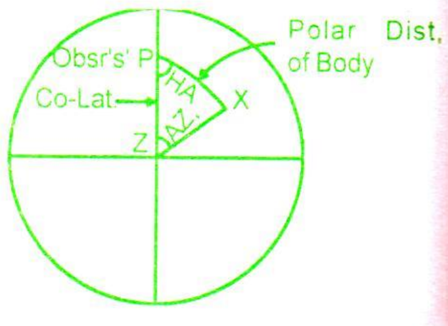
13.2 USE OF ABC TABLES TO FIND INITIAL AND FINAL COURSES

Obtaining the initial and final courses involves the finding of an angle in a spherical triangle, where two sides and an included angle are known. As can be seen from the figures above, this is similar to calculating the azimuth of the heavenly body, where two sides and the included angle of the spherical triangle PZX are known and an angle is required.

A B C tables provide a ready solution for an angle, if two sides and the included angle are known in any spherical triangle. Since this information is available in a great circle sailing problem, the ABC tables may be used in a similar manner, to find the angles at A and B in the triangle PAB.



(FIG.13.10)



(FIG.13.11)

(3) **To find the position of the vertex -** As explained earlier, the meridian of the vertex will meet the great circle track at 90°, resulting in right angle spherical triangles in which one side and an angle (calculated earlier) are known. Napier's rule may then be used to find the d'long between the meridian of the vertex and that of either departure or arrival position, from which the longitude of the vertex is obtained. The co-lat of the vertex may also be calculated using Napier's rule, to give the latitude of the vertex.

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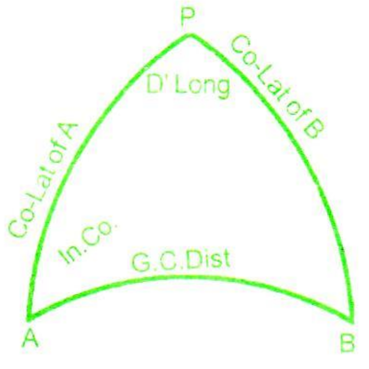
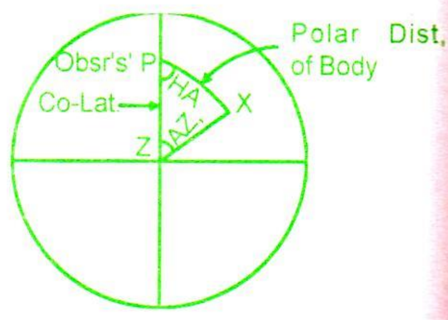


(FIG.13.10)



(FIG.13.11)

The following example will illustrate the method.

Example

Find initial and final courses from A (32°12'N, 018°15'E) to B (05°40'N) (034°20'W).

d'long = 52°35'W

To find intial course	Nories	Burtons
Using latitude of departure position (32° 12'N) as the latitude and d'long from A to B, 52°35'W as the HOUR ANGLE	A= 0.48 S	0.481 (+)
Using latitude of destination position (5° 40'N) as the DECLINATION and d'long as the HOUR ANGLE	B = 0.12 N C = 0.36 S	0.125 (-) 0.356 (+)
AZIMUTH (Since d'long used as HOUR ANGLE is West)	S 73° W	S 73¼ W
Initial Course	253° (T)	253° 15' (T)
Initial course calculated by haversine formulae	253° 11.8(T)	

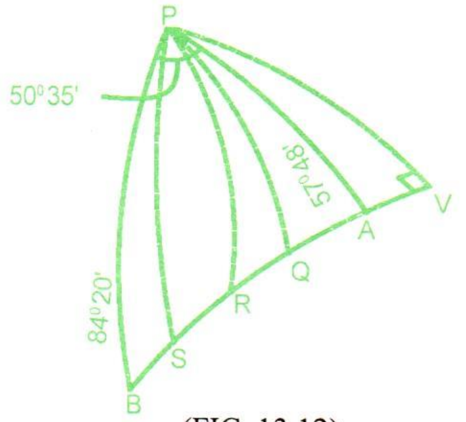
To find Final course	Nories	Burtons
Using latitude of destination position (05°40'N) as the LATITUDE and d/long from B to A. (52° 35'E) as the HOUR ANGLE	A = 0.08 S	0.076 (+)
Using latitude of departure postition (32° 12 N) as the DECLINATION and d'long as the HOUR ANGLE	B = 0.79 N C = 0.71 N	0.793 (-) 0.717 (-)
AZIMUTH (Since d'long used as HOUR ANGLE is East)	N 54.8° E	N 54½° E
This final course is found by reversing the Azimuth obtained. The azimuth is to be reversed, because we have found the course from B to A. The actual final course will obviously be the reverse of it.	S 54.8 W	S 54½° W
Final course	234.8 (T)	234. 3(T)
Final course by haversine formula	234° 29.7(T)	

The reader may note, that once he has correctly selected the values for LATITUDE, HOUR ANGLE and DECLINATION, he should strictly confirm to the rules regarding naming of A,B and Azimuth in the respective tables, as he would do in obtaining the azimuth of heavenly body.

The great cirle distance between A and B can aslo found, using ABC tables, by re-orienting the triangle PAB. An explanation of the use of this techinique is being avoided, as it may provide confusing to some students.

Examples

1. Find the distance along a great circle, the initial course and the final course from latitude 32 12'N, longitude 18 15' E to latitude 5° 40' N, long 34° 20' W. Also find he position of the vertex of the great circle track and the latitude in which the great circle track crosses longitude 10° E and further meridians 20° apart. Verify initial and final courses by use of ABC tables.



(FIG. 13.12)

	18°	15'	E		PA 57°48'
	<u>34°</u>	<u>20'</u>	<u>W</u>		<u>PB 84°20'</u>
d'long	52°	35'	W		(PA~PB) = 26°32'

hav AB = hav P. sin PA sin PB + hav (PA~PB)

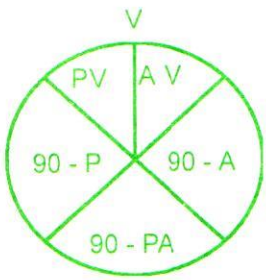
hav P	52°35'	9.29269		
sin PA	57°48'	9.92747		
sin PB	84°20'	9.99787		
			9.21803	0.16521
nat. hav (PA~PB)			26° 32'	0.05266
			55°38.9'	0.21787

= 3338.9 M

hav A = {hav PB - hav (PA~AB)} . cosec PA . cosec AB hav B = {hav PA - hav (PB~AB)} . cosec PB . cosec AB

PA	=	57° 48.0'	PB	=	84° 20.0'
AB	=	55° 38.9'	AB	=	55° 38.9'
(PA~AB)	=	02° 09.1'	(PB~AB)	=	28° 41.1'

hav PB	84° 20.0'	.45063	hav PA	57° 48.0'	.23356
-hav (PA~AB)	02° 09.1'	.00035	-hav (PB~AB)	28° 41.1'	.06136
		---------			---------
		.45028			.17220
		9.65348			9.23604
L . cosec PA	57° 48.0'	0.07253	L . cosec PB	84° 20.0'	0.00213
L . cosec AB	55° 38.9'	0.08324	L . cosec AB	55° 38.9'	0.08324
		----------			----------
		9.80925			9.32141
	A =	106°48.2'		B =	54°29.7'
Initial Course	=	**253°11.8'**	**Final Course**	=	**234°29.7'**



(FIG. 13.13)

To find position of vertex in Δ PAV

PA = 57° 48'   A = 73° 11.8'

sin PV = cos (90-A) . cos (90-PA)

= sin A . sin PA

sin A	:	73° 11.8'	9.98105
sin PA	:	57° 48'	9.92747

			9.90852
	PV	=	54°06.1'
lat. of vertex		=	**35°53.9'** N

sin (90-PA) = tan (90-P) tan (90-A)

cos PA = cot P . cot A

cot P = cos PA tan A

tan A	73° 11.8'	0.52002
cos PA	57° 48.0'	9.72663

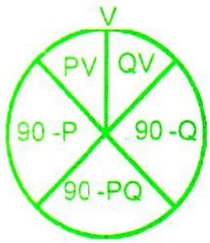
		0.24665

P = 29° 32.4' E

long. of A	=	18^{0} 15.0' E
long of vertex	**=**	**47^{0} 47.4' E**

checking initial courses & final course by ABC tables :

initial course					**final course**				
HA as d'long	=	52^{0}	35'	W	HA as d'long	=	52^{0}	35'	E
lat. as lat. of A	=	32^{0}	12'	N	lat. as lat. of B	=	05^{0}	40'	N
decl. as lat. of B	=	05^{0}	40'	N	decl. as lat. of A	=	32^{0}	12'	N
A	=	0.48	S		A	=	0.08	S	
B	=	0.12	N		B	=	0.79	N	
		------	----				------	----	
C	=	0.36	S		C	=	0.71	N	
Course S73^{0}W	=	**253^{0}(T)**			Course N54.7^{0}E	=	054.7^{0} (T)		
					final course = reverse of 054.7^{0}	=	**234.7^{0} (T)**		



(FIG. 13.14)

To find lat. various points :

Q	=	10^{0}00.0'E	R	=	10^{0}00.0'W	S	=	30^{0}00.0'W
long. of vertex		47^{0}47.4'E			47^{0}47.4'E			47^{0}47.4'E
d'long to Q	=	37^{0}47.4'	R	=	57^{0}47.4'	S	=	77^{0}47.4'

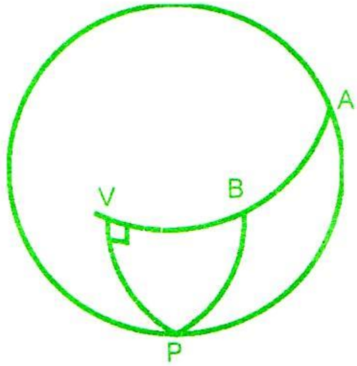
sin (90-P) = tan PV.tan (90-PQ)

cos P = tan PV.cot PQ

cot PQ = cos P. cot PV

cot PV 54^{0}06.1	9.85964	cot PV 54^{0}06.1	9.85964	cot PV 54^{0}06.1	9.85964
cos P 37^{0}47.4	9.89777	cos P 57^{0}47.4	9.72675	cos P 77^{0}47.4	9.32529
cot PQ 60^{0}13.8	9.75741	cot PR 68^{0}54.1	9.58639	cot PS 81^{0}17.8	9.18493
Pos'n of Q	29^{0}46.2'N	Pos'n of R	21^{0}05.9'N	Pos'n of S	08^{0}42'N
	10^{0}00.0'E		10^{0}00.0'W		30^{0}00'W

2 Find the great circle distance, initial course, final course and the position of the vertex of the great circle from A in latitude 08°05'N, longitude 078°10'E to B in latitude 33°55'S, longitude 025°35'E



(FIG. 13.15)

A	: 08°05'N	078°10'E	B	: 33°55'S	025°35'E		
PA	=	98°05'		78°10'E			
PB	=	56°05'		25°35'E			
(PA~PB)	=	42°00'	d'long	52°35'W			
hav AB	=	hav P.sin PA.sin PB + hav (PA~PB)					
log hav P	=	52°35'	9.29269				
log sin PA	=	98°05'	9.99566				
log sin PB	=	56°05'	9.91900				
			9.20735			.16120	
hav (PA~PB)		42°00'				.12843	
						.28963	65°07.1'

G.C. distance = 3907.1 miles

hav A = hav PB - hav (PA~AB).cosec PA.cosec AB

hav B = hav PA - hav (PB~AB).cosec PB.cosec AB

PA	98°05.0'	PB	56°05.0'			
AB	65°07.1'	AB	65°07.1'			
(PA~AB) =	32°57.9'	(PB~AB) =	09°02.1'			
nat hav PB	56°05.0'	.22101	nat hav PA		98°05.0'	.57031
-n.hav(PA~PB)32°57.9'		-.08050	-n.hav(PB~AB)		09°02.1'	.00621
		.14051				.56410
		9.14771				9.75136
cosec PA	98°05.0'	0.00434	cosec PB		56°05.0'	0.08100
cosec AB	65°07.1'	0.04230	cosec AB		65°07.1'	0.04230
		9.19435				9.874660

Angle A = 46°35.9' Angle B = 119°54.5'

Initial course = 226°35.9' Final Course = 240°05.5'

Checking initial and final courses by ABC tables :

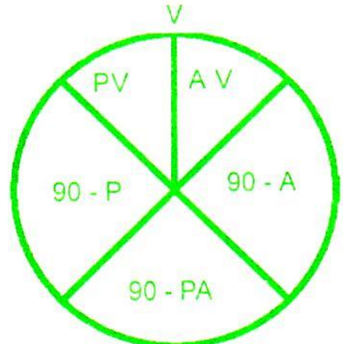
initial course **final course**

HA as d'long	=	52°35'W	HA as d'long	=	52°35'E
lat.as lat.of A	=	08°05'N	lat.as lat.of B	=	33°55'S
decl.as lat.of B	=	33°55'S	decl.as lat.of A	=	08°05'N

A	=	0.11 S	A	=	0.52 N
B	=	0.85 S	B	=	0.18 N
C	=	0.96 S	C	=	0.70 N

Course S46.5°W = **226.5°(T)**   Course N59.9°E = 059.9°(T)

final course = reverse of 059.9° = **239.9°(T)**



(FIG.13.16)

To find position of vertex in Δ PAV

$\sin PV = \cos(90-A) \cdot \cos(90-PA)$

$= \sin A \cdot \sin PA$

sin A	:	46°35.9'	9.86127
sin PA	:	98°05.0'	9.99566
			9.85693
	PV	=	46°00.0'
	lat. of vertex	=	**44°00.0'S**

$\sin(90-PA) = \tan(90-P)\tan(90-A)$

$\cos PA = \cot P \cot A$

$\cot P = \cos PA \tan A$

tan A	46°35.9'	0.02425
cos PA	98°05.0'	-9.14803

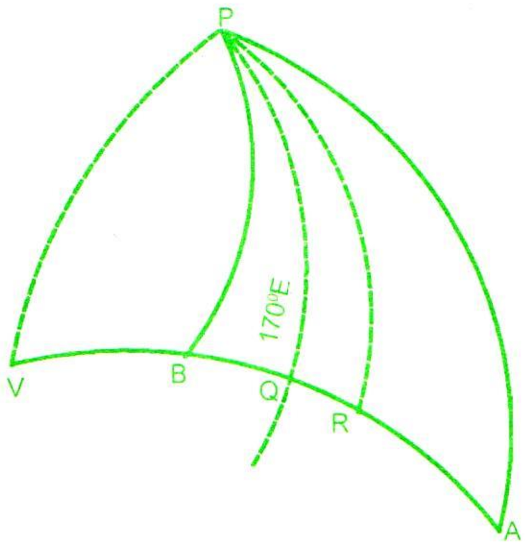
-9.17228 = 81°32.6'

P	=	(180° - 81°32.6') = 98°27.4'
long. of A	=	78°10.0'E
long. of vertex	=	**20°17.4'W**

Example

3. Find the great circle distance, the initial course and position of the

vertex from A in 24°11'N, 168°24'W to B in 47°19'N 157°47'E. Find also the latitude in which the track crosses 170°E and the longitude in which it crosses 38°N.



(FIG.13.17)

A :	24°11'N	168°24'W	B :	47°19'N	157°47'E
PA	=	65°49'	Long.A	168°24'W	
PB	=	42°41'	Long.B	157°47'E	
(PA~PB)	=	23°08'	d'long	33°49'W	
hav AB	=	hav P.sin PA.sin PB + hav (PA~PB)			
log hav P	=	33°49'	8.92731		
log sin PA	=	65°49'	9.96011		
log sin PB	=	42°41'	9.83120		
			8.71862	.05231	
hav (PA~PB)		23°08'			.04020
					.09251 35°24.9'

G.C. distance = **2124.9 miles**

hav A = hav PB - hav (PA~AB).cosec PA.cosec AB

PA		65°49.0'	
AB		35°24.9'	
(PA~AB)	=	30°24.1'	
nat hav PB		42°41.0'	.13244
-n.hav (PA~AB)		30°24.1'	-.06875

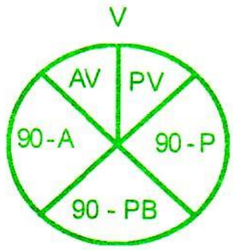
			.06369
			8.80406
L.cosec PA		65°49'	0.03989

L.cosec AB	$35^0 24.9'$	0.23695
		9.08090

Angle A = N $40^0 37.2'$W

initial course = $319^0 22.8'$

Since angle A is acute, the vertex of the great circle will lie inside the triangle or outside, on the side of B. In either case Vertex is to the westward of A. We can now solve the triangle PAV for angle P; the d'long between A and V.



(FIG.13.18)

To find position of vertex in Δ PAV

sin PV = cos (90-A) . cos (90-PA)

= sin A . sin PA

sin A	:	$40^0 37.2'$	9.81361
sin PA	:	$65^0 49.0'$	9.96011

			9.77372
		PV =	$36^0 26.1'$
		lat. of vertex =	**$53^0 33.9'$N**

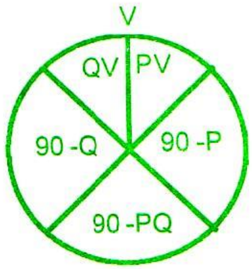
sin (90-PA) = tan (90-P) tan (90-A)

cos PA = cot P cot A

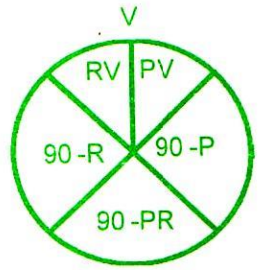
cot P = cos PA tan A

tan A	$40^0 37.2'$	9.93334
cos PA	$65^0 49.0'$	9.61242

		9.54576
	P =	70^0 38.4' W
	long. of A =	168^0 24.0' W
	=	239^0 02.4'
		360^0 00.0'
	long. of vertex =	**120^0 57.6' E**



(FIG. 13.19)



(FIG. 13.20)

long. of V	:	$120^0 57.6'$ E	
long. of Q	:	$170^0 00.0'$ E	
P (d'long)	:	$49^0 02.4'$	
sin (90-P)	=	tan PV . tan (90-PQ)	
cos P	=	tan PV . cot PQ	
cot PQ	=	cos P . cot PV	
cos P		$49^0 02.4'$	9.81659
cot PV		$36^0 26.1'$	0.13182
PQ	=	$48^0 23.7'$	9.94841

lat. in which GC crosses 170^0E
$= 90^0 - 48^0 23.7'$
$= 41^0 36.3'$N

sin (90-P) = tan PV . tan (90-PR)
cos P = tan PV . cot PR

tan PV	36^0 26.1'	9.86818
cot PR	52^0 00.0'	9.89281
P	54^0 46.7' E	9.76099
long. of vertex =	120^0 57.6' E	
	175^0 44.3' E	

long. in which GC crosses 38^0N
$= 175^0 44.3'$E

EXERCISE XIII

1. Find the distance along the great circle, the initial course, final course and the position of the vertex, when sailing from 33°50'S, 23°12'E to 20°10'S 104°00'E. Also find the latitudes in which the great circle crosses 40°E and 70°E longitude. Verify initial and final courses by ABC tables.

2. Find the great circle distance, initial and final courses, from 10°25'S, 90°12'E to 39°27'N, 55°10'E. Find also the position of the vertex and the longitude in which the GC track crosses the Equator and the course then.

3. Ship A in latitude 50°00'N, longitude 11°12'W and Ship B in latitude 50°00'N, longitude 74°42'W, proceed towards each other along the shortest track between them at 13 knots and 16 knots respectively. What distance would each ship sail before they meet ?

   (**Hint:** Triangle PAB is isosceles, a perpendicular dropped from P to AB bisects AB)

4. The rhumb line distance between two places in latitude 47°N, is 1132 miles. What is the shortest distance between them ?

   (**Hint:** Find d'long, then solve the isosceles triangle PAB by dropping a perpendicular from P)

5. The d'long between two places in the same latitude is 180°. If the great circle distance between them is 4800 miles, what is their latitude ?

   (**Hint :** Since the d'long is 180°, the GC track between them lies along two meridians across the Pole. Each place is therefore 2400 miles from the Pole.)

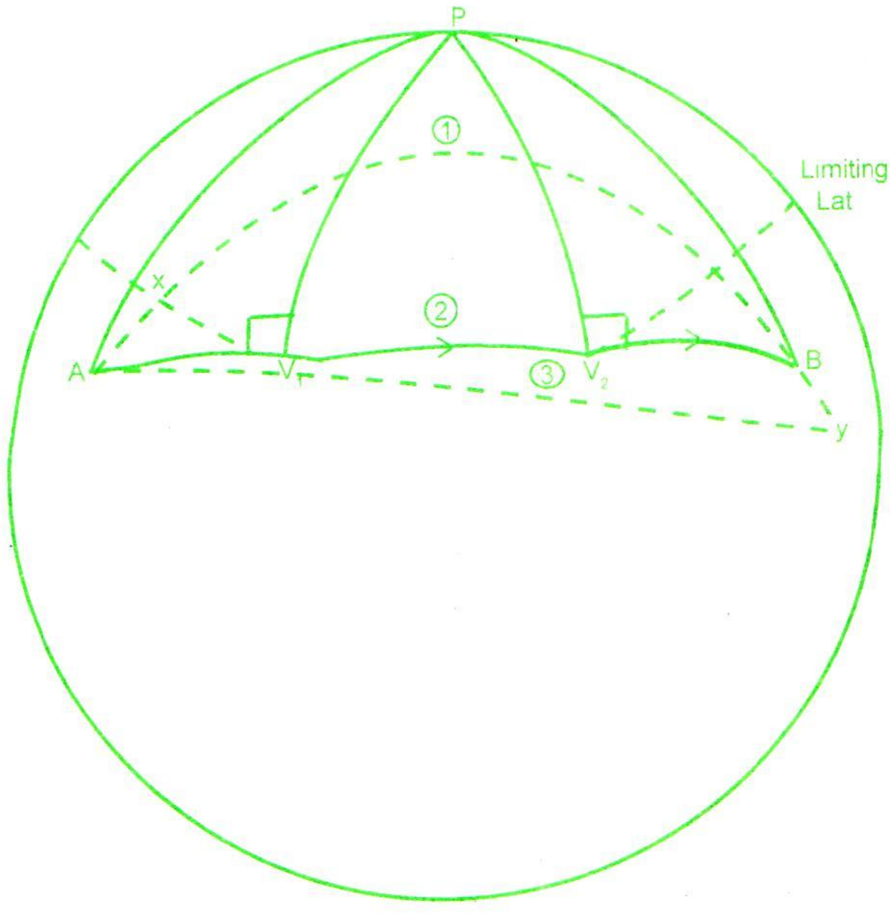
6. If the great circle distance between two places in latitude 61°S is 1384 miles, find the rhumb line distance between them.

7. A great circle crosses the Equator in longitude 30°E at an angle of 31°. What is the position of the vertex of the GC in the Northern hemisphere.

8. A vessel on a great circle track steers 090°(T) across the meridian of 150°W in latitude 46°12'N. Where would the ship cross the Equator if she continued on the same GC and what would be her course then ?

13.3 COMPOSITE TRACKS

When the great circle track between two positions passes through latitudes which are too high and therefore not desirable for many reasons, a composite track may be followed between those positions. The maximum permissible latitude is decided upon and the vessel sails between the two positions on the shortest track, under the restriction that at no time would she proceed beyond the limiting latitude.



(FIG.13.21)

To achieve this, **two separate** great circles are drawn, one from the departure position and the other from the destination position, so that their vertices lie on the limiting latitude. The ship sails along the first great circle till she reaches its vertex at the limiting latitude, then along the limiting parallel of latitude (along the arc of a small circle) till she reaches the vertex of the second great circle (also lying on the limiting latitude) and thence to the destination position along the second great circle.

The shortest distance between A and B would be track (1), which is the arc of the great circle between them. The reader should recall that there is one and only one great circle between two positions as is the case with positions A and B not situated at the extremities of a diameter of the sphere. Obviously track (1) cannot be followed, as it goes beyond the limiting latitude. Under the given condition that the ship is not to proceed beyond the limiting latitude, the shortest distance between A and B would be along the GC track AV_1, then along the rhumb line track V_1V_2 and then along the GC track V_2B.

Any other great circle track from A to the limiting latitude will have its vertex above that latitude. It may be seen from the figure that the distance AV_1 will always be shorter than the total distance from A to V_1 along any other great circle track such as AX and thence, along the rhumb line track XV_1 ($AX + XV_1 > AV_1$).

Other tracks such as track (3), if chosen, would not even meet the limiting latitude and therefore has a greater deviation from the great circle track (1) which is the shortest. The distance along track (3) would therefore obviously be greater than the distance along the composite track (2), AV_1V_2B, the deviation of which from the great circle track is lesser. It is for the above reasons that the two great circles chosen for the composite track have their vertices on the limiting latitude.

Since the two great circle tracks involved meet the limiting latitude at their vertices (course is East - West), the meridians of the vertices will meet the great circles, at those positions, at 90°, as shown in the figure. It must be noted that the triangle PV_1V_2 in the figure is not a spherical triangle as V_1V_2 is the arc of a small circle. The two spherical triangles involved PAV_1 and PV_2B may be solved using Napier's rules to obtain the initial and final courses, the distances along the great circle tracks AV_1 and V_2B and also the angles APV_1 and V_2PB the d'long between A & V_1 and V_2 & B respectively.

If the sum of the angles APV_1 and V_2PB is subtracted from the total d'long between A and B, we obtain the d'long between V_1 and V_2. The distance along the limiting latitude between V_1 and V_2 can now be calculated by the parallel sailing formula, dep. = d'long x cos lat. The total distance along the composite track = GC distance AV_1 + dist. along the parallel V_1V_2 + GC distance V_2B.

Examples

1. Find the initial course, final course and the distance along the composite track from 36°50'S, 13°40'W to 44°40'S, 146°12'E. The track is not to exceed latitude 51°S.

A 36° 50'S		013°40'W
B 44° 40'S		146°12'E
d'long		159°52'E
PA =	53° 10'	PB = 45°20'
PQ =	39° 00'	PR = 39°00'

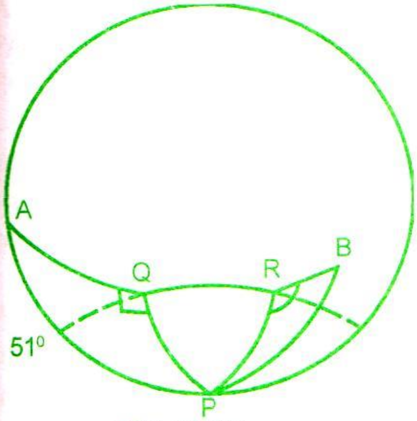
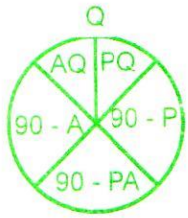
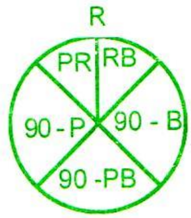


FIG.13.22)



(FIG.13.23)



(FIG.13.24)

sinPQ	=	cos(90-A).cos(90-PA)	
sin PQ	=	sin A. sin PA	
sin A	=	sin PQ. cosec PA	
sin PQ		39°00'	9.79887
cosec PA		53°10'	0.09670
			9.89557
9.94687			
A	=	51°50.3'	
Initial course	=	S51°50.3'E	
		(128°09.7')	
sin (90-PA)	=	cos AQ.cos PQ	
cos PA	=	cos AQ. cos PQ	
cos AQ	=	cos PA. sec PQ	
cos PA		53°10'	
sec PQ		39°00'	
0.10950			
9.95644			
AQ	=	339°31.2'	
sin(90-P)	=	tan PQ.tan(90-PA)	
cos P	=	tan PQ. cot PA	
tan PQ		39°00'	9.90837
cot PA		53°10'	9.87448
			9.78285
9.90332			
P	=	52° 39.6'	
Angle APQ	=	52° 39.6'	
Angle RPB	=	36° 49.7'	
		89° 29.3'	
Total d'long		159° 52.0'	

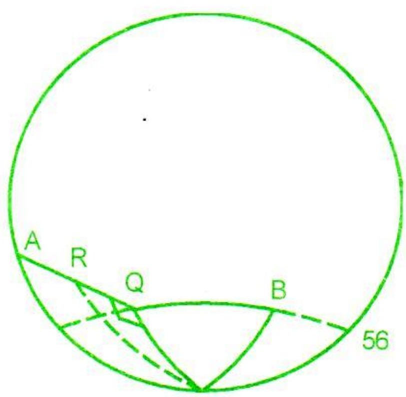
sin PR	=	cos(90-B).cos(90-PB)	
sin PR	=	sin B. sin PB	
sin B	=	sin PR. cosec PB	
sin PR		39°00'	9.79887
cosec PB		45°20'	0.14800
B	=	62°14.0'	
Final course	=	062°14.0'	
sin (90-PB)	=	cos PR. cos RB	
cos PB	=	cos PR. cos RB	
cos RB	=	cos PB. sec PR	
9.77778		cos PB 45°20' 9.84694	
0.10950		sec PR 39°00'	
9.88728			
RB	=	25°14.2'	
sin(90-P)	=	tan PR.tan(90-PB)	
cos P	=	tan PR. cot PB	
tan PR		39°00'	9.90837
cot PB		45°20'	9.99495
P	=	36°49.7'	

d'long along lat. 51°S = 70°22.7 = 4222.7'			
dep. =	d'long. cos lat.		
d'long 4222.7	3.62559		
cos lat. 51°00'	9.79887		
	3.42446		
dist.along	51°S lat.	=	2657.4M
dist.AQ :	39°31.2'	=	2371.2M
dist.RB :	25°14.2'	=	1514.2M
Composite track distance		=	6542.8M

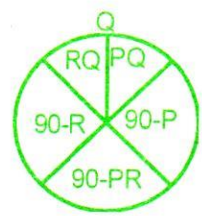
The reader would be aware of the properties of right angled and quadrantal spherical triangles, that the sides and angles opposite each other are of like affection. Since PQ and PR will always be less than 90° angles A and B will also be always less than 90°.

Example

2. Find the initial and final courses and the distance along the composite track from 40°S,180° to 56°S,64°12'W, maximum latitude 56°S. Find the latitude in which the track crosses longitude 140°W.



(FIG.13.25)



(FIG.13.26)

A	:	40° S 180°		B	:	56° S 64° 12' W	
PA	=	50°		long. of A		180°	
PQ	=	34°		long. of B		64° 12' W	
				d'long	=	115° 48' E	
sin A	=	sin PQ. cosec PA		cos AQ	=	cos PA. sec PQ	
sin PQ		34°	9.74756	sec PQ		34°	0.08143
cosec PA		50°	0.11575	cos PA		50°	9.80807
			9.86331				9.88950
A	=	46°53.1'		AQ	=		39°09.8'
Initial course	=	S46°53.1'E		dist. along GC	=		2349.8M
	=	133°06.9'					
cos P	=	tan PQ.cot PA		dep.	=	d'long cos lat.	
tan PQ		34°	9.82899	d'long		3616.2	3.55823
cot PA		50°	9.92381	cos lat.		56°	9.74756
			9.75280				3.30579
Angle APQ	=	55°31.8'		dist. along lat.		56°S =	2022.1M
Total	=	115°48.0'		dist. along GC course		=	2349.8M

		60°16.2'					
d'long along lat.		56°S =	3616.2			Total dist. along composite track = 4371.9M	
Final course		090°(T)					
long. of vertex Q		124°28.2' W		sin (90-P)	=	tan PQ.tan(90-PR)	
long. of Pos'n. R		140° 0.0' W		cos P	=	tan PQ. cot PR	
d'long P	=	15°31.8' W		cot PR	=	cos P. cot PQ	
				cos P	=	15°31.8'	9.98385
				cot PQ	=	34°00.0'	0.17101
				PR	=	34°59.7'	0.15486

lat. in which track crosses 140°W = 55°00.3'S

EXERCISE XIII (A)

1. Required the initial and final courses and the distance along the composite track from 35°N,140°E to 38°N,122°W. At no time is the ship to go above latitude 44°N.

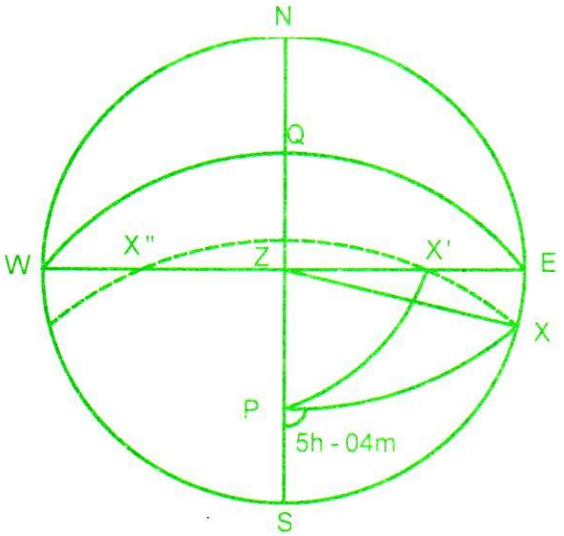
2. A ship is to sail along a composite track from 45°33'S,054°47'E to 43°12'S,134°56'E. The ship is not to proceed south of latitude 50°S. Find the initial and final courses and the total distance along the composite track.

14
CALCULATIONS IN NAUTICAL ASTRONOMY

1. In latitude by account 43°16'N, compute the approximate sextant altitude of a star of declination 08°45'N, when on the meridian. H.E. 8m, I.E. 1.5' off the arc.
(55°33.2')

2. When GHA Aries was 212°14', the easterly hour angle of the True Sun was 35° to an observer in longitude 35° West. Find the RA of the True Sun.
(14h 08m 56s)

3. In latitude 37°S, the time of theoretical sunrise was 05h 04m LAT. Find the LAT at which a sight of the Sun should be obtained so that the longitude obtained would be the same, irrespective of the DR latitude used.



(FIG.14.1)

Solar time is measured westwards from the observer's inferior meridian to the meridian of the Sun. Westerly measurements are clockwise and easterly measurements are counterclockwise, about the North Pole and vice versa about the South Pole. At theoretical sun-rise, when the Sun is at X, the LAT of 05h 04m = 76° is the angle SPX. In the quadrantal triangle PZX, angle ZPX = 104°. Using Napier's rule PX may be obtained.

For the longitude to be the same irrespective of the DR latitude used, the PL must run north south i.e. the azimuth should be east or west. The Sun should therefore be on the Prime vertical. The right angled triangle PZX' may now be solved, assuming PX = PX', to find angle P. The LAT will then be (180°-P), converted to time.
(07h 40m 52s) or (16h 19m 08s)

4. Find the True Sun's SHA at the instant when the First point of Aries crossed the meridian of 85°E, if on that day, the Sun's GHA was 60°12', when GHAγ was 255°.
(165°08.7')

5. The meridian passage time of star Canopus (SHA 264°09') was 06h 15m, as measured by the observer's sidereal clock. Find the error of the clock, measured by the same clock, if the Sun's meridian passage time was 15h 12m 30s, find the Sun's SHA.
(Error : 08m 24s slow, SHA of Sun = 129°46.5')

6. To an observer on a ship at anchor, in a Northern latitude, the Sun rose at 0559 SMT and set at 1806 SMT. Calculate the equation of time.
+ (02m 30s)

7. A vessel sailed from Bombay 18°55'N, 72°50'E, at 0220 IST, equation of time was (-) 04m 12s. Course was set 280° (T), speed 20 knots. Find the amount, the clocks should be altered to be correct for apparent Noon, the next day.
(Retard by 48m 59s)

8. A chronometer, the error on which was not known was used to calculate the longitude, in latitude 40°N. The error was assumed to be nil and longitude calculated as 50°12' West. The ship then sailed 330°(T), 100 miles when a light house in position 41°26.6'N, 51°19.2'W bore 270°(T), 10 miles off. Calculate the error of the chronometer.
(48 seconds fast)

9. In latitude 50°N, a star with an SHA of 146°10', had a true altitude of 51°36', when bearing True East. Find the local sidereal time.
(10h 51m 30s)

10. Find the GMT when Venus will rise on 23rd June, 1976, the observer being in 40°N, 070°W. (Hint - To find GMT of Venus rising, we should obtain GHA of Venus then. The GMT can thereafter be obtained by inspecting the almanac for that date).

The GHA of Venus can be obtained by applying the longitude to its LHA. To find the LHA, the declination of Venus is required for which the GMT is necessary. However, an inspection of the nautical almanac reveals that the declination of Venus changes very little on that date. We therefore obtain the declination of Venus, when rising by finding the LMT meridian passage of Venus for that date and subtracting 6 hours from it to obtain the approximate LMT of Venus rising. To this LMT, we apply LIT to obtain the approximate GMT of Venus rising. As the rate of change of declination of Venus is very small, we may use this approximate GMT to obtain the declination of Venus, when it rises.
(GMT 23d 09h 21m 17s)

11. Calculate the speed at which the geographical position of a star with declination 28°N travels across the Earth's surface.
(794.6 knots)

12. A star when on the meridian below the pole bore South with an altitude 32°06' and when on the meridian above the pole bore North with altitude 74°22'. Calculate the observer's latitude and the body's declination.
(Latitude 68°52'S, Decl. 53°14'S)

13. Star Canopus, declination 52°41.5'S had a true altitude of 18°12', when on the meridian below the pole. Find the observer's latitude and state the azimuth of the star then.
(lat. 55°30.5'S, Azimuth 180°)

14. Find the LMT of meridian passage of the Sun, Moon, Venus and Star Procyon in longitude 145°E on 14th October, 1976.
(Sun 11h 46m 03s; Moon 03h 51m 27s)
(Venus 13h 46m 20s; Procyon 06h 08m 05s)

15. Find the LMT of meridian passage of the Moon on 13th October, 1976 in longitude 76°12'W.
(03h 32m 47s)

16. To an observer in latitude 41°02'N, longitude 25°06'W, the true altitude of the Sun, East of the meridian was 62°12'. If the Sun's declination at that instant was 21°44.5' North, find the Sun's geographical position.
(lat. 21°44.5'N long. 001°21.9'W)

17. To an observer, the Sun bore 090°(T) with an altitude of 32°12', when it had a declination of 06°12'S and GHA of 44°06.2'. Find the observer's position.
(11°41.6'S, 102°26.6'W)

18. An observer in position 22°05.0'N, 034°12.5'W found the true altitude of a star with declination 10°15'N to be 41°02' west of the meridian. If GHA Aries at that instant was 223°12', find the star's SHA.
(SHA 220°43.4')

19. In DR latitude 43°S, Sun's declination 12°S, using ABC tables only, find the LAT at which a Sun's sight should be taken to obtain a position line running 330°- 150°.

Solution

To obtain a PL running N30°W, S30°E, the azimuth should be N60°E or S60°W. From the 'C' table, we find that to obtain an azimuth of 60° in latitude 43°, the 'C' value should be 0.789. 'C' is the algebraic sum of 'A' and 'B'. In this case since latitude and declination are of the same name, 'C' will be the difference between 'A' and 'B' values.

By inspection of 'A' and 'B' tables, entering 'A' with latitude 43° and 'B' with declination 12°, we obtain an hour angle of 39°42', where the difference between 'A' and 'B' exactly equals 0.789. The name of 'C' indicates that the bearing could be N60°E, when the EHA is 39°42' or N60°W when LHA is 39°42'. The azimuth N60°W does not give the PL required. The only solution which applies therefore is an EHA of 39°42'.
(LAT = 12h - 02h 38m 48s. = 09h 21m 12s)

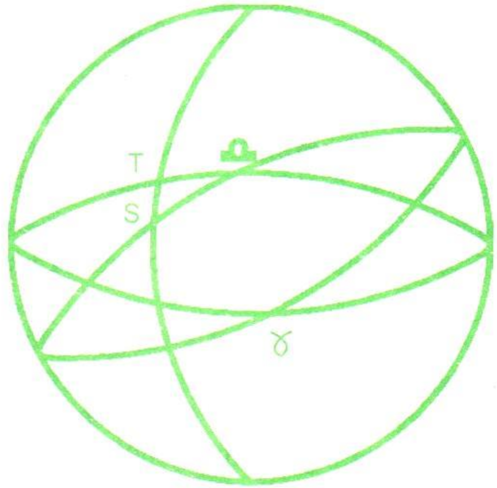
Note

A single PL is often made use of, particularly in poor visibility to pass a point at a given distance or to make a safe passage through a channel provided the direction of the channel is parallel to the PL obtained.

20. If the Sun's declination is 15°30'N, and increasing, calculate the Sun's SHA, assuming obliquity of the ecliptic to be 23°26.7'.
(320°14.8')

21. If the equation of time was + 04m 06s, when RAMS was 14h 32m 15s, calculate the Sun's declination.

Hint - Obtain RATS (14h 36m 21s) by applying equation of time to RAMS. Then solve the triangle ♎TS (FIG. 14.2)
(15°17.5'S)



(FIG.14.2)

22. SHAMS 16h 06m 10s, equation of time (-)02m 48s, obliquity of the ecliptic 23°26.7'. Calculate the Sun's declination.

    Hint - Obtain SHATS (16h 08m 58s) by applying equation of time to SHAMS. Solve triangle γST as in the previous example.
    (20°59.6'N)

23. GHA Aries 06°13', GHA Sun 270°43', Sun's declination 23°20.9'N. Calculate obliquity of the ecliptic.
    (23°26.7')

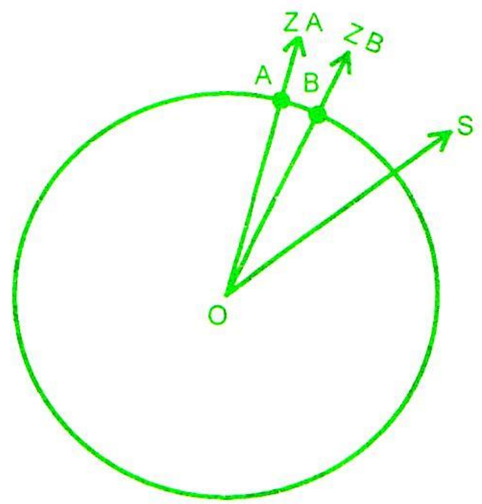
24. Given declination of the Sun 10°15'N and decreasing, SHAMS 206°36.8', Calculate the value of equation of time
    Hint - Solve triangle ♎TS (Fig.14.2) and obtain SHATS. SHAMS - SHATS = equation of time.
    (+ 07m 53s)

25. If the distance between the centres of the Earth and Moon is 30 times the diameter of the Earth, calculate the Moon's H.P.
    (00°57.3')

26. Assuming the Earth's radius to be 6373 km and the horizontal parallax of Moon to be 58.0', calculate the distance of the Moon from the Earth.
    (377750 km)

27. Two observers on the same meridian observe the meridian altitude of the Sun at the same time. The difference between the true altitude was 10°12'. Assuming the Earth to be a sphere of radius 3960 statute miles, calculate the distance between the two observers in (a) statute miles (b) nautical miles.

Hint- The difference between the meridian altitudes is equal to the difference between the meridian zenith distances. MZD for A = angle AOS, MZD for B = angle BOS
Difference between these angles = angle AOB, the angle at the centre of the Earth, between A and B. Arc AB the distance between A and B can now be evaluated.
(a) 705 (b) 612



(FIG.14.3)

28. Two observers A and B, are on the same meridian, 873 nautical miles apart. Calculate the difference between the meridian altitude of the same heavenly body as observed by them at the same time.

    Hint - Since the two observers are on the same meridian, and their distance apart is 873 miles, the d'lat between them is 14°33', which is also the angle subtended by them at the centre of the Earth. The difference between the two meridian altitudes will therefore be equal to 14°33'.
    (14°33')

29. From a vessel on a constant heading at the same position, the Sun rose bearing 086°(C), and set bearing 292°(C). If the deviation of the compass was 2°E, find the variation.
    (11°W)

30. A body of declination 'd' was at the observer's zenith, 'H' hours after rising. Prove that :
    $-\cos H = \tan^2 d$

31. At a certain time, the Sun's RA was 03h 56m 40s and GHAγ was 312°48.4'. Assuming obliquity of the ecliptic 23°27', calculate the GP of the Sun.
    (20°25.7'N, 106°21.6'E)

32. Find the GP of the First point of Aries at 02h 12m LMT on 14th Oct. 1976 in longitude 72°12'W.

Hint - Aries is on the Equinoctial and therefore its declination is always nil.
(lat. 00°00.0', long. 128°09.2'W)

33. The plane of the rational horizon of an observer in the Northern hemisphere is coincident with that of the Ecliptic. What would be the true meridian altitude of a star (declination 42°12'N) for that observer ?

Hint - Since the observer is in the Northern hemisphere and his rational horizon coincides with the ecliptic, his latitude is 66°33.3'N.
(65°38.7')

34. Assuming the 'v' of the Moon to be a constant 11', calculate the length of a lunar day.

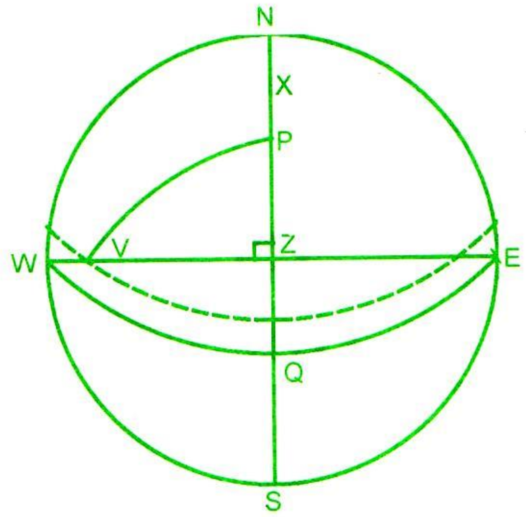
Hint - The assumed hourly increase of Moon's GHA is 14°19'. Since 'v' is 11', the actual increase of its GHA = 14°19' + 11' = 14°30'/hour. A lunar day is the interval between two successive transits of the Moon over the same meridian i.e. an increase in GHA of 360°.
Therefore duration of lunar day = 360 / 14.5.
(24h 49m 39s).

35. A vessel in latitude 50°N was on a course of 320°(T). Sun sights observed exactly 30 minutes apart gave LHA of Sun as 280°00' and 287°18', find the ship's speed.

Hint - Since the Sun's GHA increases by 15° per hour, for a stationary observer, the LHA would have increased by 07°30' in half an hour. Since the actual increase is only 07°18', the ship has made a d'long of 12' to the westward in that period. For a d'long 12', dep. = 7.71M. Distance = dep. x cosec co = 12 miles.
(Speed of ship 24 knots)

36. On 14th October, 1976 star Vega bore 270°(T) to an observer in latitude 46°30'N. At that instant another star bore 000°(T) with altitude 30°12'. Find the SHA and declination of that star.

Hint - PX gives polar distance of the star from which its declination may be obtained. From the right angled spherical triangle PZV, obtain LHA of Vega. 180 - LHA Vega gives the angle between the meridian of Vega and the meridian of the other star. This angle applied to SHA Vega, will give SHA of the other star. (FIG. 14.4)
(Decl. 73°42'N, SHA 220°36.8')



(FIG.14.4)

37. Find the angle subtended at the centre of the Earth, between star A, SHA 300°, Decl. 20°N and Star B, SHA 285°, Decl. 20°S.

Hint - The difference between SHA's gives the d'long between their GP's. Their declinations correspond to latitude of their GP's. The GC distance between their GP's gives the angle subtended by them at the centre of the Earth.
(42°36.7')

38. In latitude 38°48'N, longitude 058°15'W, a star bore 052°(T), with true altitude 32°17'. If GHAγ at that instant was 271°15' find the star's SHA.

Hint - Using haversine formula, calculate PX and hence P.
(SHA = 64°39.1')

39. Two ships A and B on the parallel of 60°S are 200 miles apart, A being west of B. Both ships set their clocks to their respective apparent solar times. They then proceed along the same parallel towards each other. They meet at 1910 hrs. by B's clock. Assuming no clocks were altered since departure, find the time by A's clock, on meeting.

Hint - Find initial d'long between the two ships. A's clock will be 26m 40s behind B's clock.
(Time by A's clock 18h 43m 20s)

40. An unknown star rose bearing 123°(T). When bearing East, it had a true altitude of 24°30'. Find the latitude of the observer and the body's declination.
(lat. 52°42.8'S, Dec. 19°.15.9'S)

15
TIDES

Tides are the periodic rise and fall in the level of seas. In mid ocean, where the depth of water is large, the tidal range is small, but near the coasts where the waters are shallow, the tidal range increases. The rise and fall of tide causes little or no horizontal movement of the waters in mid ocean, but near the coasts, a comparatively large horizontal movement of water is caused. The horizontal movement of water, due to tides, is known as **tidal streams**. Various theories have been put forward for the cause of tides. The Equilibrium Theory advanced by Sir Isaac Newton is generally accepted as the major basis on which tides occur, since the actual tides observed are normally in general agreement with the tides computed according to this theory. A brief explanation of the theory follows :

According to the equilibrium theory, every particle of water in the seas is in a state of static equilibrium under the action of the centrifugal force due to the Earth's rotation, the gravitational attraction on it by the Earth, as well as the forces of attraction on it due to the Sun and Moon.

The force of gravitational attraction between two bodies is directly proportional to their masses and inversely proportional to the square of the distance between them. Let us initially consider the Earth to be uniformly covered with water over its entire surface.

15.1 LUNAR TIDE

The Moon exerts a gravitational force on the Earth and the water surrounding it. The Earth, except for the water surrounding it, may be considered solid, and the Moon's gravitational force acting on it may be considered to act at its centre of gravity. This force acts on the solid Earth as a whole.

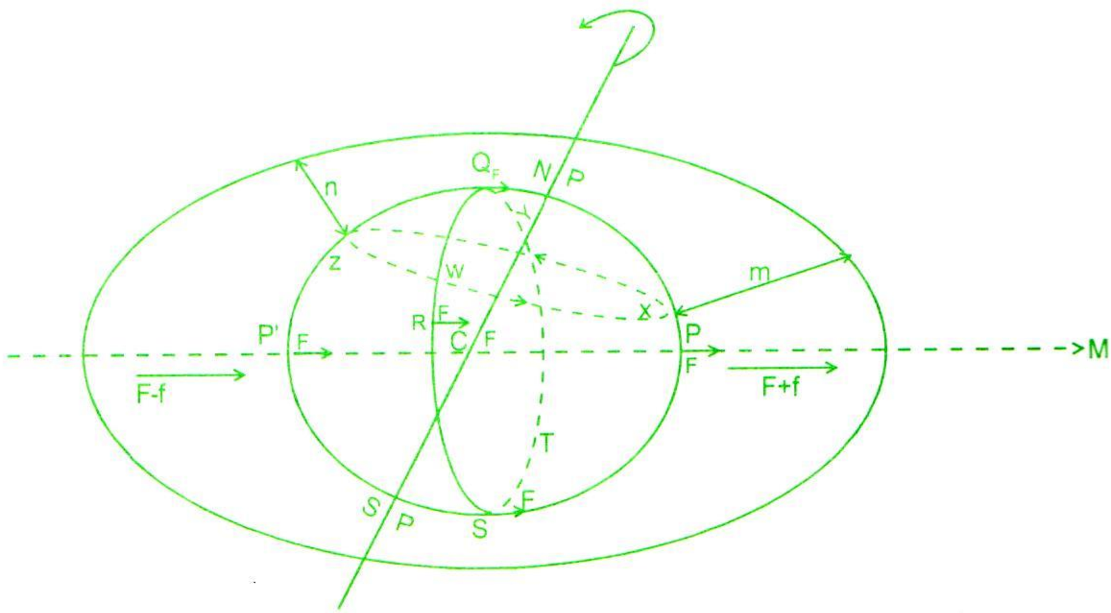
The radius of the Earth is about 4000 miles and the average distance between centre of the Earth and the centre of the Moon is about 2,40,000 miles. Thus a point P on the Earth's surface directly beneath the Moon is about 4000 miles (1/60th of the distance between their centres) closer to the Moon than the centre of the Earth and a point P' vertically opposite P is about 4000 miles further from the Moon than the Earth's centre. If we assume that the gravitational force of the Moon acting on the Earth at its centre of gravity C is F, then at point P it will be (F + f), where 'f' is the additional force due to the point P being closer to the Moon by 4000 miles.

By similar argument the force at P' will be (F - f). At points Q,R,S,T etc. the force is equal to that at C.

that is, equal to F. Considering the point P, the force of attraction on the Earth is F, while that on the water at the surface is F + f. Since the water surrounding the Earth is non-rigid, this differential force tends to raise the water away from the surface of the Earth, towards the Moon. At point P', the force of attraction on the Earth is F, while that on the water is F-f. This differential force tends to move the Earth towards the Moon leaving the water behind.

Thus the level of water above the Earth's surface, both at P and P', rises above the mean sea level. From the above explanation, it should be noted that tides are not directly caused by the attraction of the Moon on the Earth or the water surrounding it. It is caused by the differential forces of attraction on the Earth and the surrounding water.

At points such as Q,R,S,T etc. which are at the same distance from the Moon as the centre of the Earth, the differential force being nil, the tide raising force is also nil. Thus, the over all effect is to produce an ellipsoid of water around the Earth with its major axis in the direction to the Moon. As the Earth rotates within this ellipsoid, the level of water at a place would rise and fall producing **high** and **low** waters respectively. This is known as the **lunar tide.**



(FIG.15.1)

Since any position on the Earth's surface would experience two high waters and two low waters between one culmination of the Moon and the next, and as the interval between successive culminations of the Moon is about 24h 50m, the period of the lunar tide (Period between successive high waters or successive low waters) is about 12h 25m. When the Moon has a northerly or southerly declination, the major axis of the ellipsoid will lie in the direction of the Moon, that is, at an angle to the plane of the Equator. This produces unequal intervals between successive high and low waters and also unequal heights at successive high waters. The reader should refer to the last figure when reading the following explanation of the above statement.

There is a high water of height m at place X in the figure. X will complete one rotation along its parallel of latitude in 24 hours. When X has been carried round to Y by the Earth's rotation, it will have a low water. As place X rotates further to position Z, it will again have a high water, the height of which is only n. Thus, successive high waters may differ considerably in height. At W, it will again have a low water.

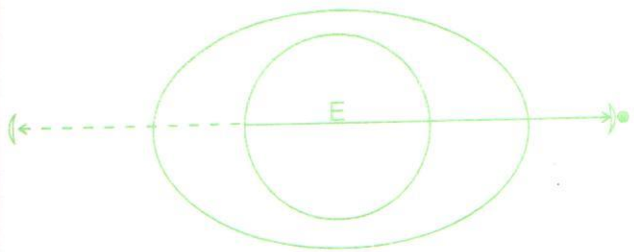
It can also be seen from the figure that the interval between the high water experienced when at Z and the low water experienced when at W is the period of rotation from Z to W. The interval between that low water and the next high water, when at X, is the period of rotation from W to X. Since this period is larger than that from Z to W, it can be seen that the interval between a high water and the succeeding low water may differ considerably from the interval between that low water and the next high water.

The Sun also has a similar effect, but to a lesser extent than the Moon. Though the force of attraction of the Sun on the Earth is about 200 times that of the Moon, the differential force of attraction of the Sun (its tide raising force) at any point on the Earth, is lesser than that of the Moon mainly because the Sun being ninety three million miles away, the 4000 miles radius of the Earth does not produce a significant difference between the Sun's distance to the Earth's centre and its surface.

The tide raising force of the Sun to that of the Moon is in the ratio of about 3:7. The Sun therefore causes another ellipsoid of water with its major axis in the direction of the Sun. But this ellipsoid is less elongated than the one produced by the Moon. As the Earth rotates within this ellipsoid, the Sun causes two high waters and two low waters in 24 hours at any place. The **solar tide** therefore has a period of 12 hours.

15.2 RELATIONSHIP BETWEEN PHASES OF THE MOON AND TIDES

The combined effect of the lunar tide raising forces and the solar tide raising forces causes the 'LUNISOLAR TIDE'. At Full and New moons when the Sun is in conjunction and opposition respectively with the Moon, these two tide raising forces act in the same line producing very high high waters and very low low waters. The range of tide then would be large. These are known as SPRING TIDES.



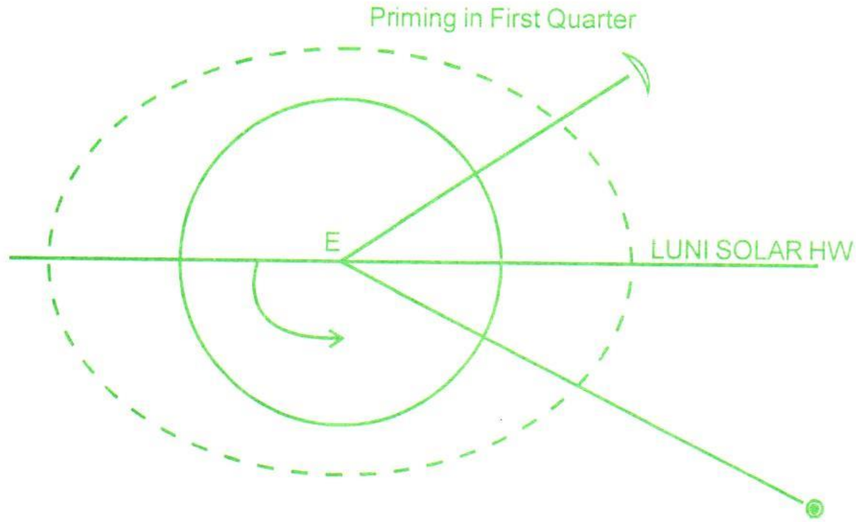
(FIG.15.2)

(FIG.15.3)

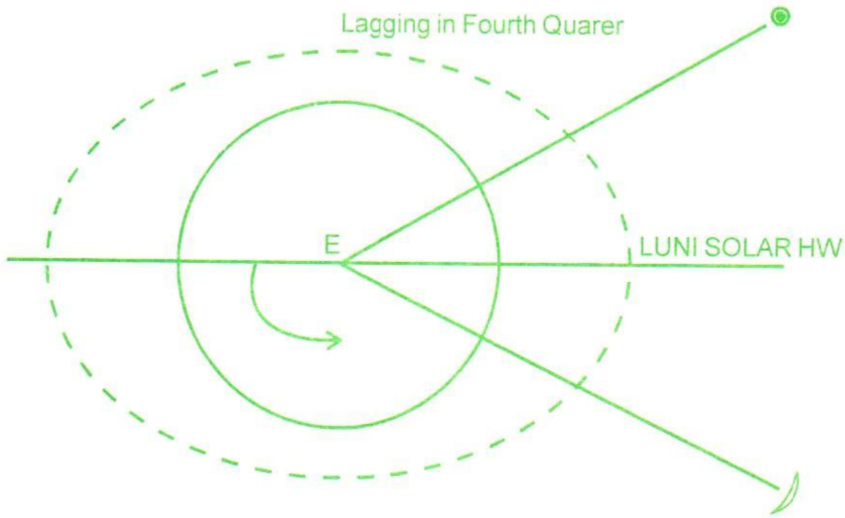
When the Moon is in quadrature, the tide raising force due to the Sun and that due to the Moon, act in direction 90° to each other. The solar tide then tends to produce a high water at points where low water occurs due to the lunar tide and vice versa. Thus at such times, the lunisolar high waters are not very high and the luni-solar low waters are not very low. Therefore the range of tide then is not very large. These are called NEAP TIDES.

At Full and New Moons since the tide raising forces of the Sun and Moon act in the same direction the lunisolar high waters would occur at the time of the Moon's upper and lower transits. When the Moon is in quadrature, the tide raising forces of the Sun and Moon act at right angles to each other, but due to the predominance of the Moon's tide raising force, the luni-solar high waters would still occur at the times of the Moon's upper and lower transits.

At intermediate positions of the Moon, the luni-solar high waters may occur before or after the upper and low transits of the Moon. When the Moon is in the first or third quarters, the solar high water occurs before the lunar high water. The luni-solar high water would therefore occur before the Moon's transit. The tide is then said to PRIME. During the second and the last quarters of the Moon, the Solar high water occurs after the lunar high water. The luni-solar high water therefore occurs after the Moon's transit. The tide is then said to LAG.



(FIG.15.4)



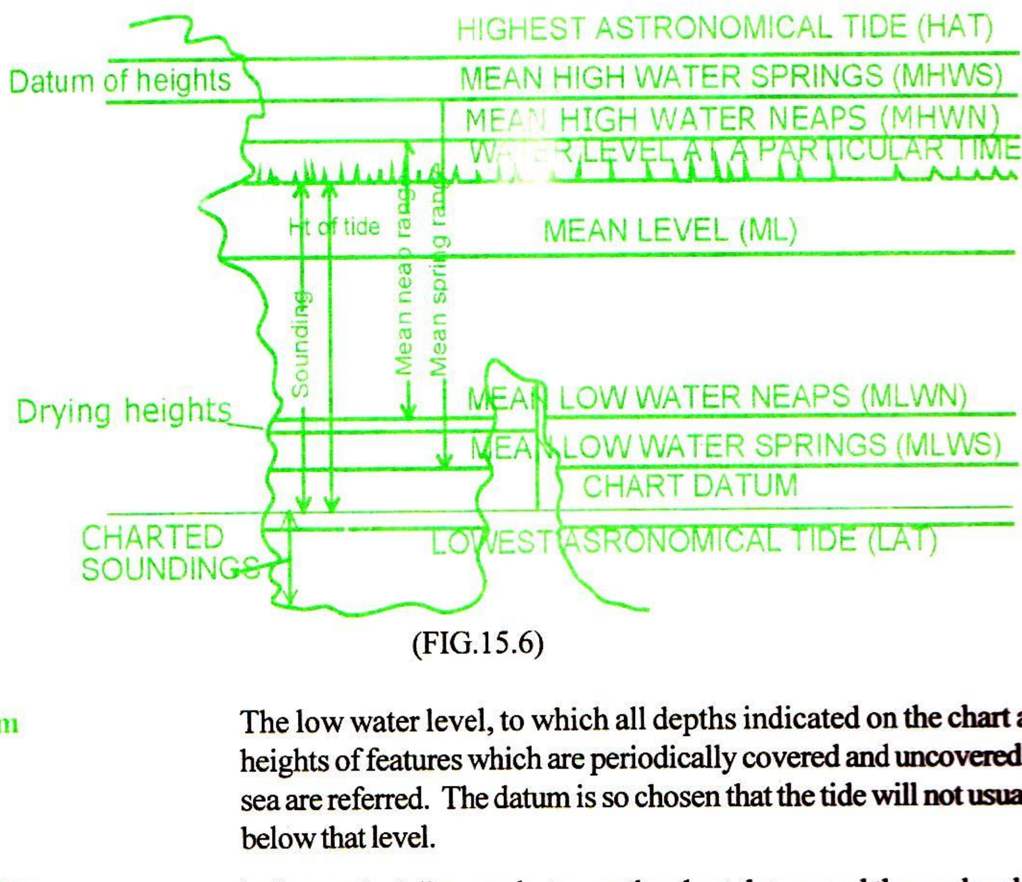
(FIG.15.5)

The above explanation of the equilibrium theory of tides was made on the assumption that the entire Earth is covered by water to a uniform height. However, due to intervening land masses causing bodies of water to have different natural periods of oscillation and due to the uneven depth of oceans and also due to the effect of the tidal waves entering shallow areas, the above described rhythmic oscillations are distorted considerably, producing more complex patterns of tide at different localities on the Earth.

It is possible that the tides are further modified by resonant oscillations of water bodies as well as by the coriolis forces due to the Earth's rotation. The fact that tides are more regional in character rather than a world phenomenon, support these ideas. The oscillations set up in the deep oceans by the tide raising forces, are of small amplitude. Thus the ebb and flow of tides are not noticeable in mid ocean. In shallow waters near the coast, the wave amplitude which was probably not more than one meter in the open ocean, increases considerably. The range of tide in certain funnel shaped estuaries of the world is as large as 10 meters.

Laplace was the first to suggest that tidal oscillations are actually composed of several harmonic motions caused by various periodic forces. The actual tides at any place are made up of a large number of harmonic constituents, some of which are diurnal and some semidiurnal. Tidal predictions are made with the help of mechanical aids or electronic computers using from 10 to 62 harmonic constituents. These predictions are tabulated in the Tide tables.

15.3 DEFINITIONS



(FIG.15.6)

Chart datum — The low water level, to which all depths indicated on the chart and all heights of features which are periodically covered and uncovered by the sea are referred. The datum is so chosen that the tide will not usually fall below that level.

Height of tide — is the vertical distance between the chart datum and the sea level at that time.

High water — The highest level reached by the sea during that tidal oscillation.

Low water — The lowest level reached by the sea during that tidal oscillation.

Range of tide — is the difference between the levels of successive high and lower waters.

Mean high water springs — (MHWS) is the average height, throughout the year, of two successive high waters during 24 hours, in each semi lunation, when the range of tide is greatest.

Mean low water springs (MLWS) is the average height, throughout the year of two successive low waters during 24 hours, in each semi lunation when the range of tide is greatest.

Mean high water neaps (MHWN) is the average height, throughout the year of two successive high waters during a period of 24 hours, in each semi lunation when the range of tide is least.

Mean low water neaps (MLWN) is the average height of two successive low waters during a period of 24 hours, in each semi lunation, when the range of tide is least.

Highest and lowest astronomical tide (HAT and LAT) are the highest and lowest tides that is possible to predict at standard ports, disregarding meteorological conditions.

Tidal stream is the periodical horizontal movement of the sea waters due to the tide raising forces of the Moon and Sun.

Slack water is the period when the tidal stream is at its weakest.

Bore is a rapid build up in the level of water due to a tidal wave of unusual height, in narrowing estuaries and rivers. The tidal wave is generated by the flood being held back by the sea ward outflow of the river. Bores occur within a few minutes of the predicted low water.

Flood tide is the inflow of water due to a rising tide.

Ebb tide is the outflow of water due to a falling tide.

A2 ALTITUDE CORRECTION TABLES 10°-90°—SUN, STARS, PLANETS

OCT.—MAR. SUN			APR.—SEPT. SUN			STARS AND PLANETS		DIP		
App. Alt.	Lower Limb	Upper Limb	App. Alt.	Lower Limb	Upper Limb	App. Alt.	Corrⁿ	Ht. of Eye	Corrⁿ	Ht. of Eye
° ′	′	′	° ′	′	′	° ′	′	m	′	ft.
9 34			9 39			9 56		2·4		8·0
	+10·8	−21·5		+10·6	−21·2		−5·3		−2·8	
9 45			9 51			10 08		2·6		8·6
	+10·9	−21·4		+10·7	−21·1		−5·2		−2·9	
9 56			10 03			10 20		2·8		9·2
	+11·0	−21·3		+10·8	−21·0		−5·1		−3·0	
10 08			10 15			10 33		3·0		9·8
	+11·1	−21·2		+10·9	−20·9		−5·0		−3·1	
10 21			10 27			10 46		3·2		10·5
	+11·2	−21·1		+11·0	−20·8		−4·9		−3·2	
10 34			10 40			11 00		3·4		11·2
	+11·3	−21·0		+11·1	−20·7		−4·8		−3·3	
10 47			10 54			11 14		3·6		11·9
	+11·4	−20·9		+11·2	−20·6		−4·7		−3·4	
11 01			11 08			11 29		3·8		12·6
	+11·5	−20·8		+11·3	−20·5		−4·6		−3·5	
11 15			11 23			11 45		4·0		13·3
	+11·6	−20·7		+11·4	−20·4		−4·5		−3·6	
11 30			11 38			12 01		4·3		14·1
	+11·7	−20·6		+11·5	−20·3		−4·4		−3·7	
11 46			11 54			12 18		4·5		14·9
	+11·8	−20·5		+11·6	−20·2		−4·3		−3·8	
12 02			12 10			12 35		4·7		15·7
	+11·9	−20·4		+11·7	−20·1		−4·2		−3·9	
12 19			12 28			12 54		5·0		16·5
	+12·0	−20·3		+11·8	−20·0		−4·1		−4·0	
12 37			12 46			13 13		5·2		17·4
	+12·1	−20·2		+11·9	−19·9		−4·0		−4·1	
12 55			13 05			13 33		5·5		18·3
	+12·2	−20·1		+12·0	−19·8		−3·9		−4·2	
13 14			13 24			13 54		5·8		19·1
	+12·3	−20·0		+12·1	−19·7		−3·8		−4·3	
13 35			13 45			14 16		6·1		20·1
	+12·4	−19·9		+12·2	−19·6		−3·7		−4·4	
13 56			14 07			14 40		6·3		21·0
	+12·5	−19·8		+12·3	−19·5		−3·6		−4·5	
14 18			14 30			15 04		6·6		22·0
	+12·6	−19·7		+12·4	−19·4		−3·5		−4·6	
14 42			14 54			15 30		6·9		22·9
	+12·7	−19·6		+12·5	−19·3		−3·4		−4·7	
15 06			15 19			15 57		7·2		23·9
	+12·8	−19·5		+12·6	−19·2		−3·3		−4·8	
15 32			15 46			16 26		7·5		24·9
	+12·9	−19·4		+12·7	−19·1		−3·2		−4·9	
15 59			16 14			16 56		7·9		26·0
	+13·0	−19·3		+12·8	−19·0		−3·1		−5·0	
16 28			16 44			17 28		8·2		27·1
	+13·1	−19·2		+12·9	−18·9		−3·0		−5·1	
16 59			17 15			18 02		8·5		28·1
	+13·2	−19·1		+13·0	−18·8		−2·9		−5·2	
17 32			17 48			18 38		8·8		29·2
	+13·3	−19·0		+13·1	−18·7		−2·8		−5·3	
18 06			18 24			19 17		9·2		30·4
	+13·4	−18·9		+13·2	−18·6		−2·7		−5·4	
18 42			19 01			19 58		9·5		31·5
	+13·5	−18·8		+13·3	−18·5		−2·6		−5·5	
19 21			19 42			20 42		9·9		32·7
	+13·6	−18·7		+13·4	−18·4		−2·5		−5·6	
20 03			20 25			21 28		10·3		33·9
	+13·7	−18·6		+13·5	−18·3		−2·4		−5·7	
20 48			21 11			22 19		10·6		35·1
	+13·8	−18·5		+13·6	−18·2		−2·3		−5·8	
21 35			22 00			23 13		11·0		36·3
	+13·9	−18·4		+13·7	−18·1		−2·2		−5·9	
22 26			22 54			24 11		11·4		37·6
	+14·0	−18·3		+13·8	−18·0		−2·1		−6·0	
23 22			23 51			25 14		11·8		38·9
	+14·1	−18·2		+13·9	−17·9		−2·0		−6·1	
24 21			24 53			26 22		12·2		40·1
	+14·2	−18·1		+14·0	−17·8		−1·9		−6·2	
25 26			26 00			27 36		12·6		41·5
	+14·3	−18·0		+14·1	−17·7		−1·8		−6·3	
26 36			27 13			28 56		13·0		42·8
	+14·4	−17·9		+14·2	−17·6		−1·7		−6·4	
27 52			28 33			30 24		13·4		44·2
	+14·5	−17·8		+14·3	−17·5		−1·6		−6·5	
29 15			30 00			32 00		13·8		45·5
	+14·6	−17·7		+14·4	−17·4		−1·5		−6·6	
30 46			31 35			33 45		14·2		46·9
	+14·7	−17·6		+14·5	−17·3		−1·4		−6·7	
32 26			33 20			35 40		14·7		48·4
	+14·8	−17·5		+14·6	−17·2		−1·3		−6·8	
34 17			35 17			37 48		15·1		49·8
	+14·9	−17·4		+14·7	−17·1		−1·2		−6·9	
36 20			37 26			40 08		15·5		51·3
	+15·0	−17·3		+14·8	−17·0		−1·1		−7·0	
38 36			39 50			42 44		16·0		52·8
	+15·1	−17·2		+14·9	−16·9		−1·0		−7·1	
41 08			42 31			45 36		16·5		54·3
	+15·2	−17·1		+15·0	−16·8		−0·9		−7·2	
43 59			45 31			48 47		16·9		55·8
	+15·3	−17·0		+15·1	−16·7		−0·8		−7·3	
47 10			48 55			52 18		17·4		57·4
	+15·4	−16·9		+15·2	−16·6		−0·7		−7·4	
50 46			52 44			56 11		17·9		58·9
	+15·5	−16·8		+15·3	−16·5		−0·6		−7·5	
54 49			57 02			60 28		18·4		60·5
	+15·6	−16·7		+15·4	−16·4		−0·5		−7·6	
59 23			61 51			65 08		18·8		62·1
	+15·7	−16·6		+15·5	−16·3		−0·4		−7·7	
64 30			67 17			70 11		19·3		63·8
	+15·8	−16·5		+15·6	−16·2		−0·3		−7·8	
70 12			73 16			75 34		19·8		65·4
	+15·9	−16·4		+15·7	−16·1		−0·2		−7·9	
76 26			79 43			81 13		20·4		67·1
	+16·0	−16·3		+15·8	−16·0		−0·1		−8·0	
83 05			86 32			87 03		20·9		68·8
	+16·1	−16·2		+15·9	−15·9		0·0		−8·1	
90 00			90 00			90 00		21·4		70·5

STARS AND PLANETS — Additional Corrⁿ

App. Alt.	Additional Corrⁿ
1976	
VENUS	
Jan. 1 — Dec. 12	
0°	
	+ 0′·1
42	
Dec. 13 — Dec. 31	
0°	
	+ 0′·2
47	
MARS	
Jan. 1 — Feb. 19	
0°	
	+ 0′·2
41	
	+ 0·1
75	
Feb. 20 — Dec. 31	
0°	
	+ 0′·1
60	

DIP

Ht. of Eye	Corrⁿ
m	′
1·0	− 1·8
1·5	− 2·2
2·0	− 2·5
2·5	− 2·8
3·0	− 3·0
See table ←	
m	′
20	− 7·9
22	− 8·3
24	− 8·6
26	− 9·0
28	− 9·3
30	− 9·6
32	−10·0
34	−10·3
36	−10·6
38	−10·8
40	−11·1
42	−11·4
44	−11·7
46	−11·9
48	−12·2
ft.	′
2	− 1·4
4	− 1·9
6	− 2·4
8	− 2·7
10	− 3·1
See table ←	
ft.	′
70	− 8·1
75	− 8·4
80	− 8·7
85	− 8·9
90	− 9·2
95	− 9·5
100	− 9·7
105	− 9·9
110	−10·2
115	−10·4
120	−10·6
125	−10·8
130	−11·1
135	−11·3
140	−11·5
145	−11·7
150	−11·9
155	−12·1

App. Alt. = Apparent altitude = Sextant altitude corrected for index error and dip.
For daylight observations of Venus, see page 260.

ALTITUDE CORRECTION TABLES 0°–10°—SUN, STARS, PLANETS A3

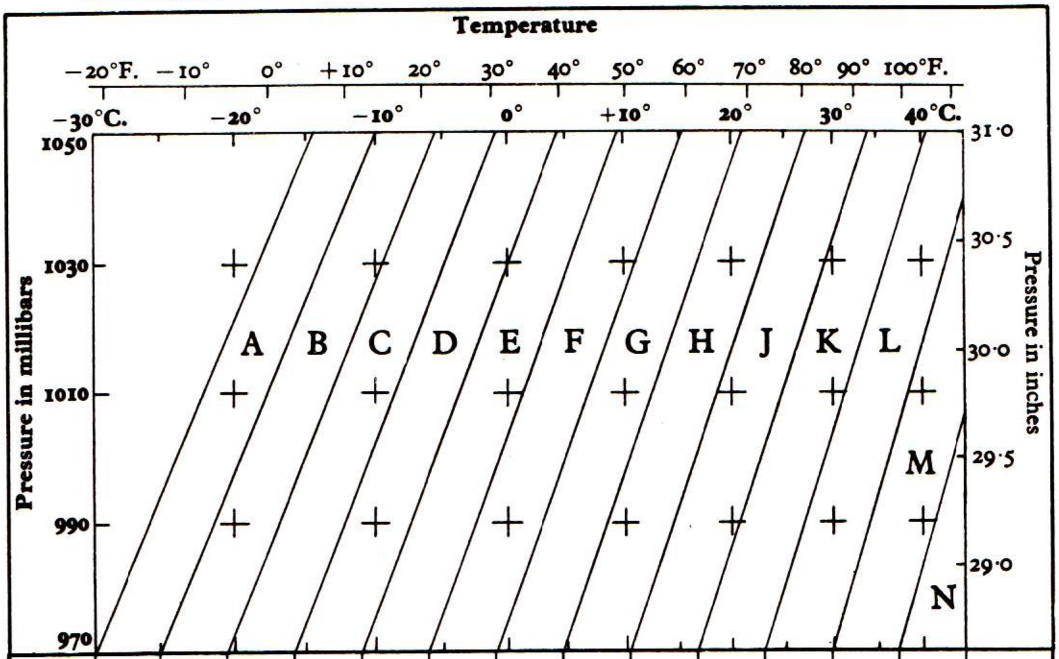
App. Alt.	SUN OCT.–MAR. Lower Limb	SUN OCT.–MAR. Upper Limb	SUN APR.–SEPT. Lower Limb	SUN APR.–SEPT. Upper Limb	STARS PLANETS
° ′	′	′	′	′	′
0 00	−18·2	−50·5	−18·4	−50·2	−34·5
03	17·5	49·8	17·8	49·6	33·8
06	16·9	49·2	17·1	48·9	33·2
09	16·3	48·6	16·5	48·3	32·6
12	15·7	48·0	15·9	47·7	32·0
15	15·1	47·4	15·3	47·1	31·4
0 18	−14·5	−46·8	−14·8	−46·6	−30·8
21	14·0	46·3	14·2	46·0	30·3
24	13·5	45·8	13·7	45·5	29·8
27	12·9	45·2	13·2	45·0	29·2
30	12·4	44·7	12·7	44·5	28·7
33	11·9	44·2	12·2	44·0	28·2
0 36	−11·5	−43·8	−11·7	−43·5	−27·8
39	11·0	43·3	11·2	43·0	27·3
42	10·5	42·8	10·8	42·6	26·8
45	10·1	42·4	10·3	42·1	26·4
48	9·6	41·9	9·9	41·7	25·9
51	9·2	41·5	9·5	41·3	25·5
0 54	− 8·8	−41·1	− 9·1	−40·9	−25·1
0 57	8·4	40·7	8·7	40·5	24·7
1 00	8·0	40·3	8·3	40·1	24·3
03	7·7	40·0	7·9	39·7	24·0
06	7·3	39·6	7·5	39·3	23·6
09	6·9	39·2	7·2	39·0	23·2
1 12	− 6·6	−38·9	− 6·8	−38·6	−22·9
15	6·2	38·5	6·5	38·3	22·5
18	5·9	38·2	6·2	38·0	22·2
21	5·6	37·9	5·8	37·6	21·9
24	5·3	37·6	5·5	37·3	21·6
27	4·9	37·2	5·2	37·0	21·2
1 30	− 4·6	−36·9	− 4·9	−36·7	−20·9
35	4·2	36·5	4·4	36·2	20·5
40	3·7	36·0	4·0	35·8	20·0
45	3·2	35·5	3·5	35·3	19·5
50	2·8	35·1	3·1	34·9	19·1
1 55	2·4	34·7	2·6	34·4	18·7
2 00	− 2·0	−34·3	− 2·2	−34·0	−18·3
05	1·6	33·9	1·8	33·6	17·9
10	1·2	33·5	1·5	33·3	17·5
15	0·9	33·2	1·1	32·9	17·2
20	0·5	32·8	0·8	32·6	16·8
25	− 0·2	32·5	0·4	32·2	16·5
2 30	+ 0·2	−32·1	− 0·1	−31·9	−16·1
35	0·5	31·8	+ 0·2	31·6	15·8
40	0·8	31·5	0·5	31·3	15·5
45	1·1	31·2	0·8	31·0	15·2
50	1·4	30·9	1·1	30·7	14·9
2 55	1·6	30·7	1·4	30·4	14·7
3 00	+ 1·9	−30·4	+ 1·7	−30·1	−14·4
05	2·2	30·1	1·9	29·9	14·1
10	2·4	29·9	2·1	29·7	13·9
15	2·6	29·7	2·4	29·4	13·7
20	2·9	29·4	2·6	29·2	13·4
25	3·1	29·2	2·9	28·9	13·2
3 30	+ 3·3	−29·0	+ 3·1	−28·7	−13·0

App. Alt.	SUN OCT.–MAR. Lower Limb	SUN OCT.–MAR. Upper Limb	SUN APR.–SEPT. Lower Limb	SUN APR.–SEPT. Upper Limb	STARS PLANETS
° ′	′	′	′	′	′
3 30	+ 3·3	−29·0	+ 3·1	−28·7	−13·0
35	3·6	28·7	3·3	28·5	12·7
40	3·8	28·5	3·5	28·3	12·5
45	4·0	28·3	3·7	28·1	12·3
50	4·2	28·1	3·9	27·9	12·1
3 55	4·4	27·9	4·1	27·7	11·9
4 00	+ 4·5	−27·8	+ 4·3	−27·5	−11·8
05	4·7	27·6	4·5	27·3	11·6
10	4·9	27·4	4·6	27·2	11·4
15	5·1	27·2	4·8	27·0	11·2
20	5·2	27·1	5·0	26·8	11·1
25	5·4	26·9	5·1	26·7	10·9
4 30	+ 5·6	−26·7	+ 5·3	−26·5	−10·7
35	5·7	26·6	5·5	26·3	10·6
40	5·9	26·4	5·6	26·2	10·4
45	6·0	26·3	5·8	26·0	10·3
50	6·2	26·1	5·9	25·9	10·1
4 55	6·3	26·0	6·0	25·8	10·0
5 00	+ 6·4	−25·9	+ 6·2	−25·6	− 9·9
05	6·6	25·7	6·3	25·5	9·7
10	6·7	25·6	6·4	25·4	9·6
15	6·8	25·5	6·6	25·2	9·5
20	6·9	25·4	6·7	25·1	9·4
25	7·1	25·2	6·8	25·0	9·2
5 30	+ 7·2	−25·1	+ 6·9	−24·9	− 9·1
35	7·3	25·0	7·0	24·8	9·0
40	7·4	24·9	7·2	24·6	8·9
45	7·5	24·8	7·3	24·5	8·8
50	7·6	24·7	7·4	24·4	8·7
5 55	7·7	24·6	7·5	24·3	8·6
6 00	+ 7·8	−24·5	+ 7·6	−24·2	− 8·5
10	8·0	24·3	7·8	24·0	8·3
20	8·2	24·1	8·0	23·8	8·1
30	8·4	23·9	8·1	23·7	7·9
40	8·6	23·7	8·3	23·5	7·7
6 50	8·7	23·6	8·5	23·3	7·6
7 00	+ 8·9	−23·4	+ 8·6	−23·2	− 7·4
10	9·1	23·2	8·8	23·0	7·2
20	9·2	23·1	9·0	22·8	7·1
30	9·3	23·0	9·1	22·7	7·0
40	9·5	22·8	9·2	22·6	6·8
7 50	9·6	22·7	9·4	22·4	6·7
8 00	+ 9·7	−22·6	+ 9·5	−22·3	− 6·6
10	9·9	22·4	9·6	22·2	6·4
20	10·0	22·3	9·7	22·1	6·3
30	10·1	22·2	9·8	22·0	6·2
40	10·2	22·1	10·0	21·8	6·1
8 50	10·3	22·0	10·1	21·7	6·0
9 00	+10·4	−21·9	+10·2	−21·6	− 5·9
10	10·5	21·8	10·3	21·5	5·8
20	10·6	21·7	10·4	21·4	5·7
30	10·7	21·6	10·5	21·3	5·6
40	10·8	21·5	10·6	21·2	5·5
9 50	10·9	21·4	10·6	21·2	5·4
10 00	+11·0	−21·3	+10·7	−21·1	− 5·3

Additional corrections for temperature and pressure are given on the following page.
For bubble sextant observations ignore dip and use the star corrections for Sun, planets, and stars.

A4 ALTITUDE CORRECTION TABLES—ADDITIONAL CORRECTIONS

ADDITIONAL REFRACTION CORRECTIONS FOR NON-STANDARD CONDITIONS



App. Alt.	A	B	C	D	E	F	G	H	J	K	L	M	N	App. Alt.
° ′	′	′	′	′	′	′	′	′	′	′	′	′	′	°
0 00	−6·9	−5·7	−4·6	−3·4	−2·3	−1·1	0·0	+1·1	+2·3	+3·4	+4·6	+5·7	+6·9	0 00
0 30	5·2	4·4	3·5	2·6	1·7	0·9	0·0	0·9	1·7	2·6	3·5	4·4	5·2	0 30
1 00	4·3	3·5	2·8	2·1	1·4	0·7	0·0	0·7	1·4	2·1	2·8	3·5	4·3	1 00
1 30	3·5	2·9	2·4	1·8	1·2	0·6	0·0	0·6	1·2	1·8	2·4	2·9	3·5	1 30
2 00	3·0	2·5	2·0	1·5	1·0	0·5	0·0	0·5	1·0	1·5	2·0	2·5	3·0	2 00
2 30	−2·5	−2·1	−1·6	−1·2	−0·8	−0·4	0·0	+0·4	+0·8	+1·2	+1·6	+2·1	+2·5	2 30
3 00	2·2	1·8	1·5	1·1	0·7	0·4	0·0	0·4	0·7	1·1	1·5	1·8	2·2	3 00
3 30	2·0	1·6	1·3	1·0	0·7	0·3	0·0	0·3	0·7	1·0	1·3	1·6	2·0	3 30
4 00	1·8	1·5	1·2	0·9	0·6	0·3	0·0	0·3	0·6	0·9	1·2	1·5	1·8	4 00
4 30	1·6	1·4	1·1	0·8	0·5	0·3	0·0	0·3	0·5	0·8	1·1	1·4	1·6	4 30
5 00	−1·5	−1·3	−1·0	−0·8	−0·5	−0·2	0·0	+0·2	+0·5	+0·8	+1·0	+1·3	+1·5	5 00
6	1·3	1·1	0·9	0·6	0·4	0·2	0·0	0·2	0·4	0·6	0·9	1·1	1·3	6
7	1·1	0·9	0·7	0·6	0·4	0·2	0·0	0·2	0·4	0·6	0·7	0·9	1·1	7
8	1·0	0·8	0·7	0·5	0·3	0·2	0·0	0·2	0·3	0·5	0·7	0·8	1·0	8
9	0·9	0·7	0·6	0·4	0·3	0·1	0·0	0·1	0·3	0·4	0·6	0·7	0·9	9
10 00	−0·8	−0·7	−0·5	−0·4	−0·3	−0·1	0·0	+0·1	+0·3	+0·4	+0·5	+0·7	+0·8	10 00
12	0·7	0·6	0·5	0·3	0·2	0·1	0·0	0·1	0·2	0·3	0·5	0·6	0·7	12
14	0·6	0·5	0·4	0·3	0·2	0·1	0·0	0·1	0·2	0·3	0·4	0·5	0·6	14
16	0·5	0·4	0·3	0·3	0·2	0·1	0·0	0·1	0·2	0·3	0·3	0·4	0·5	16
18	0·4	0·4	0·3	0·2	0·2	0·1	0·0	0·1	0·2	0·2	0·3	0·4	0·4	18
20 00	−0·4	−0·3	−0·3	−0·2	−0·1	−0·1	0·0	+0·1	+0·1	+0·2	+0·3	+0·3	+0·4	20 00
25	0·3	0·3	0·2	0·2	0·1	−0·1	0·0	+0·1	0·1	0·2	0·2	0·3	0·3	25
30	0·3	0·2	0·2	0·1	0·1	0·0	0·0	0·0	0·1	0·1	0·2	0·2	0·3	30
35	0·2	0·2	0·1	0·1	0·1	0·0	0·0	0·0	0·1	0·1	0·1	0·2	0·2	35
40	0·2	0·1	0·1	0·1	−0·1	0·0	0·0	0·0	+0·1	0·1	0·1	0·1	0·2	40
50 00	−0·1	−0·1	−0·1	−0·1	0·0	0·0	0·0	0·0	0·0	+0·1	+0·1	+0·1	+0·1	50 00

The graph is entered with arguments temperature and pressure to find a zone letter; using as arguments this zone letter and apparent altitude (sextant altitude corrected for dip), a correction is taken from the table. This correction is to be applied to the sextant altitude in addition to the corrections for standard conditions (for the Sun, stars and planets from page A2 and for the Moon from pages xxxiv and xxxv).

1976 JUNE 23, 24, 25 (WED., THURS. FRI.)

	G.M.T.	ARIES G.H.A.	VENUS −3.5 G.H.A.	VENUS Dec.	MARS +1.8 G.H.A.	MARS Dec.	JUPITER −1.7 G.H.A.	JUPITER Dec.	SATURN +0.5 G.H.A.	SATURN Dec.	STARS Name	S.H.A.	Dec.
	d h	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′		° ′	° ′
	23 00	271 17.3	178 02.3	N23 51.0	126 51.5	N15 28.7	222 58.9	N16 56.5	146 56.9	N20 10.3	Acamar	315 39.7	S40 23.8
	01	286 19.8	193 01.4	51.1	141 52.5	28.2	238 00.9	56.7	161 59.1	10.3	Achernar	335 47.7	S57 21.1
	02	301 22.2	208 00.5	51.1	156 53.6	27.7	253 02.8	56.8	177 01.3	10.2	Acrux	173 40.3	S62 58.5
	03	316 24.7	222 59.6	·· 51.1	171 54.6	·· 27.2	268 04.8	·· 57.0	192 03.4	·· 10.1	Adhara	255 34.7	S28 56.5
	04	331 27.2	237 58.7	51.1	186 55.6	26.7	283 06.7	57.1	207 05.6	10.1	Aldebaran	291 21.6	N16 27.7
	05	346 29.6	252 57.8	51.2	201 56.6	26.2	298 08.6	57.2	222 07.7	10.0			
	06	1 32.1	267 56.9	N23 51.2	216 57.6	N15 25.7	313 10.6	N16 57.4	237 09.9	N20 09.9	Alioth	166 45.0	N56 05.5
W	07	16 34.5	282 56.0	51.2	231 58.7	25.2	328 12.5	57.5	252 12.1	09.9	Alkaid	153 20.6	N49 26.0
E	08	31 37.0	297 55.1	51.2	246 59.7	24.7	343 14.5	57.6	267 14.2	09.8	Al Na'ir	28 18.3	S47 04.2
D	09	46 39.5	312 54.2	·· 51.2	262 00.7	·· 24.2	358 16.4	·· 57.8	282 16.4	·· 09.7	Alnilam	276 14.9	S 1 13.1
N	10	61 41.9	327 53.3	51.2	277 01.7	23.7	13 18.4	57.9	297 18.5	09.7	Alphard	218 23.6	S 8 33.5
E	11	76 44.4	342 52.4	51.2	292 02.7	23.2	28 20.3	58.0	312 20.7	09.6			
S	12	91 46.9	357 51.5	N23 51.3	307 03.7	N15 22.7	43 22.3	N16 58.2	327 22.8	N20 09.6	Alphecca	126 34.2	N26 47.7
D	13	106 49.3	12 50.6	51.3	322 04.8	22.2	58 24.2	58.3	342 25.0	09.5	Alpheratz	358 12.2	N28 57.6
A	14	121 51.8	27 49.7	51.3	337 05.8	21.7	73 26.1	58.4	357 27.2	09.4	Altair	62 35.0	N 8 48.4
Y	15	136 54.3	42 48.8	·· 51.3	352 06.8	·· 21.2	88 28.1	·· 58.6	12 29.3	·· 09.4	Ankaa	353 43.1	S42 25.7
	16	151 56.7	57 47.9	51.3	7 07.8	20.7	103 30.0	58.7	27 31.5	09.3	Antares	113 00.0	S26 22.8
	17	166 59.2	72 47.0	51.3	22 08.8	20.2	118 32.0	58.8	42 33.6	09.2			
	18	182 01.6	87 46.1	N23 51.3	37 09.9	N15 19.6	133 33.9	N16 59.0	57 35.8	N20 09.2	Arcturus	146 20.9	N19 18.4
	19	197 04.1	102 45.2	51.3	52 10.9	19.1	148 35.9	59.1	72 37.9	09.1	Atria	108 26.2	S68 59.2
	20	212 06.6	117 44.3	51.3	67 11.9	18.6	163 37.8	59.2	87 40.1	09.0	Avior	234 29.9	S59 26.3
	21	227 09.0	132 43.4	·· 51.3	82 12.9	·· 18.1	178 39.8	·· 59.4	102 42.3	·· 09.0	Bellatrix	279 02.1	N 6 19.7
	22	242 11.5	147 42.5	51.3	97 13.9	17.6	193 41.7	59.5	117 44.4	08.9	Betelgeuse	271 31.7	N 7 24.1
	23	257 14.0	162 41.7	51.3	112 14.9	17.1	208 43.6	59.6	132 46.6	08.9			
	24 00	272 16.4	177 40.8	N23 51.3	127 16.0	N15 16.6	223 45.6	N16 59.8	147 48.7	N20 08.8	Canopus	264 09.0	S52 41.1
	01	287 18.9	192 39.9	51.3	142 17.0	16.1	238 47.5	16 59.9	162 50.9	08.7	Capella	281 16.0	N45 58.4
	02	302 21.4	207 39.0	51.3	157 18.0	15.6	253 49.5	17 00.0	177 53.0	08.7	Deneb	49 50.0	N45 11.7
	03	317 23.8	222 38.1	·· 51.3	172 19.0	·· 15.1	268 51.4	·· 00.2	192 55.2	·· 08.6	Denebola	183 02.0	N14 42.2
	04	332 26.3	237 37.2	51.3	187 20.0	14.6	283 53.4	00.3	207 57.4	08.5	Diphda	349 23.8	S18 06.8
	05	347 28.8	252 36.3	51.3	202 21.1	14.1	298 55.3	00.4	222 59.5	08.5			
	06	2 31.2	267 35.4	N23 51.2	217 22.1	N15 13.6	313 57.3	N17 00.6	238 01.7	N20 08.4	Dubhe	194 25.9	N61 52.9
	07	17 33.7	282 34.5	51.2	232 23.1	13.1	328 59.2	00.7	253 03.8	08.3	Elnath	278 48.1	N28 35.2
T	08	32 36.1	297 33.6	51.2	247 24.1	12.5	344 01.2	00.8	268 06.0	08.3	Eltanin	90 58.5	N51 29.6
H	09	47 38.6	312 32.7	·· 51.2	262 25.1	·· 12.0	359 03.1	·· 01.0	283 08.1	·· 08.2	Enif	34 14.2	N 9 46.1
U	10	62 41.1	327 31.8	51.2	277 26.1	11.5	14 05.0	01.1	298 10.3	08.1	Fomalhaut	15 54.5	S29 44.6
R	11	77 43.5	342 30.9	51.2	292 27.2	11.0	29 07.0	01.2	313 12.5	08.1			
S	12	92 46.0	357 30.0	N23 51.1	307 28.2	N15 10.5	44 08.9	N17 01.4	328 14.6	N20 08.0	Gacrux	172 31.8	S56 59.2
D	13	107 48.5	12 29.1	51.1	322 29.2	10.0	59 10.9	01.5	343 16.8	08.0	Gienah	176 20.9	S17 24.8
A	14	122 50.9	27 28.2	51.1	337 30.2	09.5	74 12.8	01.6	358 18.9	07.9	Hadar	149 27.0	S60 15.9
Y	15	137 53.4	42 27.3	·· 51.1	352 31.2	·· 09.0	89 14.8	·· 01.8	13 21.1	·· 07.8	Hamal	328 32.3	N23 21.0
	16	152 55.9	57 26.4	51.1	7 32.3	08.5	104 16.7	01.9	28 23.2	07.8	Kaus Aust.	84 20.2	S34 23.7
	17	167 58.3	72 25.5	51.0	22 33.3	08.0	119 18.7	02.0	43 25.4	07.7			
	18	183 00.8	87 24.6	N23 51.0	37 34.3	N15 07.5	134 20.6	N17 02.2	58 27.5	N20 07.6	Kochab	137 18.2	N74 15.4
	19	198 03.3	102 23.7	51.0	52 35.3	06.9	149 22.6	02.3	73 29.7	07.6	Markab	14 05.9	N15 04.7
	20	213 05.7	117 22.8	51.0	67 36.3	06.4	164 24.5	02.4	88 31.9	07.5	Menkar	314 44.3	N 3 59.8
	21	228 08.2	132 21.9	·· 50.9	82 37.4	·· 05.9	179 26.5	·· 02.6	103 34.0	·· 07.4	Menkent	148 40.1	S36 15.5
	22	243 10.6	147 21.0	50.9	97 38.4	05.4	194 28.4	02.7	118 36.2	07.4	Miaplacidus	221 46.1	S69 37.6
	23	258 13.1	162 20.1	50.9	112 39.4	04.9	209 30.4	02.8	133 38.3	07.3			
	25 00	273 15.6	177 19.2	N23 50.8	127 40.4	N15 04.4	224 32.3	N17 03.0	148 40.5	N20 07.2	Mirfak	309 20.5	N49 46.5
	01	288 18.0	192 18.3	50.8	142 41.4	03.9	239 34.3	03.1	163 42.6	07.2	Nunki	76 32.3	S26 19.5
	02	303 20.5	207 17.4	50.8	157 42.4	03.4	254 36.2	03.2	178 44.8	07.1	Peacock	54 02.4	S56 48.4
	03	318 23.0	222 16.5	·· 50.7	172 43.5	·· 02.9	269 38.2	·· 03.4	193 46.9	·· 07.1	Pollux	244 02.0	N28 05.0
	04	333 25.4	237 15.7	50.7	187 44.5	02.4	284 40.1	03.5	208 49.1	07.0	Procyon	245 29.1	N 5 17.1
	05	348 27.9	252 14.8	50.6	202 45.5	01.8	299 42.1	03.6	223 51.3	06.9			
	06	3 30.4	267 13.9	N23 50.6	217 46.5	N15 01.3	314 44.0	N17 03.8	238 53.4	N20 06.9	Rasalhague	96 31.9	N12 34.7
	07	18 32.8	282 13.0	50.6	232 47.5	00.8	329 45.9	03.9	253 55.6	06.8	Regulus	208 13.2	N12 04.9
	08	33 35.3	297 12.1	50.5	247 48.6	15 00.3	344 47.9	04.0	268 57.7	06.7	Rigel	281 39.1	S 8 13.8
F	09	48 37.8	312 11.2	·· 50.5	262 49.6	14 59.8	359 49.8	·· 04.1	283 59.9	·· 06.7	Rigil Kent.	140 29.2	S60 44.5
R	10	63 40.2	327 10.3	50.4	277 50.6	59.3	14 51.8	04.3	299 02.0	06.6	Sabik	102 44.1	S15 41.7
I	11	78 42.7	342 09.4	50.4	292 51.6	58.8	29 53.7	04.4	314 04.2	06.5			
D	12	93 45.1	357 08.5	N23 50.3	307 52.6	N14 58.3	44 55.7	N17 04.5	329 06.3	N20 06.5	Schedar	350 12.3	N56 24.3
A	13	108 47.6	12 07.6	50.3	322 53.7	57.7	59 57.6	04.7	344 08.5	06.4	Shaula	96 59.2	S37 05.2
Y	14	123 50.1	27 06.7	50.2	337 54.7	57.2	74 59.6	04.8	359 10.7	06.3	Sirius	258 58.6	S16 41.2
	15	138 52.5	42 05.8	·· 50.2	352 55.7	·· 56.7	90 01.5	·· 04.9	14 12.8	·· 06.3	Spica	159 00.4	S11 02.4
	16	153 55.0	57 04.9	50.1	7 56.7	56.2	105 03.5	05.1	29 15.0	06.2	Suhail	223 13.2	S43 20.5
	17	168 57.5	72 04.0	50.1	22 57.7	55.7	120 05.4	05.2	44 17.1	06.2			
	18	183 59.9	87 03.1	N23 50.0	37 58.8	N14 55.2	135 07.4	N17 05.3	59 19.3	N20 06.1	Vega	80 57.3	N38 45.8
	19	199 02.4	102 02.2	50.0	52 59.8	54.7	150 09.3	05.5	74 21.4	06.0	Zuben'ubi	137 36.0	S15 56.7
	20	214 04.9	117 01.3	49.9	68 00.8	54.1	165 11.3	05.6	89 23.6	06.0			
	21	229 07.3	132 00.4	·· 49.8	83 01.8	·· 53.6	180 13.2	·· 05.7	104 25.7	·· 05.9			
	22	244 09.8	146 59.5	49.8	98 02.8	53.1	195 15.2	05.9	119 27.9	05.8			
	23	259 12.2	161 58.6	49.7	113 03.8	52.6	210 17.1	06.0	134 30.0	05.8			
	Mer. Pass.	h m 5 49.9	v −0.9	d 0.0	v 1.0	d 0.5	v 1.9	d 0.1	v 2.2	d 0.1			

	S.H.A. ° ′	Mer. Pass. h m
Venus	265 24.3	12 10
Mars	214 59.5	15 30
Jupiter	311 29.2	9 04
Saturn	235 32.3	14 07

	G.M.T.	SUN G.H.A.	SUN Dec.	MOON G.H.A.	MOON v	MOON Dec.	MOON d	MOON H.P.
	d h	° ′	° ′	° ′	′	° ′	′	′
	23 00	179 29.1	N23 25.8	234 33.3	13.4	N14 45.6	6.9	54.3
	01	194 28.9	25.8	249 05.7	13.5	14 52.5	6.8	54.3
	02	209 28.8	25.7	263 38.2	13.3	14 59.3	6.8	54.3
	03	224 28.7	·· 25.7	278 10.5	13.4	15 06.1	6.7	54.4
	04	239 28.5	25.7	292 42.9	13.3	15 12.8	6.6	54.4
	05	254 28.4	25.6	307 15.2	13.2	15 19.4	6.6	54.4
	06	269 28.3	N23 25.6	321 47.4	13.2	N15 26.0	6.5	54.4
W	07	284 28.1	25.6	336 19.6	13.2	15 32.5	6.5	54.4
E	08	299 28.0	25.5	350 51.8	13.1	15 39.0	6.4	54.4
D	09	314 27.9	·· 25.5	5 23.9	13.1	15 45.4	6.3	54.4
N	10	329 27.7	25.5	19 56.0	13.0	15 51.7	6.2	54.4
E	11	344 27.6	25.4	34 28.0	13.0	15 57.9	6.2	54.4
S	12	359 27.5	N23 25.4	49 00.0	12.9	N16 04.1	6.1	54.4
D	13	14 27.3	25.3	63 31.9	12.9	16 10.2	6.1	54.5
A	14	29 27.2	25.3	78 03.8	12.8	16 16.3	5.9	54.5
Y	15	44 27.1	·· 25.3	92 35.6	12.8	16 22.2	5.9	54.5
	16	59 26.9	25.2	107 07.4	12.7	16 28.1	5.9	54.5
	17	74 26.8	25.2	121 39.1	12.7	16 34.0	5.7	54.5
	18	89 26.6	N23 25.1	136 10.8	12.7	N16 39.7	5.7	54.5
	19	104 26.5	25.1	150 42.5	12.6	16 45.4	5.6	54.5
	20	119 26.4	25.0	165 14.1	12.5	16 51.0	5.6	54.5
	21	134 26.2	·· 25.0	179 45.6	12.5	16 56.6	5.4	54.5
	22	149 26.1	25.0	194 17.1	12.5	17 02.0	5.4	54.6
	23	164 26.0	24.9	208 48.6	12.4	17 07.4	5.3	54.6
	24 00	179 25.8	N23 24.9	223 20.0	12.4	N17 12.7	5.3	54.6
	01	194 25.7	24.8	237 51.4	12.3	17 18.0	5.2	54.6
	02	209 25.6	24.8	252 22.7	12.3	17 23.2	5.0	54.6
	03	224 25.4	·· 24.7	266 54.0	12.2	17 28.2	5.0	54.6
	04	239 25.3	24.7	281 25.2	12.2	17 33.2	5.0	54.6
	05	254 25.2	24.6	295 56.4	12.1	17 38.2	4.8	54.7
	06	269 25.0	N23 24.6	310 27.5	12.1	N17 43.0	4.8	54.7
	07	284 24.9	24.5	324 58.6	12.0	17 47.8	4.7	54.7
T	08	299 24.8	24.5	339 29.6	12.0	17 52.5	4.6	54.7
H	09	314 24.6	·· 24.4	354 00.6	11.9	17 57.1	4.5	54.7
U	10	329 24.5	24.4	8 31.5	11.9	18 01.6	4.5	54.7
R	11	344 24.4	24.3	23 02.4	11.9	18 06.1	4.3	54.7
S	12	359 24.2	N23 24.2	37 33.3	11.8	N18 10.4	4.3	54.8
D	13	14 24.1	24.2	52 04.1	11.7	18 14.7	4.2	54.8
A	14	29 24.0	24.1	66 34.8	11.7	18 18.9	4.1	54.8
Y	15	44 23.8	·· 24.1	81 05.5	11.7	18 23.0	4.0	54.8
	16	59 23.7	24.0	95 36.2	11.6	18 27.0	4.0	54.8
	17	74 23.6	24.0	110 06.8	11.6	18 31.0	3.8	54.8
	18	89 23.4	N23 23.9	124 37.4	11.5	N18 34.8	3.8	54.9
	19	104 23.3	23.8	139 07.9	11.5	18 38.6	3.7	54.9
	20	119 23.2	23.8	153 38.4	11.4	18 42.3	3.6	54.9
	21	134 23.0	·· 23.7	168 08.8	11.4	18 45.9	3.5	54.9
	22	149 22.9	23.7	182 39.2	11.4	18 49.4	3.4	54.9
	23	164 22.8	23.6	197 09.6	11.3	18 52.8	3.3	54.9
	25 00	179 22.6	N23 23.5	211 39.9	11.2	N18 56.1	3.3	55.0
	01	194 22.5	23.5	226 10.1	11.3	18 59.4	3.1	55.0
	02	209 22.3	23.4	240 40.4	11.1	19 02.5	3.1	55.0
	03	224 22.2	·· 23.3	255 10.5	11.2	19 05.6	2.9	55.0
	04	239 22.1	23.3	269 40.7	11.0	19 08.5	2.9	55.0
	05	254 21.9	23.2	284 10.7	11.1	19 11.4	2.8	55.1
	06	269 21.8	N23 23.1	298 40.8	11.0	N19 14.2	2.7	55.1
	07	284 21.7	23.1	313 10.8	11.0	19 16.9	2.6	55.1
	08	299 21.5	23.0	327 40.8	10.9	19 19.5	2.5	55.1
F	09	314 21.4	·· 22.9	342 10.7	10.9	19 22.0	2.4	55.1
R	10	329 21.3	22.8	356 40.6	10.8	19 24.4	2.3	55.1
I	11	344 21.1	22.8	11 10.4	10.8	19 26.7	2.2	55.2
D	12	359 21.0	N23 22.7	25 40.2	10.8	N19 28.9	2.1	55.2
A	13	14 20.9	22.6	40 10.0	10.7	19 31.0	2.0	55.2
Y	14	29 20.7	22.6	54 39.7	10.7	19 33.0	2.0	55.2
	15	44 20.6	·· 22.5	69 09.4	10.6	19 35.0	1.8	55.2
	16	59 20.5	22.4	83 39.0	10.6	19 36.8	1.7	55.3
	17	74 20.4	22.3	98 08.6	10.6	19 38.5	1.6	55.3
	18	89 20.2	N23 22.3	112 38.2	10.5	N19 40.1	1.6	55.3
	19	104 20.1	22.2	127 07.7	10.5	19 41.7	1.4	55.3
	20	119 20.0	22.1	141 37.2	10.5	19 43.1	1.4	55.3
	21	134 19.8	·· 22.0	156 06.7	10.4	19 44.5	1.2	55.4
	22	149 19.7	21.9	170 36.1	10.4	19 45.7	1.1	55.4
	23	164 19.6	21.9	185 05.5	10.4	19 46.8	1.1	55.4
		S.D. 15.8	*d* 0.1	S.D. 14.8		14.9		15.0

Lat.	Twilight Naut.	Twilight Civil	Sunrise	Moonrise 23	Moonrise 24	Moonrise 25	Moonrise 26
°	h m	h m	h m	h m	h m	h m	h m
N 72	□	□	□	22 24	□	□	□
N 70	□	□	□	23 16	23 15	23 23	24 21
68	□	□	□	23 48	24 03	00 03	00 33
66	□	□	□	24 12	00 12	00 34	01 10
64	////	////	01 32	00 12	00 31	00 57	01 36
62	////	////	02 10	00 24	00 46	01 16	01 56
60	////	00 50	02 37	00 35	00 59	01 31	02 13
N 58	////	01 41	02 57	00 44	01 10	01 44	02 27
56	////	02 11	03 14	00 51	01 20	01 55	02 39
54	00 46	02 34	03 28	00 59	01 28	02 05	02 49
52	01 33	02 52	03 40	01 05	01 36	02 14	02 59
50	02 01	03 07	03 51	01 11	01 43	02 21	03 07
45	02 47	03 36	04 14	01 23	01 58	02 38	03 25
N 40	03 17	03 59	04 32	01 33	02 10	02 52	03 39
35	03 40	04 17	04 47	01 42	02 21	03 04	03 52
30	03 59	04 33	05 00	01 50	02 30	03 14	04 02
20	04 28	04 58	05 22	02 04	02 46	03 32	04 21
N 10	04 51	05 18	05 41	02 15	03 00	03 47	04 37
0	05 10	05 36	05 59	02 27	03 13	04 02	04 52
S 10	05 27	05 53	06 16	02 38	03 26	04 16	05 07
20	05 43	06 11	06 35	02 50	03 41	04 32	05 23
30	06 00	06 30	06 56	03 04	03 57	04 50	05 42
35	06 08	06 40	07 08	03 12	04 06	05 00	05 53
40	06 18	06 52	07 22	03 21	04 17	05 12	06 05
45	06 28	07 05	07 39	03 32	04 30	05 26	06 20
S 50	06 40	07 21	08 00	03 45	04 45	05 44	06 38
52	06 45	07 29	08 10	03 51	04 53	05 52	06 47
54	06 51	07 37	08 21	03 58	05 01	06 01	06 56
56	06 57	07 46	08 34	04 05	05 10	06 11	07 07
58	07 04	07 57	08 48	04 14	05 20	06 23	07 19
S 60	07 11	08 08	09 06	04 23	05 32	06 37	07 33

Lat.	Sunset	Twilight Civil	Twilight Naut.	Moonset 23	Moonset 24	Moonset 25	Moonset 26
°	h m	h m	h m	h m	h m	h m	h m
N 72	□	□	□	19 28	□	□	□
N 70	□	□	□	18 37	20 17	21 53	22 42
68	□	□	□	18 05	19 29	20 43	21 34
66	□	□	□	17 42	18 59	20 06	20 58
64	22 32	////	////	17 24	18 36	19 40	20 32
62	21 54	////	////	17 09	18 18	19 20	20 12
60	21 28	23 13	////	16 56	18 03	19 04	19 56
N 58	21 07	22 23	////	16 46	17 50	18 50	19 42
56	20 51	21 53	////	16 36	17 39	18 38	19 30
54	20 36	21 31	23 18	16 28	17 30	18 27	19 19
52	20 24	21 13	22 31	16 21	17 21	18 18	19 10
50	20 13	20 58	22 03	16 14	17 13	18 10	19 02
45	19 51	20 28	21 18	16 00	16 57	17 52	18 44
N 40	19 33	20 06	20 47	15 48	16 44	17 38	18 30
35	19 18	19 47	20 24	15 38	16 32	17 26	18 17
30	19 05	19 32	20 05	15 29	16 22	17 15	18 06
20	18 43	19 07	19 36	15 14	16 05	16 57	17 48
N 10	18 24	18 47	19 14	15 01	15 50	16 41	17 32
0	18 06	18 29	18 55	14 49	15 36	16 26	17 17
S 10	17 49	18 11	18 38	14 37	15 22	16 11	17 02
20	17 30	17 54	18 22	14 24	15 08	15 55	16 45
30	17 09	17 35	18 05	14 09	14 51	15 37	16 27
35	16 56	17 25	17 56	14 00	14 41	15 26	16 16
40	16 42	17 13	17 47	13 50	14 29	15 14	16 04
45	16 25	16 59	17 37	13 39	14 16	14 59	15 49
S 50	16 05	16 43	17 25	13 25	14 00	14 42	15 31
52	15 55	16 36	17 19	13 18	13 52	14 33	15 22
54	15 44	16 28	17 14	13 11	13 44	14 24	15 13
56	15 31	16 18	17 08	13 03	13 35	14 14	15 02
58	15 16	16 08	17 01	12 54	13 24	14 02	14 50
S 60	14 59	15 57	16 53	12 44	13 12	13 48	14 36

Day	SUN Eqn. of Time 00ʰ	SUN Eqn. of Time 12ʰ	SUN Mer. Pass.	MOON Mer. Pass. Upper	MOON Mer. Pass. Lower	MOON Age	MOON Phase
	m s	m s	h m	h m	h m	d	
23	02 03	02 10	12 02	08 38	21 01	25	
24	02 16	02 23	12 02	09 25	21 49	26	◐
25	02 29	02 36	12 03	10 14	22 39	27	

1976 SEPTEMBER 21, 22, 23 (TUES., WED., THURS.)

G.M.T. d h	ARIES G.H.A.	VENUS −3.3 G.H.A.	VENUS Dec.	MARS +1.9 G.H.A.	MARS Dec.	JUPITER −2.2 G.H.A.	JUPITER Dec.	SATURN +0.6 G.H.A.	SATURN Dec.	STARS Name	S.H.A.	Dec.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′		° ′	° ′
21 00 (TUESDAY)	359 59.8	157 56.0	S 8 45.2	163 11.9	S 6 43.7	300 40.8	N19 19.0	224 21.5	N17 29.5	Acamar	315 39.0	S40 23.6
01	15 02.2	172 55.6	46.5	178 12.8	44.3	315 43.3	19.0	239 23.7	29.4	Achernar	335 46.8	S57 21.1
02	30 04.7	187 55.3	47.7	193 13.8	45.0	330 45.8	19.0	254 25.9	29.3	Acrux	173 40.8	S62 58.2
03	45 07.2	202 54.9	·· 48.9	208 14.7	·· 45.6	345 48.3	·· 19.0	269 28.1	·· 29.3	Adhara	255 34.3	S28 56.2
04	60 09.6	217 54.5	50.1	223 15.6	46.3	0 50.7	19.0	284 30.3	29.2	Aldebaran	291 21.0	N16 27.8
05	75 12.1	232 54.1	51.4	238 16.6	46.9	15 53.2	19.0	299 32.5	29.1			
06	90 14.6	247 53.7	S 8 52.6	253 17.5	S 6 47.6	30 55.7	N19 18.9	314 34.7	N17 29.1	Alioth	166 45.5	N56 05.3
07	105 17.0	262 53.4	53.8	268 18.4	48.2	45 58.2	18.9	329 36.9	29.0	Alkaid	153 21.[illegible]	N49 25.9
08	120 19.5	277 53.0	55.0	283 19.4	48.9	61 00.6	18.9	344 39.1	28.9	Al Na'ir	28 17.9	S47 04.4
09	135 22.0	292 52.6	·· 56.3	298 20.3	·· 49.5	76 03.1	·· 18.9	359 41.3	·· 28.9	Alnilam	276 14.4	S 1 12.9
10	150 24.4	307 52.2	57.5	313 21.2	50.2	91 05.6	18.9	14 43.5	28.8	Alphard	[illegible] 23.4	S 8 33.4
11	165 26.9	322 51.8	8 58.7	328 22.1	50.8	106 08.1	18.9	29 45.7	28.7			
12	180 29.3	337 51.4	S 9 00.0	343 23.1	S 6 51.5	121 10.6	N19 18.9	44 47.9	N17 28.7	Alphecca	126 34.6	N26 47.8
13	195 31.8	352 51.1	01.2	358 24.0	52.1	136 13.0	18.9	59 50.1	28.6	Alpheratz	358 11.7	N28 57.9
14	210 34.3	7 50.7	02.4	13 24.9	52.8	151 15.5	18.9	74 52.3	28.5	Altair	62 35.0	N 8 48.7
15	225 36.7	22 50.3	·· 03.6	28 25.9	·· 53.4	166 18.0	·· 18.9	89 54.5	·· 28.5	Ankaa	353 42.5	S42 25.8
16	240 39.2	37 49.9	04.9	43 26.8	54.1	181 20.5	18.9	104 56.7	28.4	Antares	113 00.3	S26 22.8
17	255 41.7	52 49.5	06.1	58 27.7	54.8	196 23.0	18.9	119 58.9	28.3			
18	270 44.1	67 49.1	S 9 07.3	73 28.7	S 6 55.4	211 25.4	N19 18.9	135 01.1	N17 28.2	Arcturus	146 21.2	N19 18.4
19	285 46.6	82 48.7	08.5	88 29.6	56.1	226 27.9	18.9	150 03.3	28.2	Atria	108 27.0	S68 59.4
20	300 49.1	97 48.4	09.7	103 30.5	56.7	241 30.4	18.8	165 05.5	28.1	Avior	234 29.6	S59 25.9
21	315 51.5	112 48.0	·· 11.0	118 31.5	·· 57.4	256 32.9	·· 18.8	180 07.7	·· 28.0	Bellatrix	279 01.6	N 6 19.8
22	330 54.0	127 47.6	12.2	133 32.4	58.0	271 35.4	18.8	195 09.9	28.0	Betelgeuse	271 31.2	N 7 24.2
23	345 56.5	142 47.2	13.4	148 33.3	58.7	286 37.8	18.8	210 12.2	27.9			
22 00 (WEDNESDAY)	0 58.9	157 46.8	S 9 14.6	163 34.2	S 6 59.3	301 40.3	N19 18.8	225 14.4	N17 27.8	Canopus	264 08.4	S52 40.7
01	16 01.4	172 46.4	15.9	178 35.2	7 00.0	316 42.8	18.8	240 16.6	27.8	Capella	281 15.2	N45 58.3
02	31 03.8	187 46.0	17.1	193 36.1	00.6	331 45.3	18.8	255 18.8	27.7	Deneb	49 50.0	N45 12.2
03	46 06.3	202 45.7	·· 18.3	208 37.0	·· 01.3	346 47.8	·· 18.8	270 21.0	·· 27.6	Denebola	183 02.1	N14 42.2
04	61 08.8	217 45.3	19.5	223 38.0	01.9	1 50.3	18.8	285 23.2	27.6	Diphda	349 23.2	S18 06.7
05	76 11.2	232 44.9	20.7	238 38.9	02.6	16 52.7	18.8	300 25.4	27.5			
06	91 13.7	247 44.5	S 9 22.0	253 39.8	S 7 03.2	31 55.2	N19 18.8	315 27.6	N17 27.4	Dubhe	194 26.1	N61 52.5
07	106 16.2	262 44.1	23.2	268 40.7	03.9	46 57.7	18.7	330 29.8	27.4	Elnath	278 47.5	N28 35.2
08	121 18.6	277 43.7	24.4	283 41.7	04.5	62 00.2	18.7	345 32.0	27.3	Eltanin	90 59.0	N51 29.9
09	136 21.1	292 43.3	·· 25.6	298 42.6	·· 05.2	77 02.7	·· 18.7	0 34.2	·· 27.2	Enif	34 13.9	N 9 46.3
10	151 23.6	307 42.9	26.8	313 43.5	05.8	92 05.2	18.7	15 36.4	27.2	Fomalhaut	15 54.0	S29 44.6
11	166 26.0	322 42.5	28.1	328 44.5	06.5	107 07.7	18.7	30 38.6	27.1			
12	181 28.5	337 42.2	S 9 29.3	343 45.4	S 7 07.1	122 10.1	N19 18.7	45 40.8	N17 27.0	Gacrux	172 32.2	S56 59.0
13	196 30.9	352 41.8	30.5	358 46.3	07.8	137 12.6	18.7	60 43.0	27.0	Gienah	176 21.0	S17 24.7
14	211 33.4	7 41.4	31.7	13 47.2	08.4	152 15.1	18.7	75 45.2	26.9	Hadar	149 27.6	S60 15.8
15	226 35.9	22 41.0	·· 32.9	28 48.2	·· 09.1	167 17.6	·· 18.7	90 47.4	·· 26.8	Hamal	328 31.6	N23 21.2
16	241 38.3	37 40.6	34.1	43 49.1	09.7	182 20.1	18.7	105 49.6	26.8	Kaus Aust.	84 20.4	S34 23.8
17	256 40.8	52 40.2	35.4	58 50.0	10.4	197 22.6	18.7	120 51.9	26.7			
18	271 43.3	67 39.8	S 9 36.6	73 50.9	S 7 11.0	212 25.1	N19 18.6	135 54.1	N17 26.6	Kochab	137 19.8	N74 15.3
19	286 45.7	82 39.4	37.8	88 51.9	11.7	227 27.6	18.6	150 56.3	26.6	Markab	14 05.5	N15 05.0
20	301 48.2	97 39.0	39.0	103 52.8	12.3	242 30.0	18.6	165 58.5	26.5	Menkar	314 43.7	N 4 00.0
21	316 50.7	112 38.6	·· 40.2	118 53.7	·· 13.0	257 32.5	·· 18.6	181 00.7	·· 26.4	Menkent	148 40.5	S36 15.4
22	331 53.1	127 38.2	41.4	133 54.6	13.6	272 35.0	18.6	196 02.9	26.4	Miaplacidus	221 46.1	S69 37.2
23	346 55.6	142 37.8	42.6	148 55.6	14.3	287 37.5	18.6	211 05.1	26.3			
23 00 (THURSDAY)	1 58.1	157 37.4	S 9 43.9	163 56.5	S 7 14.9	302 40.0	N19 18.6	226 07.3	N17 26.2	Mirfak	309 19.6	N49 46.6
01	17 00.5	172 37.0	45.1	178 57.4	15.6	317 42.5	18.6	241 09.5	26.2	Nunki	76 32.4	S26 19.5
02	32 03.0	187 36.6	46.3	193 58.3	16.2	332 45.0	18.6	256 11.7	26.1	Peacock	54 02.3	S56 48.7
03	47 05.4	202 36.3	·· 47.5	208 59.3	·· 16.9	347 47.5	·· 18.6	271 13.9	·· 26.0	Pollux	244 01.6	N28 04.9
04	62 07.9	217 35.9	48.7	224 00.2	17.5	2 50.0	18.5	286 16.1	25.9	Procyon	245 28.7	N 5 17.1
05	77 10.4	232 35.5	49.9	239 01.1	18.2	17 52.5	18.5	301 18.3	25.9			
06	92 12.8	247 35.1	S 9 51.1	254 02.0	S 7 18.8	32 55.0	N19 18.5	316 20.5	N17 25.8	Rasalhague	96 32.1	N12 34.9
07	107 15.3	262 34.7	52.3	269 03.0	19.5	47 57.5	18.5	331 22.8	25.7	Regulus	208 13.1	N12 04.9
08	122 17.8	277 34.3	53.5	284 03.9	20.1	63 00.0	18.5	346 25.0	25.7	Rigel	281 38.5	S 8 13.6
09	137 20.2	292 33.9	·· 54.8	299 04.8	·· 20.8	78 02.4	·· 18.5	1 27.2	·· 25.6	Rigil Kent.	140 29.9	S60 44.4
10	152 22.7	307 33.5	56.0	314 05.7	21.4	93 04.9	18.5	16 29.4	25.5	Sabik	102 44.3	S15 41.7
11	167 25.2	322 33.1	57.2	329 06.7	22.1	108 07.4	18.5	31 31.6	25.5			
12	182 27.6	337 32.7	S 9 58.4	344 07.6	S 7 22.7	123 09.9	N19 18.5	46 33.8	N17 25.4	Schedar	350 11.5	N56 24.7
13	197 30.1	352 32.3	9 59.6	359 08.5	23.4	138 12.4	18.4	61 36.0	25.3	Shaula	96 59.4	S37 05.3
14	212 32.5	7 31.9	10 00.8	14 09.4	24.0	153 14.9	18.4	76 38.2	25.3	Sirius	258 58.1	S16 40.9
15	227 35.0	22 31.5	·· 02.0	29 10.4	·· 24.7	168 17.4	·· 18.4	91 40.4	·· 25.2	Spica	159 00.7	S11 02.3
16	242 37.5	37 31.1	03.2	44 11.3	25.3	183 19.9	18.4	106 42.6	25.1	Suhail	223 13.0	S43 20.2
17	257 39.9	52 30.7	04.4	59 12.2	26.0	198 22.4	18.4	121 44.8	25.1			
18	272 42.4	67 30.3	S10 05.6	74 13.1	S 7 26.6	213 24.9	N19 18.4	136 47.1	N17 25.0	Vega	80 57.6	N38 46.1
19	287 44.9	82 29.9	06.8	89 14.1	27.3	228 27.4	18.4	151 49.3	24.9	Zuben'ubi	137 36.2	S15 56.7
20	302 47.3	97 29.5	08.0	104 15.0	27.9	243 29.9	18.4	166 51.5	24.9			
21	317 49.8	112 29.1	·· 09.2	119 15.9	·· 28.6	258 32.4	·· 18.4	181 53.7	·· 24.8			
22	332 52.3	127 28.7	10.4	134 16.8	29.2	273 34.9	18.3	196 55.9	24.7			
23	347 54.7	142 28.3	11.7	149 17.7	29.9	288 37.4	18.3	211 58.1	24.7			
Mer. Pass.	h m 23 52.2	v −0.4	d 1.2	v 0.9	d 0.7	v 2.5	d 0.0	v 2.2	d 0.1			

	S.H.A. ° ′	Mer. Pass. h m
Venus	156 47.9	13 29
Mars	162 35.3	13 05
Jupiter	300 41.4	3 53
Saturn	224 15.4	8 58

1976 SEPTEMBER 21, 22, 23 (TUES., WED., THURS.)

G.M.T. d h	SUN G.H.A. ° ′	SUN Dec. ° ′	MOON G.H.A. ° ′	v ′	MOON Dec. ° ′	d ′	H.P. ′
21 00	181 42.7	N 0 44.6	219 21.4	9.5	N10 07.0	10.0	58.7
01	196 42.9	43.6	233 49.9	9.5	9 57.0	10.0	58.8
02	211 43.2	42.6	248 18.4	9.5	9 47.0	10.1	58.8
03	226 43.4	·· 41.7	262 46.9	9.5	9 36.9	10.2	58.8
04	241 43.6	40.7	277 15.4	9.5	9 26.7	10.3	58.9
05	256 43.8	39.7	291 43.9	9.4	9 16.4	10.3	58.9
06	271 44.0	N 0 38.7	306 12.3	9.5	N 9 06.1	10.4	58.9
07	286 44.3	37.8	320 40.8	9.4	8 55.7	10.4	59.0
T 08	301 44.5	36.8	335 09.2	9.5	8 45.3	10.6	59.0
U 09	316 44.7	·· 35.8	349 37.7	9.4	8 34.7	10.5	59.1
E 10	331 44.9	34.8	4 06.1	9.4	8 24.2	10.7	59.1
S 11	346 45.1	33.9	18 34.5	9.4	8 13.5	10.7	59.1
D 12	1 45.4	N 0 32.9	33 02.9	9.4	N 8 02.8	10.7	59.2
A 13	16 45.6	31.9	47 31.3	9.3	7 52.1	10.9	59.2
Y 14	31 45.8	31.0	61 59.6	9.4	7 41.2	10.8	59.2
15	46 46.0	·· 30.0	76 28.0	9.3	7 30.4	11.0	59.3
16	61 46.2	29.0	90 56.3	9.4	7 19.4	11.0	59.3
17	76 46.5	28.0	105 24.7	9.3	7 08.4	11.0	59.3
18	91 46.7	N 0 27.1	119 53.0	9.3	N 6 57.4	11.1	59.4
19	106 46.9	26.1	134 21.3	9.2	6 46.3	11.2	59.4
20	121 47.1	25.1	148 49.5	9.3	6 35.1	11.2	59.4
21	136 47.3	·· 24.1	163 17.8	9.2	6 23.9	11.2	59.5
22	151 47.6	23.2	177 46.0	9.3	6 12.7	11.4	59.5
23	166 47.8	22.2	192 14.3	9.2	6 01.3	11.3	59.5
22 00	181 48.0	N 0 21.2	206 42.5	9.2	N 5 50.0	11.4	59.6
01	196 48.2	20.3	221 10.7	9.2	5 38.6	11.5	59.6
02	211 48.4	19.3	235 38.9	9.1	5 27.1	11.5	59.6
03	226 48.7	·· 18.3	250 07.0	9.2	5 15.6	11.5	59.7
04	241 48.9	17.3	264 35.2	9.1	5 04.1	11.6	59.7
05	256 49.1	16.4	279 03.3	9.1	4 52.5	11.6	59.7
06	271 49.3	N 0 15.4	293 31.4	9.1	N 4 40.9	11.7	59.8
W 07	286 49.5	14.4	307 59.5	9.1	4 29.2	11.7	59.8
E 08	301 49.7	13.4	322 27.6	9.0	4 17.5	11.7	59.8
D 09	316 50.0	·· 12.5	336 55.6	9.1	4 05.8	11.8	59.9
N 10	331 50.2	11.5	351 23.7	9.0	3 54.0	11.8	59.9
E 11	346 50.4	10.5	5 51.7	9.0	3 42.2	11.9	59.9
S 12	1 50.6	N 0 09.5	20 19.7	8.9	N 3 30.3	11.9	59.9
D 13	16 50.8	08.6	34 47.6	9.0	3 18.4	11.9	60.0
A 14	31 51.1	07.6	49 15.6	8.9	3 06.5	11.9	60.0
Y 15	46 51.3	·· 06.6	63 43.5	8.9	2 54.6	12.0	60.0
16	61 51.5	05.7	78 11.4	8.9	2 42.6	12.0	60.1
17	76 51.7	04.7	92 39.3	8.8	2 30.6	12.0	60.1
18	91 51.9	N 0 03.7	107 07.1	8.9	N 2 18.6	12.1	60.1
19	106 52.1	02.7	121 35.0	8.8	2 06.5	12.0	60.1
20	121 52.4	01.8	136 02.8	8.8	1 54.5	12.1	60.2
21	136 52.6	N 0 00.8	150 30.6	8.8	1 42.4	12.2	60.2
22	151 52.8	S 0 00.2	164 58.4	8.7	1 30.2	12.1	60.2
23	166 53.0	01.2	179 26.1	8.7	1 18.1	12.2	60.2
23 00	181 53.2	S 0 02.1	193 53.8	8.7	N 1 05.9	12.1	60.3
01	196 53.5	03.1	208 21.5	8.7	0 53.8	12.2	60.3
02	211 53.7	04.1	222 49.2	8.6	0 41.6	12.2	60.3
03	226 53.9	·· 05.1	237 16.8	8.6	0 29.4	12.3	60.3
04	241 54.1	06.0	251 44.4	8.6	0 17.1	12.2	60.4
05	256 54.3	07.0	266 12.0	8.6	N 0 04.9	12.2	60.4
06	271 54.5	S 0 08.0	280 39.6	8.5	S 0 07.3	12.3	60.4
07	286 54.8	09.0	295 07.1	8.5	0 19.6	12.2	60.4
T 08	301 55.0	09.9	309 34.6	8.5	0 31.8	12.3	60.5
H 09	316 55.2	·· 10.9	324 02.1	8.5	0 44.1	12.2	60.5
U 10	331 55.4	11.9	338 29.6	8.4	0 56.3	12.3	60.5
R 11	346 55.6	12.9	352 57.0	8.4	1 08.6	12.3	60.5
S 12	1 55.8	S 0 13.8	7 24.4	8.3	S 1 20.9	12.2	60.5
D 13	16 56.1	14.8	21 51.7	8.4	1 33.1	12.3	60.6
A 14	31 56.3	15.8	36 19.1	8.3	1 45.4	12.3	60.6
Y 15	46 56.5	·· 16.7	50 46.4	8.3	1 57.7	12.2	60.6
16	61 56.7	17.7	65 13.7	8.2	2 09.9	12.3	60.6
17	76 56.9	18.7	79 40.9	8.2	2 22.2	12.2	60.6
18	91 57.1	S 0 19.7	94 08.1	8.2	S 2 34.4	12.2	60.7
19	106 57.4	20.6	108 35.3	8.2	2 46.6	12.3	60.7
20	121 57.6	21.6	123 02.5	8.1	2 58.9	12.2	60.7
21	136 57.8	·· 22.6	137 29.6	8.1	3 11.1	12.2	60.7
22	151 58.0	23.6	151 56.7	8.0	3 23.3	12.1	60.7
23	166 58.2	24.5	166 23.7	8.0	3 35.4	12.2	60.7
	S.D. 16.0	d 1.0	S.D. 16.1		16.3		16.5

Lat.	Twilight Naut. h m	Twilight Civil h m	Sunrise h m	Moonrise 21 h m	Moonrise 22 h m	Moonrise 23 h m	Moonrise 24 h m
N 72	02 58	04 30	05 38	01 18	03 17	05 18	07 21
N 70	03 18	04 38	05 40	01 34	03 25	05 17	07 13
68	03 34	04 45	05 41	01 47	03 31	05 17	07 06
66	03 46	04 50	05 42	01 57	03 36	05 17	07 01
64	03 5[illegible]	04 55	05 43	02 06	03 40	05 17	06 56
62	04 05	04 59	05 44	02 13	03 44	05 17	06 52
60	04 12	05 02	05 44	02 20	03 47	05 17	06 49
N 58	04 18	05 05	05 45	02 25	03 50	05 17	06 46
56	04 24	05 08	05 45	02 30	03 52	05 17	06 43
54	04 28	05 10	05 46	02 35	03 54	05 17	06 41
52	04 32	05 12	05 46	02 39	03 56	05 17	06 39
50	04 36	05 14	05 46	02 42	03 58	05 17	06 37
45	04 43	05 18	05 47	02 50	04 02	05 17	06 33
N 40	04 49	05 20	05 47	02 57	04 06	05 17	06 29
35	04 53	05 23	05 48	03 02	04 09	05 17	06 26
30	04 56	05 24	05 48	03 07	04 11	05 16	06 23
20	05 01	05 27	05 49	03 16	04 16	05 16	06 19
N 10	05 04	05 28	05 49	03 23	04 19	05 16	06 15
0	05 05	05 29	05 50	03 31	04 23	05 16	06 11
S 10	05 04	05 29	05 50	03 38	04 27	05 16	06 07
20	05 02	05 28	05 50	03 45	04 31	05 17	06 03
30	04 58	05 26	05 50	03 54	04 35	05 17	05 59
35	04 55	05 24	05 49	03 58	04 38	05 17	05 57
40	04 51	05 22	05 49	04 04	04 41	05 17	05 54
45	04 46	05 20	05 49	04 10	04 44	05 17	05 51
S 50	04 39	05 17	05 49	04 18	04 48	05 17	05 47
52	04 35	05 15	05 49	04 22	04 50	05 17	05 45
54	04 32	05 13	05 49	04 25	04 52	05 17	05 43
56	04 28	05 12	05 49	04 30	04 54	05 17	05 41
58	04 23	05 09	05 48	04 34	04 56	05 17	05 39
S 60	04 17	05 07	05 48	04 40	04 59	05 17	05 36

Lat.	Sunset h m	Twilight Civil h m	Twilight Naut. h m	Moonset 21 h m	Moonset 22 h m	Moonset 23 h m	Moonset 24 h m
N 72	18 05	19 13	20 43	17 27	17 18	17 10	17 01
N 70	18 03	19 05	20 23	17 17	17 15	17 13	17 11
68	18 02	18 58	20 08	17 09	17 13	17 16	17 20
66	18 02	18 53	19 56	17 02	17 10	17 18	17 27
64	18 01	18 49	19 47	16 56	17 09	17 20	17 33
62	18 00	18 45	19 38	16 51	17 07	17 22	17 39
60	18 00	18 42	19 32	16 47	17 06	17 24	17 43
N 58	18 00	18 39	19 26	16 43	17 04	17 25	17 47
56	17 59	18 36	19 20	16 40	17 03	17 26	17 51
54	17 59	18 34	19 16	16 37	17 02	17 28	17 54
52	17 58	18 32	19 12	16 34	17 01	17 29	17 57
50	17 58	18 31	19 08	16 31	17 00	17 29	18 00
45	17 58	18 27	19 01	16 25	16 58	17 32	18 06
N 40	17 57	18 24	18 56	16 21	16 57	17 33	18 11
35	17 57	18 22	18 52	16 17	16 56	17 35	18 15
30	17 57	18 21	18 48	16 13	16 54	17 36	18 19
20	17 56	18 18	18 44	16 07	16 52	17 38	18 26
N 10	17 56	18 17	18 41	16 01	16 50	17 40	18 32
0	17 56	18 17	18 41	15 55	16 48	17 42	18 37
S 10	17 56	18 17	18 41	15 50	16 46	17 44	18 43
20	17 56	18 18	18 44	15 44	16 44	17 46	18 49
30	17 56	18 20	18 48	15 38	16 42	17 48	18 55
35	17 56	18 22	18 51	15 34	16 41	17 49	18 59
40	17 57	18 24	18 55	15 29	16 39	17 51	19 04
45	17 57	18 26	19 01	15 24	16 37	17 52	19 09
S 50	17 57	18 30	19 08	15 18	16 35	17 54	19 15
52	17 58	18 31	19 11	15 15	16 34	17 55	19 18
54	17 58	18 33	19 15	15 12	16 33	17 56	19 21
56	17 58	18 35	19 20	15 08	16 32	17 57	19 24
58	17 58	18 38	19 25	15 04	16 30	17 59	19 28
S 60	17 59	18 40	19 30	15 00	16 29	18 00	19 32

Day	SUN Eqn. of Time 00ʰ m s	SUN Eqn. of Time 12ʰ m s	SUN Mer. Pass. h m	MOON Mer. Pass. Upper h m	MOON Mer. Pass. Lower h m	MOON Age d	MOON Phase
21	06 50	07 01	11 53	09 43	22 09	27	
22	07 12	07 22	11 53	10 36	23 02	28	●
23	07 33	07 43	11 52	11 29	23 56	29	

1976 OCTOBER 12, 13, 14 (TUES., WED., THURS.)

G.M.T. d h	ARIES G.H.A.	VENUS −3.4 G.H.A.	Dec.	MARS +1.8 G.H.A.	Dec.	JUPITER −2.3 G.H.A.	Dec.	SATURN +0.6 G.H.A.	Dec.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
12 00	20 41.7	153 56.4	S18 04.6	170 40.5	S12 03.0	322 13.2	N19 06.9	243 06.3	N16 58.8
01	35 44.1	168 55.9	05.5	185 41.4	03.6	337 15.9	06.9	258 08.6	58.7
02	50 46.6	183 55.3	06.5	200 42.3	04.2	352 18.5	06.9	273 10.8	58.7
03	65 49.1	198 54.7	·· 07.4	215 43.1	·· 04.8	7 21.2	·· 06.8	288 13.1	·· 58.6
04	80 51.5	213 54.1	08.4	230 44.0	05.4	22 23.8	06.8	303 15.4	58.6
05	95 54.0	228 53.5	09.3	245 44.8	06.0	37 26.5	06.7	318 17.6	58.5
06	110 56.5	243 52.9	S18 10.3	260 45.6	S12 06.7	52 29.1	N19 06.7	333 19.9	N16 58.5
07	125 58.9	258 52.3	11.2	275 46.5	07.3	67 31.8	06.7	348 22.2	58.4
T 08	141 01.4	273 51.7	12.1	290 47.3	07.9	82 34.5	06.6	3 24.4	58.4
U 09	156 03.8	288 51.1	·· 13.1	305 48.2	·· 08.5	97 37.1	·· 06.6	18 26.7	·· 58.3
E 10	171 06.3	303 50.5	14.0	320 49.0	09.1	112 39.8	06.5	33 28.9	58.2
S 11	186 08.8	318 49.9	15.0	335 49.8	09.7	127 42.4	06.5	48 31.2	58.2
D 12	201 11.2	333 49.3	S18 15.9	350 50.7	S12 10.3	142 45.1	N19 06.5	63 33.5	N16 58.1
A 13	216 13.7	348 48.7	16.9	5 51.5	10.9	157 47.7	06.4	78 35.7	58.1
Y 14	231 16.2	3 48.1	17.8	20 52.4	11.5	172 50.4	06.4	93 38.0	58.0
15	246 18.6	18 47.5	·· 18.7	35 53.2	·· 12.1	187 53.0	·· 06.3	108 40.3	·· 58.0
16	261 21.1	33 46.9	19.7	50 54.0	12.7	202 55.7	06.3	123 42.5	57.9
17	276 23.6	48 46.3	20.6	65 54.9	13.3	217 58.4	06.2	138 44.8	57.9
18	291 26.0	63 45.7	S18 21.6	80 55.7	S12 13.9	233 01.0	N19 06.2	153 47.1	N16 57.8
19	306 28.5	78 45.1	22.5	95 56.6	14.5	248 03.7	06.2	168 49.3	57.8
20	321 31.0	93 44.5	23.4	110 57.4	15.1	263 06.3	06.1	183 51.6	57.7
21	336 33.4	108 43.9	·· 24.4	125 58.2	·· 15.7	278 09.0	·· 06.1	198 53.9	·· 57.7
22	351 35.9	123 43.3	25.3	140 59.1	16.4	293 11.6	06.0	213 56.2	57.6
23	6 38.3	138 42.7	26.2	155 59.9	17.0	308 14.3	06.0	228 58.4	57.6
13 00	21 40.8	153 42.1	S18 27.2	171 00.7	S12 17.6	323 17.0	N19 06.0	244 00.7	N16 57.5
01	36 43.3	168 41.5	28.1	186 01.6	18.2	338 19.6	05.9	259 03.0	57.5
02	51 45.7	183 40.9	29.0	201 02.4	18.8	353 22.3	05.9	274 05.2	57.4
03	66 48.2	198 40.3	·· 30.0	216 03.3	·· 19.4	8 24.9	·· 05.8	289 07.5	·· 57.4
04	81 50.7	213 39.7	30.9	231 04.1	20.0	23 27.6	05.8	304 09.8	57.3
05	96 53.1	228 39.1	31.8	246 04.9	20.6	38 30.3	05.7	319 12.0	57.3
06	111 55.6	243 38.5	S18 32.7	261 05.8	S12 21.2	53 32.9	N19 05.7	334 14.3	N16 57.2
W 07	126 58.1	258 37.9	33.7	276 06.6	21.8	68 35.6	05.7	349 16.6	57.2
E 08	142 00.5	273 37.3	34.6	291 07.4	22.4	83 38.2	05.6	4 18.8	57.1
D 09	157 03.0	288 36.7	·· 35.5	306 08.3	·· 23.0	98 40.9	·· 05.6	19 21.1	·· 57.1
N 10	172 05.5	303 36.1	36.4	321 09.1	23.6	113 43.6	05.5	34 23.4	57.0
E 11	187 07.9	318 35.5	37.4	336 09.9	24.2	128 46.2	05.5	49 25.6	57.0
S 12	202 10.4	333 34.9	S18 38.3	351 10.8	S12 24.8	143 48.9	N19 05.4	64 27.9	N16 56.9
D 13	217 12.8	348 34.3	39.2	6 11.6	25.4	158 51.5	05.4	79 30.2	56.9
A 14	232 15.3	3 33.6	40.1	21 12.5	26.0	173 54.2	05.4	94 32.5	56.8
Y 15	247 17.8	18 33.0	·· 41.0	36 13.3	·· 26.6	188 56.9	·· 05.3	109 34.7	·· 56.8
16	262 20.2	33 32.4	42.0	51 14.1	27.2	203 59.5	05.3	124 37.0	56.7
17	277 22.7	48 31.8	42.9	66 15.0	27.8	219 02.2	05.2	139 39.3	56.7
18	292 25.2	63 31.2	S18 43.8	81 15.8	S12 28.4	234 04.9	N19 05.2	154 41.5	N16 56.6
19	307 27.6	78 30.6	44.7	96 16.6	29.0	249 07.5	05.1	169 43.8	56.6
20	322 30.1	93 30.0	45.6	111 17.5	29.6	264 10.2	05.1	184 46.1	56.5
21	337 32.6	108 29.4	·· 46.5	126 18.3	·· 30.2	279 12.9	·· 05.1	199 48.3	·· 56.5
22	352 35.0	123 28.8	47.5	141 19.1	30.8	294 15.5	05.0	214 50.6	56.4
23	7 37.5	138 28.1	48.4	156 20.0	31.4	309 18.2	05.0	229 52.9	56.4
14 00	22 39.9	153 27.5	S18 49.3	171 20.8	S12 32.0	324 20.9	N19 04.9	244 55.2	N16 56.3
01	37 42.4	168 26.9	50.2	186 21.6	32.6	339 23.5	04.9	259 57.4	56.3
02	52 44.9	183 26.3	51.1	201 22.5	33.3	354 26.2	04.8	274 59.7	56.2
03	67 47.3	198 25.7	·· 52.0	216 23.3	·· 33.9	9 28.9	·· 04.8	290 02.0	·· 56.2
04	82 49.8	213 25.1	52.9	231 24.1	34.5	24 31.5	04.8	305 04.2	56.1
05	97 52.3	228 24.4	53.8	246 24.9	35.1	39 34.2	04.7	320 06.5	56.1
06	112 54.7	243 23.8	S18 54.7	261 25.8	S12 35.7	54 36.9	N19 04.7	335 08.8	N16 56.0
07	127 57.2	258 23.2	55.7	276 26.6	36.3	69 39.5	04.6	350 11.1	56.0
T 08	142 59.7	273 22.6	56.6	291 27.4	36.9	84 42.2	04.6	5 13.3	55.9
H 09	158 02.1	288 22.0	·· 57.5	306 28.3	·· 37.5	99 44.9	·· 04.5	20 15.6	·· 55.9
U 10	173 04.6	303 21.3	58.4	321 29.1	38.1	114 47.5	04.5	35 17.9	55.8
R 11	188 07.1	318 20.7	18 59.3	336 29.9	38.7	129 50.2	04.5	50 20.2	55.8
S 12	203 09.5	333 20.1	S19 00.2	351 30.8	S12 39.3	144 52.9	N19 04.4	65 22.4	N16 55.7
D 13	218 12.0	348 19.5	01.1	6 31.6	39.9	159 55.5	04.4	80 24.7	55.7
A 14	233 14.4	3 18.9	02.0	21 32.4	40.5	174 58.2	04.3	95 27.0	55.6
Y 15	248 16.9	18 18.2	·· 02.9	36 33.2	·· 41.1	190 00.9	·· 04.3	110 29.2	·· 55.6
16	263 19.4	33 17.6	03.8	51 34.1	41.7	205 03.6	04.2	125 31.5	55.5
17	278 21.8	48 17.0	04.7	66 34.9	42.3	220 06.2	04.2	140 33.8	55.5
18	293 24.3	63 16.4	S19 05.6	81 35.7	S12 42.9	235 08.9	N19 04.1	155 36.1	N16 55.4
19	308 26.8	78 15.7	06.5	96 36.6	43.5	250 11.6	04.1	170 38.3	55.4
20	323 29.2	93 15.1	07.4	111 37.4	44.1	265 14.3	04.1	185 40.6	55.3
21	338 31.7	108 14.5	·· 08.3	126 38.2	·· 44.7	280 16.9	·· 04.0	200 42.9	·· 55.3
22	353 34.2	123 13.9	09.2	141 39.0	45.3	295 19.6	04.0	215 45.2	55.2
23	8 36.6	138 13.2	10.0	156 39.9	45.9	310 22.3	03.9	230 47.4	55.2
Mer. Pass.	h m 22 29.6	*v* −0.6	*d* 0.9	*v* 0.8	*d* 0.6	*v* 2.7	*d* 0.0	*v* 2.3	*d* 0.1

STARS

Name	S.H.A. ° ′	Dec. ° ′
Acamar	315 38.9	S40 23.7
Achernar	335 46.7	S57 21.2
Acrux	173 40.8	S62 58.1
Adhara	255 34.1	S28 56.2
Aldebaran	291 20.8	N16 27.8
Alioth	166 45.5	N56 05.1
Alkaid	153 21.1	N49 25.8
Al Na'ir	28 18.0	S47 04.4
Alnilam	276 14.2	S 1 12.9
Alphard	218 23.3	S 8 33.4
Alphecca	126 34.7	N26 47.8
Alpheratz	358 11.7	N28 58.0
Altair	62 35.1	N 8 48.7
Ankaa	353 42.5	S42 25.8
Antares	113 00.4	S26 22.8
Arcturus	146 21.2	N19 18.3
Atria	108 27.3	S68 59.3
Avior	234 29.4	S59 25.9
Bellatrix	279 01.4	N 6 19.8
Betelgeuse	271 31.0	N 7 24.2
Canopus	264 08.2	S52 40.8
Capella	281 15.0	N45 58.4
Deneb	49 50.1	N45 12.2
Denebola	183 02.1	N14 42.1
Diphda	349 23.2	S18 06.7
Dubhe	194 26.0	N61 52.4
Elnath	278 47.3	N28 35.2
Eltanin	90 59.2	N51 29.9
Enif	34 14.0	N 9 46.4
Fomalhaut	15 54.1	S29 44.6
Gacrux	172 32.2	S56 58.9
Gienah	176 21.0	S17 24.7
Hadar	149 27.7	S60 15.7
Hamal	328 31.6	N23 21.3
Kaus Aust.	84 20.5	S34 23.8
Kochab	137 20.1	N74 15.2
Markab	14 05.5	N15 05.1
Menkar	314 43.6	N 4 00.0
Menkent	148 40.5	S36 15.3
Miaplacidus	221 45.8	S69 37.1
Mirfak	309 19.4	N49 46.7
Nunki	76 32.5	S26 19.5
Peacock	54 02.5	S56 48.7
Pollux	244 01.4	N28 04.8
Procyon	245 28.6	N 5 17.1
Rasalhague	96 32.2	N12 34.9
Regulus	208 13.0	N12 04.8
Rigel	281 38.4	S 8 13.6
Rigil Kent.	140 30.0	S60 44.3
Sabik	102 44.4	S15 41.7
Schedar	350 11.4	N56 24.8
Shaula	96 59.6	S37 05.2
Sirius	258 58.0	S16 41.0
Spica	159 00.7	S11 02.3
Suhail	223 12.9	S43 20.1
Vega	80 57.7	N38 46.1
Zuben'ubi	137 36.3	S15 56.6

	S.H.A. ° ′	Mer. Pass. h m
Venus	132 01.3	13 46
Mars	149 19.9	12 35
Jupiter	301 36.1	2 26
Saturn	222 19.9	7 43

1976 OCTOBER 12, 13, 14 (TUES., WED., THURS.)

G.M.T. d	h	SUN G.H.A. ° ′	SUN Dec. ° ′	MOON G.H.A. ° ′	v ′	MOON Dec. ° ′	d ′	H.P. ′
12 (Tuesday)	00	183 21.6	S 7 21.7	322 28.4	12.6	N17 49.6	3.7	54.1
	01	198 21.8	22.6	337 00.0	12.5	17 53.3	3.7	54.1
	02	213 21.9	23.6	351 31.5	12.6	17 57.0	3.6	54.1
	03	228 22.1	·· 24.5	6 03.1	12.5	18 00.6	3.5	54.1
	04	243 22.3	25.4	20 34.6	12.5	18 04.1	3.4	54.1
	05	258 22.4	26.4	35 06.1	12.4	18 07.5	3.4	54.2
	06	273 22.6	S 7 27.3	49 37.5	12.5	N18 10.9	3.3	54.2
	07	288 22.7	28.2	64 09.0	12.4	18 14.2	3.1	54.2
	08	303 22.9	29.2	78 40.4	12.3	18 17.3	3.1	54.2
	09	318 23.0	·· 30.1	93 11.7	12.4	18 20.4	3.1	54.2
	10	333 23.2	31.1	107 43.1	12.3	18 23.5	2.9	54.2
	11	348 23.3	32.0	122 14.4	12.3	18 26.4	2.9	54.2
	12	3 23.5	S 7 32.9	136 45.7	12.3	N18 29.3	2.7	54.2
	13	18 23.6	33.9	151 17.0	12.2	18 32.0	2.7	54.2
	14	33 23.8	34.8	165 48.2	12.3	18 34.7	2.6	54.2
	15	48 24.0	·· 35.7	180 19.5	12.2	18 37.3	2.5	54.2
	16	63 24.1	36.7	194 50.7	12.1	18 39.8	2.4	54.3
	17	78 24.3	37.6	209 21.8	12.2	18 42.2	2.4	54.3
	18	93 24.4	S 7 38.6	223 53.0	12.1	N18 44.6	2.2	54.3
	19	108 24.6	39.5	238 24.1	12.1	18 46.8	2.2	54.3
	20	123 24.7	40.4	252 55.2	12.1	18 49.0	2.1	54.3
	21	138 24.9	·· 41.4	267 26.3	12.1	18 51.1	2.0	54.3
	22	153 25.0	42.3	281 57.4	12.0	18 53.1	1.9	54.3
	23	168 25.2	43.2	296 28.4	12.0	18 55.0	1.8	54.3
13 (Wednesday)	00	183 25.3	S 7 44.2	310 59.4	12.0	N18 56.8	1.7	54.3
	01	198 25.5	45.1	325 30.4	11.9	18 58.5	1.7	54.4
	02	213 25.6	46.0	340 01.3	12.0	19 00.2	1.5	54.4
	03	228 25.8	·· 47.0	354 32.3	11.9	19 01.7	1.5	54.4
	04	243 25.9	47.9	9 03.2	11.9	19 03.2	1.4	54.4
	05	258 26.1	48.8	23 34.1	11.9	19 04.6	1.3	54.4
	06	273 26.2	S 7 49.8	38 05.0	11.8	N19 05.9	1.2	54.4
	07	288 26.4	50.7	52 35.8	11.8	19 07.1	1.1	54.4
	08	303 26.5	51.6	67 06.6	11.9	19 08.2	1.0	54.5
	09	318 26.7	·· 52.6	81 37.5	11.7	19 09.2	0.9	54.5
	10	333 26.8	53.5	96 08.2	11.8	19 10.1	0.9	54.5
	11	348 27.0	54.4	110 39.0	11.8	19 11.0	0.7	54.5
	12	3 27.1	S 7 55.4	125 09.8	11.7	N19 11.7	0.7	54.5
	13	18 27.3	56.3	139 40.5	11.7	19 12.4	0.5	54.5
	14	33 27.4	57.2	154 11.2	11.7	19 12.9	0.5	54.5
	15	48 27.6	·· 58.2	168 41.9	11.6	19 13.4	0.4	54.6
	16	63 27.7	7 59.1	183 12.5	11.7	19 13.8	0.3	54.6
	17	78 27.9	8 00.0	197 43.2	11.6	19 14.1	0.2	54.6
	18	93 28.0	S 8 01.0	212 13.8	11.6	N19 14.3	0.1	54.6
	19	108 28.2	01.9	226 44.4	11.6	19 14.4	0.0	54.6
	20	123 28.3	02.8	241 15.0	11.6	19 14.4	0.1	54.6
	21	138 28.5	·· 03.8	255 45.6	11.6	19 14.3	0.1	54.7
	22	153 28.6	04.7	270 16.2	11.5	19 14.2	0.3	54.7
	23	168 28.7	05.6	284 46.7	11.5	19 13.9	0.3	54.7
14 (Thursday)	00	183 28.9	S 8 06.6	299 17.2	11.5	N19 13.6	0.5	54.7
	01	198 29.0	07.5	313 47.7	11.5	19 13.1	0.5	54.7
	02	213 29.2	08.4	328 18.2	11.5	19 12.6	0.6	54.8
	03	228 29.3	·· 09.3	342 48.7	11.5	19 12.0	0.8	54.8
	04	243 29.5	10.3	357 19.2	11.4	19 11.2	0.8	54.8
	05	258 29.6	11.2	11 49.6	11.4	19 10.4	0.9	54.8
	06	273 29.8	S 8 12.1	26 20.0	11.4	N19 09.5	1.0	54.8
	07	288 29.9	13.1	40 50.4	11.4	19 08.5	1.1	54.9
	08	303 30.0	14.0	55 20.8	11.4	19 07.4	1.2	54.9
	09	318 30.2	·· 14.9	69 51.2	11.4	19 06.2	1.3	54.9
	10	333 30.3	15.9	84 21.6	11.3	19 04.9	1.3	54.9
	11	348 30.5	16.8	98 51.9	11.4	19 03.6	1.5	54.9
	12	3 30.6	S 8 17.7	113 22.3	11.3	N19 02.1	1.6	55.0
	13	18 30.8	18.6	127 52.6	11.3	19 00.5	1.6	55.0
	14	33 30.9	19.6	142 22.9	11.3	18 58.9	1.8	55.0
	15	48 31.0	·· 20.5	156 53.2	11.3	18 57.1	1.8	55.0
	16	63 31.2	21.4	171 23.5	11.3	18 55.3	2.0	55.1
	17	78 31.3	22.3	185 53.8	11.2	18 53.3	2.0	55.1
	18	93 31.5	S 8 23.3	200 24.0	11.3	N18 51.3	2.1	55.1
	19	108 31.6	24.2	214 54.3	11.2	18 49.2	2.2	55.1
	20	123 31.8	25.1	229 24.5	11.3	18 47.0	2.3	55.1
	21	138 31.9	·· 26.1	243 54.8	11.2	18 44.7	2.5	55.2
	22	153 32.0	27.0	258 25.0	11.2	18 42.2	2.5	55.2
	23	168 32.2	27.9	272 55.2	11.2	18 39.7	2.6	55.2
		S.D. 16.1	d 0.9	S.D. 14.8		14.9		15.0

Lat. °	Twilight Naut. h m	Twilight Civil h m	Sunrise h m	Moonrise 12 h m	Moonrise 13 h m	Moonrise 14 h m	Moonrise 15 h m
N 72	04 46	06 05	07 15	□	□	□	18 27
N 70	04 51	06 02	07 05	16 31	16 59	18 03	19 33
68	04 55	06 00	06 56	17 17	17 54	18 52	20 09
66	04 58	05 57	06 50	17 47	18 27	19 23	20 34
64	05 01	05 56	06 44	18 10	18 51	19 46	20 54
62	05 03	05 54	06 39	18 28	19 10	20 05	21 10
60	05 04	05 52	06 34	18 43	19 26	20 20	21 23
N 58	05 06	05 51	06 31	18 55	19 39	20 33	21 35
56	05 07	05 50	06 27	19 06	19 51	20 44	21 45
54	05 08	05 48	06 24	19 16	20 01	20 54	21 53
52	05 08	05 47	06 21	19 25	20 10	21 02	22 01
50	05 09	05 46	06 19	19 32	20 18	21 10	22 08
45	05 09	05 44	06 13	19 49	20 35	21 27	22 23
N 40	05 10	05 41	06 08	20 02	20 49	21 40	22 36
35	05 09	05 39	06 04	20 14	21 01	21 52	22 46
30	05 09	05 36	06 01	20 24	21 11	22 02	22 55
20	05 06	05 32	05 54	20 42	21 29	22 19	23 11
N 10	05 03	05 27	05 48	20 57	21 45	22 34	23 25
0	04 58	05 22	05 43	21 11	21 59	22 48	23 38
S 10	04 51	05 16	05 37	21 25	22 14	23 02	23 [illegible]
20	04 43	05 09	05 31	21 41	22 30	23 18	24 [illegible]
30	04 31	05 00	05 24	21 58	22 47	23 35	24 2[illegible]
35	04 24	04 54	05 20	22 08	22 58	23 45	24 29
40	04 15	04 48	05 16	22 20	23 10	23 56	24 40
45	04 04	04 40	05 10	22 34	23 24	24 10	00 10
S 50	03 49	04 30	05 03	22 51	23 41	24 26	00 26
52	03 42	04 25	05 00	22 59	23 49	24 34	00 34
54	03 34	04 20	04 57	23 08	23 58	24 43	00 43
56	03 25	04 14	04 53	23 18	24 08	00 08	00 53
58	03 15	04 08	04 49	23 29	24 20	00 20	01 03
S 60	03 03	04 00	04 45	23 42	24 34	00 34	01 16

Lat. °	Sunset h m	Twilight Civil h m	Twilight Naut. h m	Moonset 12 h m	Moonset 13 h m	Moonset 14 h m	Moonset 15 h m
N 72	16 16	17 26	18 44	□	□	□	16 31
N 70	16 26	17 29	18 39	13 22	14 34	15 12	15 25
68	16 35	17 31	18 35	12 36	13 39	14 22	14 48
66	16 42	17 34	18 33	12 06	13 06	13 51	14 22
64	16 48	17 36	18 30	11 44	12 42	13 28	14 02
62	16 53	17 37	18 28	11 26	12 23	13 09	13 46
60	16 57	17 39	18 27	11 12	12 07	12 54	13 32
N 58	17 01	17 41	18 26	10 59	11 54	12 41	13 20
56	17 04	17 42	18 25	10 48	11 42	12 30	13 10
54	17 08	17 43	18 24	10 39	11 32	12 20	13 01
52	17 10	17 44	18 23	10 30	11 23	12 11	12 53
50	17 13	17 46	18 23	10 23	11 15	12 03	12 46
45	17 19	17 48	18 22	10 06	10 58	11 46	12 30
N 40	17 24	17 51	18 22	09 53	10 44	11 32	12 17
35	17 28	17 53	18 23	09 42	10 32	11 21	12 06
30	17 32	17 56	18 23	09 32	10 22	11 10	11 57
20	17 38	18 00	18 26	09 15	10 04	10 53	11 40
N 10	17 44	18 05	18 30	09 00	09 49	10 37	11 26
0	17 50	18 10	18 35	08 46	09 34	10 23	11 12
S 10	17 55	18 17	18 41	08 33	09 20	10 08	10 59
20	18 01	18 24	18 50	08 18	09 04	09 53	10 44
30	18 09	18 33	19 02	08 01	08 46	09 35	10 27
35	18 13	18 39	19 09	07 51	08 36	09 25	10 17
40	18 18	18 45	19 18	07 40	08 24	09 13	10 06
45	18 23	18 54	19 30	07 27	08 10	08 59	09 53
S 50	18 30	19 04	19 45	07 11	07 53	08 42	09 37
52	18 33	19 09	19 52	07 03	07 45	08 34	09 29
54	18 37	19 14	20 00	06 55	07 36	08 25	09 21
56	18 40	19 20	20 09	06 45	07 26	08 14	09 11
58	18 45	19 27	20 20	06 35	07 14	08 03	09 01
S 60	18 49	19 34	20 33	06 22	07 01	07 50	08 48

Day	SUN Eqn. of Time 00ʰ m s	SUN Eqn. of Time 12ʰ m s	SUN Mer. Pass. h m	MOON Mer. Pass. Upper h m	MOON Mer. Pass. Lower h m	Age d	Phase
12	13 26	13 34	11 46	02 35	14 59	19	◑
13	13 41	13 48	11 46	03 23	15 47	20	
14	13 55	14 02	11 46	04 11	16 36	21	

POLARIS (POLE STAR) TABLES, 1976

FOR DETERMINING LATITUDE FROM SEXTANT ALTITUDE AND FOR AZIMUTH

L.H.A. ARIES	0°–9°	10°–19°	20°–29°	30°–39°	40°–49°	50°–59°	60°–69°	70°–79°	80°–89°	90°–99°	100°–109°	110°–119°
	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	0 16·3	0 12·2	0 09·5	0 08·3	0 08·8	0 10·7	0 14·2	0 19·0	0 25·0	0 32·1	0 40·0	0 48·4
1	15·8	11·8	09·3	08·3	08·9	11·0	14·6	19·5	25·7	32·8	40·8	49·3
2	15·3	11·5	09·1	08·3	09·0	11·3	15·0	20·1	26·4	33·6	41·6	50·2
3	14·9	11·2	09·0	08·3	09·2	11·6	15·5	20·7	27·0	34·4	42·5	51·0
4	14·5	10·9	08·8	08·3	09·4	11·9	15·9	21·3	27·7	35·2	43·3	51·9
5	0 14·0	0 10·6	0 08·7	0 08·4	0 09·5	0 12·3	0 16·4	0 21·9	0 28·4	0 35·9	0 44·1	0 52·8
6	13·6	10·4	08·6	08·4	09·8	12·6	16·9	22·5	29·2	36·7	45·0	53·7
7	13·2	10·1	08·5	08·5	10·0	13·0	17·4	23·1	29·9	37·5	45·8	54·5
8	12·9	09·9	08·4	08·5	10·2	13·4	17·9	23·7	30·6	38·3	46·7	55·4
9	12·5	09·7	08·4	08·6	10·5	13·8	18·4	24·4	31·3	39·2	47·6	56·3
10	0 12·2	0 09·5	0 08·3	0 08·8	0 10·7	0 14·2	0 19·0	0 25·0	0 32·1	0 40·0	0 48·4	0 57·2
Lat.	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1
°	′	′	′	′	′	′	′	′	′	′	′	′
0	0·5	0·6	0·6	0·6	0·6	0·5	0·5	0·4	0·3	0·3	0·2	0·2
10	·5	·6	·6	·6	·6	·5	·5	·4	·4	·3	·3	·2
20	·5	·6	·6	·6	·6	·6	·5	·5	·4	·4	·3	·3
30	·6	·6	·6	·6	·6	·6	·5	·5	·5	·4	·4	·4
40	0·6	0·6	0·6	0·6	·6	0·6	0·6	0·5	0·5	0·5	0·5	0·5
45	·6	·6	·6	·6	·6	·6	·6	·6	·6	·5	·5	·5
50	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6
55	·6	·6	·6	·6	·6	·6	·6	·6	·7	·7	·7	·7
60	·6	·6	·6	·6	·6	·6	·7	·7	·7	·8	·8	·8
62	0·7	0·6	0·6	0·6	0·6	0·6	0·7	0·7	0·8	0·8	0·8	0·9
64	·7	·6	·6	·6	·6	·6	·7	·7	·8	·9	0·9	0·9
66	·7	·6	·6	·6	·6	·7	·7	·8	·8	0·9	1·0	1·0
68	0·7	0·6	0·6	0·6	0·6	0·7	0·7	0·8	0·9	1·0	1·0	1·1
Month	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2
	′	′	′	′	′	′	′	′	′	′	′	′
Jan.	0·7	0·7	0·7	0·7	0·7	0·7	0·7	0·7	0·7	0·7	0·7	0·7
Feb.	·6	·7	·7	·7	·8	·8	·8	·8	·8	·8	·8	·8
Mar.	·5	·5	·6	·6	·7	·8	·8	·8	·9	·9	·9	·9
Apr.	0·3	0·4	0·4	0·5	0·6	0·6	0·7	0·8	0·8	0·9	0·9	0·9
May	2	·3	·3	·4	·4	·5	·6	·6	·7	·8	·8	·9
June	·2	·2	·2	·3	·3	·4	·4	·5	·5	·6	·7	·8
July	0·2	0·2	0·2	0·2	0·2	0·3	0·3	0·3	0·4	0·5	0·5	0·6
Aug.	·4	·3	·3	·3	·2	·2	·3	·3	·3	·3	·4	·4
Sept.	·5	·5	·4	·4	·3	·3	·3	·3	·3	·3	·3	·3
Oct.	0·7	0·7	0·6	0·5	0·5	0·4	0·4	0·3	0·3	0·3	0·3	0·2
Nov.	0·9	0·8	·8	·7	·7	·6	·5	·5	·4	·3	·3	·3
Dec.	1·0	1·0	0·9	0·9	0·8	0·8	0·7	0·6	0·5	0·5	0·4	0·3
Lat.	AZIMUTH											
°	°	°	°	°	°	°	°	°	°	°	°	°
0	0·4	0·3	0·1	0·0	359·8	359·7	359·5	359·4	359·3	359·3	359·2	359·2
20	0·4	0·3	0·1	0·0	359·8	359·7	359·5	359·4	359·3	359·2	359·1	359·1
40	0·5	0·3	0·1	359·9	359·8	359·6	359·4	359·2	359·1	359·0	358·9	358·9
50	0·6	0·4	0·2	359·9	359·7	359·5	359·3	359·1	358·9	358·8	358·7	358·7
55	0·7	0·4	0·2	359·9	359·7	359·4	359·2	359·0	358·8	358·7	358·6	358·5
60	0·8	0·5	0·2	359·9	359·6	359·3	359·1	358·8	358·6	358·5	358·4	358·3
65	0·9	0·6	0·3	359·9	359·6	359·2	358·9	358·6	358·4	358·2	358·1	358·0

Latitude = Apparent altitude (corrected for refraction) − 1° + a_0 + a_1 + a_2

The table is entered with L.H.A. Aries to determine the column to be used; each column refers to a range of 10°. a_0 is taken, with mental interpolation, from the upper table with the units of L.H.A. Aries in degrees as argument; a_1, a_2 are taken, without interpolation, from the second and third tables with arguments latitude and month respectively. a_0, a_1, a_2 are always positive. The final table gives the azimuth of *Polaris*.

POLARIS (POLE STAR) TABLES, 1976

FOR DETERMINING LATITUDE FROM SEXTANT ALTITUDE AND FOR AZIMUTH

L.H.A. ARIES	120°–129°	130°–139°	140°–149°	150°–159°	160°–169°	170°–179°	180°–189°	190°–199°	200°–209°	210°–219°	220°–229°	230°–239°
	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	0 57·2	1 06·0	1 14·5	1 22·6	1 29·9	1 36·3	1 41·6	1 45·6	1 48·2	1 49·3	1 48·9	1 47·0
1	58·1	06·8	15·3	23·4	30·6	36·9	42·0	45·9	48·3	49·3	48·7	46·7
2	58·9	07·7	16·2	24·1	31·3	37·5	42·5	46·2	48·5	49·3	48·6	46·4
3	0 59·8	08·6	17·0	24·9	32·0	38·0	42·9	46·5	48·6	49·3	48·4	46·1
4	1 00·7	09·4	17·8	25·6	32·6	38·6	43·3	46·8	48·8	49·3	48·3	45·8
5	1 01·6	1 10·3	1 18·6	1 26·4	1 33·3	1 39·1	1 43·7	1 47·0	1 48·9	1 49·2	1 48·1	1 45·5
6	02·5	11·1	19·4	27·1	33·9	39·6	44·1	47·3	49·0	49·2	47·9	45·1
7	03·3	12·0	20·2	27·8	34·5	40·1	44·5	47·5	49·1	49·1	47·7	44·8
8	04·2	12·8	21·0	28·5	35·1	40·6	44·9	47·8	49·2	49·1	47·5	44·4
9	05·1	13·7	21·8	29·2	35·7	41·1	45·2	48·0	49·2	49·0	47·2	44·0
10	1 06·0	1 14·5	1 22·6	1 29·9	1 36·3	1 41·6	1 45·6	1 48·2	1 49·3	1 48·9	1 47·0	1 43·6
Lat.	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1
°	′	′	′	′	′	′	′	′	′	′	′	′
0	0·2	0·2	0·2	0·3	0·4	0·4	0·5	0·6	0·6	0·6	0·6	0·5
10	·2	·2	·3	·3	·4	·5	·5	·6	·6	·6	·6	·5
20	·3	·3	·3	·4	·4	·5	·5	·6	·6	·6	·6	·6
30	·4	·4	·4	·4	·5	·5	·6	·6	·6	·6	·6	·6
40	0·5	0·5	0·5	0·5	0·5	0·6	0·5	0·6	0·6	0·6	0·6	0·6
45	·5	·5	·5	·5	·6	·6	·6	·6	·6	·6	·6	·6
50	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6
55	·7	·7	·7	·7	·6	·6	·6	·6	·6	·6	·6	·6
60	·8	·8	·8	·7	·7	·7	·6	·6	·6	·6	·6	·6
62	0·9	0·8	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·6	0·6	0·6
64	0·9	0·9	·9	·8	·8	·7	·7	·6	·6	·6	·6	·6
66	1·0	1·0	0·9	·9	·8	·7	·7	·6	·6	·6	·6	·7
68	1·1	1·1	1·0	0·9	0·9	0·8	0·7	0·6	0·6	0·6	0·6	0·7
Month	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2
	′	′	′	′	′	′	′	′	′	′	′	′
Jan.	0·6	0·6	0·6	0·6	0·5	0·5	0·5	0·5	0·5	0·5	0·5	0·5
Feb.	·8	·8	·7	·7	·7	·6	·6	·5	·5	·5	·4	·4
Mar.	0·9	0·9	0·9	0·8	·8	·8	·7	·7	·6	·6	·5	·4
Apr.	1·0	1·0	1·0	1·0	0·9	0·9	0·9	0·8	0·8	0·7	0·6	0·6
May	0·9	1·0	1·0	1·0	1·0	1·0	1·0	0·9	0·9	·8	·8	·7
June	·8	0·9	0·9	1·0	1·0	1·0	1·0	1·0	1·0	0·9	0·9	·8
July	0·7	0·7	0·8	0·8	0·9	0·9	1·0	1·0	1·0	1·0	1·0	0·9
Aug.	·5	·6	·6	·7	·7	·8	0·8	0·9	0·9	0·9	1·0	1·0
Sept.	·3	·4	·4	·5	·6	·6	·7	·7	·8	·8	0·9	0·9
Oct.	0·3	0·3	0·3	0·3	0·4	0·4	0·5	0·5	0·6	0·7	0·7	0·8
Nov.	·2	·2	·2	·2	·2	·3	·3	·4	·4	·5	·5	·6
Dec.	0·3	0·3	0·2	0·2	0·2	0·2	0·2	0·2	0·3	0·3	0·4	0·4
Lat.	AZIMUTH											
°	°	°	°	°	°	°	°	°	°	°	°	°
0	359·2	359·2	359·2	359·3	359·4	359·5	359·6	359·7	359·9	0·0	0·2	0·3
20	359·1	359·1	359·2	359·2	359·3	359·5	359·6	359·7	359·9	0·0	0·2	0·3
40	358·9	358·9	359·0	359·1	359·2	359·3	359·5	359·7	359·9	0·1	0·2	0·4
50	358·7	358·7	358·8	358·9	359·0	359·2	359·4	359·6	359·8	0·1	0·3	0·5
55	358·5	358·6	358·7	358·8	358·9	359·1	359·3	359·6	359·8	0·1	0·3	0·6
60	358·3	358·4	358·5	358·6	358·8	359·0	359·2	359·5	359·8	0·1	0·4	0·6
65	358·0	358·1	358·2	358·4	358·6	358·8	359·1	359·4	359·8	0·1	0·4	0·7

ILLUSTRATION

On 1976 May 22 at G.M.T. 23ʰ 18ᵐ 56ˢ in longitude W. 37° 14′ the corrected sextant altitude of *Polaris* was 49° 31′·6.

From the daily pages:	° ′
G.H.A. Aries (23ʰ)	225 41·5
Increment (18ᵐ 56ˢ)	4 44·8
Longitude (west)	−37 14
L.H.A. Aries	193 12

	° ′
Corr. Sext. Alt.	49 31·6
a_0 (argument 193° 12′)	1 46·6
a_1 (lat. 50° approx.)	0·6
a_2 (May)	0·9
Sum −1° = Lat. =	50 19·7

POLARIS (POLE STAR) TABLES, 1976

FOR DETERMINING LATITUDE FROM SEXTANT ALTITUDE AND FOR AZIMUTH

L.H.A. ARIES	240°– 249°	250°– 259°	260°– 269°	270°– 279°	280°– 289°	290°– 299°	300°– 309°	310°– 319°	320°– 329°	330°– 339°	340°– 349°	350°– 359°
	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	1 43·6	1 38·9	1 33·1	1 26·1	1 18·4	1 10·0	1 01·3	0 52·5	0 43·9	0 35·7	0 28·2	0 21·7
1	43·2	38·4	32·4	25·4	17·6	09·2	1 00·4	51·6	43·0	34·9	27·5	21·1
2	42·8	37·9	31·8	24·6	16·8	08·3	0 59·6	50·8	42·2	34·2	26·8	20·5
3	42·4	37·3	31·1	23·9	15·9	07·4	58·7	49·9	41·4	33·4	26·2	19·9
4	41·9	36·7	30·4	23·1	15·1	06·6	57·8	49·0	40·6	32·6	25·5	19·4
5	1 41·4	1 36·1	1 29·7	1 22·3	1 14·3	1 05·7	0 56·9	0 48·2	0 39·7	0 31·9	0 24·8	0 18·8
6	41·0	35·6	29·0	21·6	13·4	04·8	56·0	47·3	38·9	31·1	24·2	18·3
7	40·5	34·9	28·3	20·8	12·6	03·9	55·1	46·4	38·1	30·4	23·5	17·8
8	40·0	34·3	27·6	20·0	11·7	03·1	54·3	45·6	37·3	29·7	22·9	17·2
9	39·5	33·7	26·9	19·2	10·9	02·2	53·4	44·7	36·5	28·9	22·3	16·7
10	1 38·9	1 33·1	1 26·1	1 18·4	1 10·0	1 01·3	0 52·5	0 43·9	0 35·7	0 28·2	0 21·7	0 16·3
Lat.	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1
°	′	′	′	′	′	′	′	′	′	′	′	′
0	0·5	0·4	0·3	0·3	0·2	0·2	0·2	0·2	0·2	0·3	0·4	0·4
10	·5	·4	·4	·3	·3	·2	·2	·2	·3	·3	·4	·5
20	·5	·5	·4	·4	·3	·3	·3	·3	·3	·4	·4	·5
30	·5	·5	·5	·4	·4	·4	·4	·4	·4	·4	·5	·5
40	0·6	0·5	0·5	0·5	0·5	0·5	0·5	0·5	0·5	0·5	0·5	0·6
45	·6	·6	·6	·5	·5	·5	·5	·5	·5	·5	·6	·6
50	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6
55	·6	·6	·7	·7	·7	·7	·7	·7	·7	·7	·6	·6
60	·7	·7	·7	·8	·8	·8	·8	·8	·8	·7	·7	·7
62	0·7	0·7	0·8	0·8	0·8	0·9	0·9	0·8	0·8	0·8	0·7	0·7
64	·7	·7	·8	·9	0·9	0·9	0·9	0·9	·9	·8	·8	·7
66	·7	·8	·8	0·9	1·0	1·0	1·0	1·0	0·9	·9	·8	·7
68	0·7	0·8	0·9	1·0	1·0	1·1	1·1	1·1	1·0	0·9	0·9	0·8
Month	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2
	′	′	′	′	′	′	′	′	′	′	′	′
Jan.	0·5	0·5	0·5	0·5	0·5	0·5	0·6	0·6	0·6	0·6	0·7	0·7
Feb.	·4	·4	·4	·4	·4	·4	·4	·4	·5	·5	·5	·6
Mar.	·4	·4	·3	·3	·3	·3	·3	·3	·3	·4	·4	·4
Apr.	0·5	0·4	0·4	0·3	0·3	0·3	0·2	0·2	0·2	0·2	0·3	0·3
May	·6	·6	·5	·4	·4	·3	·3	·2	·2	·2	·2	·2
June	·8	·7	·7	·6	·5	·4	·4	·3	·3	·2	·2	·2
July	0·9	0·9	0·8	0·7	0·7	0·6	0·5	0·5	0·4	0·4	0·3	0·3
Aug.	·9	·9	·9	·9	·8	·8	·7	·6	·6	·5	·5	·4
Sept.	·9	·9	·9	·9	·9	0·9	·9	·8	·8	·7	·6	·6
Oct.	0·8	0·9	0·9	0·9	0·9	1·0	0·9	0·9	0·9	0·9	0·8	0·8
Nov.	·7	·7	·8	·9	·9	0·9	1·0	1·0	1·0	1·0	1·0	0·9
Dec.	0·5	0·6	0·7	0·7	0·8	0·9	0·9	0·9	1·0	1·0	1·0	1·0
Lat.	AZIMUTH											
°	°	°	°	°	°	°	°	°	°	°	°	°
0	0·5	0·6	0·7	0·7	0·8	0·8	0·8	0·8	0·8	0·7	0·6	0·5
20	0·5	0·6	0·7	0·8	0·9	0·9	0·9	0·9	0·8	0·8	0·7	0·5
40	0·6	0·7	0·9	1·0	1·0	1·1	1·1	1·1	1·0	0·9	0·8	0·7
50	0·7	0·9	1·0	1·2	1·2	1·3	1·3	1·3	1·2	1·1	1·0	0·8
55	0·8	1·0	1·2	1·3	1·4	1·5	1·5	1·4	1·4	1·2	1·1	0·9
60	0·9	1·1	1·3	1·5	1·6	1·7	1·7	1·7	1·6	1·4	1·3	1·0
65	1·0	1·3	1·6	1·7	1·9	2·0	2·0	2·0	1·9	1·7	1·5	1·2

Latitude = Apparent altitude (corrected for refraction) $-1° + a_0 + a_1 + a_2$

The table is entered with L.H.A. Aries to determine the column to be used; each column refers to a range of 10°. a_0 is taken, with mental interpolation, from the upper table with the units of L.H.A. Aries in degrees as argument; a_1, a_2 are taken, without interpolation, from the second and third tables with arguments latitude and month respectively. a_0, a_1, a_2 are always positive. The final table gives the azimuth of *Polaris*.

CONVERSION OF ARC TO TIME

0°–59°		60°–119°		120°–179°		180°–239°		240°–299°		300°–359°			0′.00	0′.25	0′.50	0′.75
°	h m	°	h m	°	h m	°	h m	°	h m	°	h m	′	m s	m s	m s	m s
0	0 00	60	4 00	120	8 00	180	12 00	240	16 00	300	20 00	0	0 00	0 01	0 02	0 03
1	0 04	61	4 04	121	8 04	181	12 04	241	16 04	301	20 04	1	0 04	0 05	0 06	0 07
2	0 08	62	4 08	122	8 08	182	12 08	242	16 08	302	20 08	2	0 08	0 09	0 10	0 11
3	0 12	63	4 12	123	8 12	183	12 12	243	16 12	303	20 12	3	0 12	0 13	0 14	0 15
4	0 16	64	4 16	124	8 16	184	12 16	244	16 16	304	20 16	4	0 16	0 17	0 18	0 19
5	0 20	65	4 20	125	8 20	185	12 20	245	16 20	305	20 20	5	0 20	0 21	0 22	0 23
6	0 24	66	4 24	126	8 24	186	12 24	246	16 24	306	20 24	6	0 24	0 25	0 26	0 27
7	0 28	67	4 28	127	8 28	187	12 28	247	16 28	307	20 28	7	0 28	0 29	0 30	0 31
8	0 32	68	4 32	128	8 32	188	12 32	248	16 32	308	20 32	8	0 32	0 33	0 34	0 35
9	0 36	69	4 36	129	8 36	189	12 36	249	16 36	309	20 36	9	0 36	0 37	0 38	0 39
10	0 40	70	4 40	130	8 40	190	12 40	250	16 40	310	20 40	10	0 40	0 41	0 42	0 43
11	0 44	71	4 44	131	8 44	191	12 44	251	16 44	311	20 44	11	0 44	0 45	0 46	0 47
12	0 48	72	4 48	132	8 48	192	12 48	252	16 48	312	20 48	12	0 48	0 49	0 50	0 51
13	0 52	73	4 52	133	8 52	193	12 52	253	16 52	313	20 52	13	0 52	0 53	0 54	0 55
14	0 56	74	4 56	134	8 56	194	12 56	254	16 56	314	20 56	14	0 56	0 57	0 58	0 59
15	1 00	75	5 00	135	9 00	195	13 00	255	17 00	315	21 00	15	1 00	1 01	1 02	1 03
16	1 04	76	5 04	136	9 04	196	13 04	256	17 04	316	21 04	16	1 04	1 05	1 06	1 07
17	1 08	77	5 08	137	9 08	197	13 08	257	17 08	317	21 08	17	1 08	1 09	1 10	1 11
18	1 12	78	5 12	138	9 12	198	13 12	258	17 12	318	21 12	18	1 12	1 13	1 14	1 15
19	1 16	79	5 16	139	9 16	199	13 16	259	17 16	319	21 16	19	1 16	1 17	1 18	1 19
20	1 20	80	5 20	140	9 20	200	13 20	260	17 20	320	21 20	20	1 20	1 21	1 22	1 23
21	1 24	81	5 24	141	9 24	201	13 24	261	17 24	321	21 24	21	1 24	1 25	1 26	1 27
22	1 28	82	5 28	142	9 28	202	13 28	262	17 28	322	21 28	22	1 28	1 29	1 30	1 31
23	1 32	83	5 32	143	9 32	203	13 32	263	17 32	323	21 32	23	1 32	1 33	1 34	1 35
24	1 36	84	5 36	144	9 36	204	13 36	264	17 36	324	21 36	24	1 36	1 37	1 38	1 39
25	1 40	85	5 40	145	9 40	205	13 40	265	17 40	325	21 40	25	1 40	1 41	1 42	1 43
26	1 44	86	5 44	146	9 44	206	13 44	266	17 44	326	21 44	26	1 44	1 45	1 46	1 47
27	1 48	87	5 48	147	9 48	207	13 48	267	17 48	327	21 48	27	1 48	1 49	1 50	1 51
28	1 52	88	5 52	148	9 52	208	13 52	268	17 52	328	21 52	28	1 52	1 53	1 54	1 55
29	1 56	89	5 56	149	9 56	209	13 56	269	17 56	329	21 56	29	1 56	1 57	1 58	1 59
30	2 00	90	6 00	150	10 00	210	14 00	270	18 00	330	22 00	30	2 00	2 01	2 02	2 03
31	2 04	91	6 04	151	10 04	211	14 04	271	18 04	331	22 04	31	2 04	2 05	2 06	2 07
32	2 08	92	6 08	152	10 08	212	14 08	272	18 08	332	22 08	32	2 08	2 09	2 10	2 11
33	2 12	93	6 12	153	10 12	213	14 12	273	18 12	333	22 12	33	2 12	2 13	2 14	2 15
34	2 16	94	6 16	154	10 16	214	14 16	274	18 16	334	22 16	34	2 16	2 17	2 18	2 19
35	2 20	95	6 20	155	10 20	215	14 20	275	18 20	335	22 20	35	2 20	2 21	2 22	2 23
36	2 24	96	6 24	156	10 24	216	14 24	276	18 24	336	22 24	36	2 24	2 25	2 26	2 27
37	2 28	97	6 28	157	10 28	217	14 28	277	18 28	337	22 28	37	2 28	2 29	2 30	2 31
38	2 32	98	6 32	158	10 32	218	14 32	278	18 32	338	22 32	38	2 32	2 33	2 34	2 35
39	2 36	99	6 36	159	10 36	219	14 36	279	18 36	339	22 36	39	2 36	2 37	2 38	2 39
40	2 40	100	6 40	160	10 40	220	14 40	280	18 40	340	22 40	40	2 40	2 41	2 42	2 43
41	2 44	101	6 44	161	10 44	221	14 44	281	18 44	341	22 44	41	2 44	2 45	2 46	2 47
42	2 48	102	6 48	162	10 48	222	14 48	282	18 48	342	22 48	42	2 48	2 49	2 50	2 51
43	2 52	103	6 52	163	10 52	223	14 52	283	18 52	343	22 52	43	2 52	2 53	2 54	2 55
44	2 56	104	6 56	164	10 56	224	14 56	284	18 56	344	22 56	44	2 56	2 57	2 58	2 59
45	3 00	105	7 00	165	11 00	225	15 00	285	19 00	345	23 00	45	3 00	3 01	3 02	3 03
46	3 04	106	7 04	166	11 04	226	15 04	286	19 04	346	23 04	46	3 04	3 05	3 06	3 07
47	3 08	107	7 08	167	11 08	227	15 08	287	19 08	347	23 08	47	3 08	3 09	3 10	3 11
48	3 12	108	7 12	168	11 12	228	15 12	288	19 12	348	23 12	48	3 12	3 13	3 14	3 15
49	3 16	109	7 16	169	11 16	229	15 16	289	19 16	349	23 16	49	3 16	3 17	3 18	3 19
50	3 20	110	7 20	170	11 20	230	15 20	290	19 20	350	23 20	50	3 20	3 21	3 22	3 23
51	3 24	111	7 24	171	11 24	231	15 24	291	19 24	351	23 24	51	3 24	3 25	3 26	3 27
52	3 28	112	7 28	172	11 28	232	15 28	292	19 28	352	23 28	52	3 28	3 29	3 30	3 31
53	3 32	113	7 32	173	11 32	233	15 32	293	19 32	353	23 32	53	3 32	3 33	3 34	3 35
54	3 36	114	7 36	174	11 36	234	15 36	294	19 36	354	23 36	54	3 36	3 37	3 38	3 39
55	3 40	115	7 40	175	11 40	235	15 40	295	19 40	355	23 40	55	3 40	3 41	3 42	3 43
56	3 44	116	7 44	176	11 44	236	15 44	296	19 44	356	23 44	56	3 44	3 45	3 46	3 47
57	3 48	117	7 48	177	11 48	237	15 48	297	19 48	357	23 48	57	3 48	3 49	3 50	3 51
58	3 52	118	7 52	178	11 52	238	15 52	298	19 52	358	23 52	58	3 52	3 53	3 54	3 55
59	3 56	119	7 56	179	11 56	239	15 56	299	19 56	359	23 56	59	3 56	3 57	3 58	3 59

The above table is for converting expressions in arc to their equivalent in time; its main use in this Almanac is for the conversion of longitude for application to L.M.T. (*added* if *west*, *subtracted* if *east*) to give G.M.T. or vice versa, particularly in the case of sunrise, sunset, etc.

INDEX TO SELECTED STARS, 1976

Name	No.	Mag.	S.H.A.	Dec.
Acamar	7	3·1	316°	S. 40°
Achernar	5	0·6	336	S. 57
Acrux	30	1·1	174	S. 63
Adhara	19	1·6	256	S. 29
Aldebaran	10	1·1	291	N. 16
Alioth	32	1·7	167	N. 56
Alkaid	34	1·9	153	N. 49
Al Na'ir	55	2·2	28	S. 47
Alnilam	15	1·8	276	S. 1
Alphard	25	2·2	218	S. 9
Alphecca	41	2·3	127	N. 27
Alpheratz	1	2·2	358	N. 29
Altair	51	0·9	63	N. 9
Ankaa	2	2·4	354	S. 42
Antares	42	1·2	113	S. 26
Arcturus	37	0·2	146	N. 19
Atria	43	1·9	108	S. 69
Avior	22	1·7	234	S. 59
Bellatrix	13	1·7	279	N. 6
Betelgeuse	16	Var.*	272	N. 7
Canopus	17	−0·9	264	S. 53
Capella	12	0·2	281	N. 46
Deneb	53	1·3	50	N. 45
Denebola	28	2·2	183	N. 15
Diphda	4	2·2	349	S. 18
Dubhe	27	2·0	194	N. 62
Elnath	14	1·8	279	N. 29
Eltanin	47	2·4	91	N. 51
Enif	54	2·5	34	N. 10
Fomalhaut	56	1·3	16	S. 30
Gacrux	31	1·6	173	S. 57
Gienah	29	2·8	176	S. 17
Hadar	35	0·9	149	S. 60
Hamal	6	2·2	329	N. 23
Kaus Australis	48	2·0	84	S. 34
Kochab	40	2·2	137	N. 74
Markab	57	2·6	14	N. 15
Menkar	8	2·8	315	N. 4
Menkent	36	2·3	149	S. 36
Miaplacidus	24	1·8	222	S. 70
Mirfak	9	1·9	309	N. 50
Nunki	50	2·1	77	S. 26
Peacock	52	2·1	54	S. 57
Pollux	21	1·2	244	N. 28
Procyon	20	0·5	245	N. 5
Rasalhague	46	2·1	97	N. 13
Regulus	26	1·3	208	N. 12
Rigel	11	0·3	282	S. 8
Rigil Kentaurus	38	0·1	140	S. 61
Sabik	44	2·6	103	S. 16
Schedar	3	2·5	350	N. 56
Shaula	45	1·7	97	S. 37
Sirius	18	−1·6	259	S. 17
Spica	33	1·2	159	S. 11
Suhail	23	2·2	223	S. 43
Vega	49	0·1	81	N. 39
Zubenelgenubi	39	2·9	138	S. 16

No.	Name	Mag.	S.H.A.	Dec.
1	*Alpheratz*	2·2	358°	N. 29°
2	*Ankaa*	2·4	354	S. 42
3	*Schedar*	2·5	350	N. 56
4	*Diphda*	2·2	349	S. 18
5	*Achernar*	0·6	336	S. 57
6	*Hamal*	2·2	329	N. 23
7	*Acamar*	3·1	316	S. 40
8	*Menkar*	2·8	315	N. 4
9	*Mirfak*	1·9	309	N. 50
10	*Aldebaran*	1·1	291	N. 16
11	*Rigel*	0·3	282	S. 8
12	*Capella*	0·2	281	N. 46
13	*Bellatrix*	1·7	279	N. 6
14	*Elnath*	1·8	279	N. 29
15	*Alnilam*	1·8	276	S. 1
16	*Betelgeuse*	Var.*	272	N. 7
17	*Canopus*	−0·9	264	S. 53
18	*Sirius*	−1·6	259	S. 17
19	*Adhara*	1·6	256	S. 29
20	*Procyon*	0·5	245	N. 5
21	*Pollux*	1·2	244	N. 28
22	*Avior*	1·7	234	S. 59
23	*Suhail*	2·2	223	S. 43
24	*Miaplacidus*	1·8	222	S. 70
25	*Alphard*	2·2	218	S. 9
26	*Regulus*	1·3	208	N. 12
27	*Dubhe*	2·0	194	N. 62
28	*Denebola*	2·2	183	N. 15
29	*Gienah*	2·8	176	S. 17
30	*Acrux*	1·1	174	S. 63
31	*Gacrux*	1·6	173	S. 57
32	*Alioth*	1·7	167	N. 56
33	*Spica*	1·2	159	S. 11
34	*Alkaid*	1·9	153	N. 49
35	*Hadar*	0·9	149	S. 60
36	*Menkent*	2·3	149	S. 36
37	*Arcturus*	0·2	146	N. 19
38	*Rigil Kentaurus*	0·1	140	S. 61
39	*Zubenelgenubi*	2·9	138	S. 16
40	*Kochab*	2·2	137	N. 74
41	*Alphecca*	2·3	127	N. 27
42	*Antares*	1·2	113	S. 26
43	*Atria*	1·9	108	S. 69
44	*Sabik*	2·6	103	S. 16
45	*Shaula*	1·7	97	S. 37
46	*Rasalhague*	2·1	97	N. 13
47	*Eltanin*	2·4	91	N. 51
48	*Kaus Australis*	2·0	84	S. 34
49	*Vega*	0·1	81	N. 39
50	*Nunki*	2·1	77	S. 26
51	*Altair*	0·9	63	N. 9
52	*Peacock*	2·1	54	S. 57
53	*Deneb*	1·3	50	N. 45
54	*Enif*	2·5	34	N. 10
55	*Al Na'ir*	2·2	28	S. 47
56	*Fomalhaut*	1·3	16	S. 30
57	*Markab*	2·6	14	N. 15

ALTITUDE CORRECTION TABLES 0°–35°—MOON

App. Alt.	0°–4° Corrⁿ	5°–9° Corrⁿ	10°–14° Corrⁿ	15°–19° Corrⁿ	20°–24° Corrⁿ	25°–29° Corrⁿ	30°–34° Corrⁿ	App. Alt.
′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	′
00	0 33·8	5 58·2	10 62·1	15 62·8	20 62·2	25 60·8	30 58·9	00
10	35·9	58·5	62·2	62·8	62·1	60·8	58·8	10
20	37·8	58·7	62·2	62·8	62·1	60·7	58·8	20
30	39·6	58·9	62·3	62·8	62·1	60·7	58·7	30
40	41·2	59·1	62·3	62·8	62·0	60·6	58·6	40
50	42·6	59·3	62·4	62·7	62·0	60·6	58·5	50
00	1 44·0	6 59·5	11 62·4	16 62·7	21 62·0	26 60·5	31 58·5	00
10	45·2	59·7	62·4	62·7	61·9	60·4	58·4	10
20	46·3	59·9	62·5	62·7	61·9	60·4	58·3	20
30	47·3	60·0	62·5	62·7	61·9	60·3	58·2	30
40	48·3	60·2	62·5	62·7	61·8	60·3	58·2	40
50	49·2	60·3	62·6	62·7	61·8	60·2	58·1	50
00	2 50·0	7 60·5	12 62·6	17 62·7	22 61·7	27 60·1	32 58·0	00
10	50·8	60·6	62·6	62·6	61·7	60·1	57·9	10
20	51·4	60·7	62·6	62·6	61·6	60·0	57·8	20
30	52·1	60·9	62·7	62·6	61·6	59·9	57·8	30
40	52·7	61·0	62·7	62·6	61·5	59·9	57·7	40
50	53·3	61·1	62·7	62·6	61·5	59·8	57·6	50
00	3 53·8	8 61·2	13 62·7	18 62·5	23 61·5	28 59·7	33 57·5	00
10	54·3	61·3	62·7	62·5	61·4	59·7	57·4	10
20	54·8	61·4	62·7	62·5	61·4	59·6	57·4	20
30	55·2	61·5	62·8	62·5	61·3	59·6	57·3	30
40	55·6	61·6	62·8	62·4	61·3	59·5	57·2	40
50	56·0	61·6	62·8	62·4	61·2	59·4	57·1	50
00	4 56·4	9 61·7	14 62·8	19 62·4	24 61·2	29 59·3	34 57·0	00
10	56·7	61·8	62·8	62·3	61·1	59·3	56·9	10
20	57·1	61·9	62·8	62·3	61·1	59·2	56·9	20
30	57·4	61·9	62·8	62·3	61·0	59·1	56·8	30
40	57·7	62·0	62·8	62·2	60·9	59·1	56·7	40
50	57·9	62·1	62·8	62·2	60·9	59·0	56·6	50

H.P.	L	U	L	U	L	U	L	U	L	U	L	U	L	U	H.P.
′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′
54·0	0·3	0·9	0·3	0·9	0·4	1·0	0·5	1·1	0·6	1·2	0·7	1·3	0·9	1·5	54·0
54·3	0·7	1·1	0·7	1·2	0·7	1·2	0·8	1·3	0·9	1·4	1·1	1·5	1·2	1·7	54·3
54·6	1·1	1·4	1·1	1·4	1·1	1·4	1·2	1·5	1·3	1·6	1·4	1·7	1·5	1·8	54·6
54·9	1·4	1·6	1·5	1·6	1·5	1·6	1·6	1·7	1·6	1·8	1·8	1·9	1·9	2·0	54·9
55·2	1·8	1·8	1·8	1·8	1·9	1·9	1·9	1·9	2·0	2·0	2·1	2·1	2·2	2·2	55·2
55·5	2·2	2·0	2·2	2·0	2·3	2·1	2·3	2·1	2·4	2·2	2·4	2·3	2·5	2·4	55·5
55·8	2·6	2·2	2·6	2·2	2·6	2·3	2·7	2·3	2·7	2·4	2·8	2·4	2·9	2·5	55·8
56·1	3·0	2·4	3·0	2·5	3·0	2·5	3·0	2·5	3·1	2·6	3·1	2·6	3·2	2·7	56·1
56·4	3·4	2·7	3·4	2·7	3·4	2·7	3·4	2·7	3·4	2·8	3·5	2·8	3·5	2·9	56·4
56·7	3·7	2·9	3·7	2·9	3·8	2·9	3·8	2·9	3·8	3·0	3·8	3·0	3·9	3·0	56·7
57·0	4·1	3·1	4·1	3·1	4·1	3·1	4·1	3·1	4·2	3·1	4·2	3·2	4·2	3·2	57·0
57·3	4·5	3·3	4·5	3·3	4·5	3·3	4·5	3·3	4·5	3·3	4·5	3·4	4·6	3·4	57·3
57·6	4·9	3·5	4·9	3·5	4·9	3·5	4·9	3·5	4·9	3·5	4·9	3·5	4·9	3·6	57·6
57·9	5·3	3·8	5·3	3·8	5·2	3·8	5·2	3·7	5·2	3·7	5·2	3·7	5·2	3·7	57·9
58·2	5·6	4·0	5·6	4·0	5·6	4·0	5·6	4·0	5·6	3·9	5·6	3·9	5·6	3·9	58·2
58·5	6·0	4·2	6·0	4·2	6·0	4·2	6·0	4·2	6·0	4·1	5·9	4·1	5·9	4·1	58·5
58·8	6·4	4·4	6·4	4·4	6·4	4·4	6·3	4·4	6·3	4·3	6·3	4·3	6·2	4·2	58·8
59·1	6·8	4·6	6·8	4·6	6·7	4·6	6·7	4·6	6·7	4·5	6·6	4·5	6·6	4·4	59·1
59·4	7·2	4·8	7·1	4·8	7·1	4·8	7·1	4·8	7·0	4·7	7·0	4·7	6·9	4·6	59·4
59·7	7·5	5·1	7·5	5·0	7·5	5·0	7·5	5·0	7·4	4·9	7·3	4·8	7·2	4·7	59·7
60·0	7·9	5·3	7·9	5·3	7·9	5·2	7·8	5·2	7·8	5·1	7·7	5·0	7·6	4·9	60·0
60·3	8·3	5·5	8·3	5·5	8·2	5·4	8·2	5·4	8·1	5·3	8·0	5·2	7·9	5·1	60·3
60·6	8·7	5·7	8·7	5·7	8·6	5·7	8·6	5·6	8·5	5·5	8·4	5·4	8·2	5·3	60·6
60·9	9·1	5·9	9·0	5·9	9·0	5·9	8·9	5·8	8·8	5·7	8·7	5·6	8·6	5·4	60·9
61·2	9·5	6·2	9·4	6·1	9·4	6·1	9·3	6·0	9·2	5·9	9·1	5·8	8·9	5·6	61·2
61·5	9·8	6·4	9·8	6·3	9·7	6·3	9·7	6·2	9·5	6·1	9·4	5·9	9·2	5·8	61·5

DIP

Ht. of Eye	Corrⁿ	Ht. of Eye	Ht. of Eye	Corrⁿ	Ht. of Eye
m	′	ft.	m	′	ft.
2·4		8·0	9·5		31·5
	−2·8			−5·5	
2·6		8·6	9·9		32·7
	−2·9			−5·6	
2·8		9·2	10·3		33·9
	−3·0			−5·7	
3·0		9·8	10·6		35·1
	−3·1			−5·8	
3·2		10·5	11·0		36·3
	−3·2			−5·9	
3·4		11·2	11·4		37·6
	−3·3			−6·0	
3·6		11·9	11·8		38·9
	−3·4			−6·1	
3·8		12·6	12·2		40·1
	−3·5			−6·2	
4·0		13·3	12·6		41·5
	−3·6			−6·3	
4·3		14·1	13·0		42·8
	−3·7			−6·4	
4·5		14·9	13·4		44·2
	−3·8			−6·5	
4·7		15·7	13·8		45·5
	−3·9			−6·6	
5·0		16·5	14·2		46·9
	−4·0			−6·7	
5·2		17·4	14·7		48·4
	−4·1			−6·8	
5·5		18·3	15·1		49·8
	−4·2			−6·9	
5·8		19·1	15·5		51·3
	−4·3			−7·0	
6·1		20·1	16·0		52·8
	−4·4			−7·1	
6·3		21·0	16·5		54·3
	−4·5			−7·2	
6·6		22·0	16·9		55·8
	−4·6			−7·3	
6·9		22·9	17·4		57·4
	−4·7			−7·4	
7·2		23·9	17·9		58·9
	−4·8			−7·5	
7·5		24·9	18·4		60·5
	−4·9			−7·6	
7·9		26·0	18·8		62·1
	−5·0			−7·7	
8·2		27·1	19·3		63·8
	−5·1			−7·8	
8·5		28·1	19·8		65·4
	−5·2			−7·9	
8·8		29·2	20·4		67·1
	−5·3			−8·0	
9·2		30·4	20·9		68·8
	−5·4			−8·1	
9·5		31·5	21·4		70·5

MOON CORRECTION TABLE

The correction is in two parts; the first correction is taken from the upper part of the table with argument apparent altitude, and the second from the lower part, with argument H.P., in the same column as that from which the first correction was taken. Separate corrections are given in the lower part for lower (L) and upper (U) limbs. All corrections are to be **added** to apparent altitude, *but 30′ is to be subtracted from the altitude of the upper limb.*

For corrections for pressure and temperature see page A4.

For bubble sextant observations ignore dip, take the mean of upper and lower limb corrections and subtract 15′ from the altitude.

App. Alt. = Apparent altitude = Sextant altitude corrected for index error and dip.

ALTITUDE CORRECTION TABLES 35°-90°—MOON

App. Alt.	35°–39° Corr^{n}	40°–44° Corr^{n}	45°–49° Corr^{n}	50°–54° Corr^{n}	55°–59° Corr^{n}	60°–64° Corr^{n}	65°–69° Corr^{n}	70°–74° Corr^{n}	75°–79° Corr^{n}	80°–84° Corr^{n}	85°–89° Corr^{n}	App. Alt.
00	35° 56′.5	40° 53′.7	45° 50′.5	50° 46′.9	55° 43′.1	60° 38′.9	65° 34′.6	70° 30′.1	75° 25′.3	80° 20′.5	85° 15′.6	00
10	56.4	53.6	50.4	46.8	42.9	38.8	34.4	29.9	25.2	20.4	15.5	10
20	56.3	53.5	50.2	46.7	42.8	38.7	34.3	29.7	25.0	20.2	15.3	20
30	56.2	53.4	50.1	46.5	42.7	38.5	34.1	29.6	24.9	20.0	15.1	30
40	56.2	53.3	50.0	46.4	42.5	38.4	34.0	29.4	24.7	19.9	15.0	40
50	56.1	53.2	49.9	46.3	42.4	38.2	33.8	29.3	24.5	19.7	14.8	50
00	36 56.0	41 53.1	46 49.8	51 46.2	56 42.3	61 38.1	66 33.7	71 29.1	76 24.4	81 19.6	86 14.6	00
10	55.9	53.0	49.7	46.0	42.1	37.9	33.5	29.0	24.2	19.4	14.5	10
20	55.8	52.8	49.5	45.9	42.0	37.8	33.4	28.8	24.1	19.2	14.3	20
30	55.7	52.7	49.4	45.8	41.8	37.7	33.2	28.7	23.9	19.1	14.1	30
40	55.6	52.6	49.3	45.7	41.7	37.5	33.1	28.5	23.8	18.9	14.0	40
50	55.5	52.5	49.2	45.5	41.6	37.4	32.9	28.3	23.6	18.7	13.8	50
00	37 55.4	42 52.4	47 49.1	52 45.4	57 41.4	62 37.2	67 32.8	72 28.2	77 23.4	82 18.6	87 13.7	00
10	55.3	52.3	49.0	45.3	41.3	37.1	32.6	28.0	23.3	18.4	13.5	10
20	55.2	52.2	48.8	45.2	41.2	36.9	32.5	27.9	23.1	18.2	13.3	20
30	55.1	52.1	48.7	45.0	41.0	36.8	32.3	27.7	22.9	18.1	13.2	30
40	55.0	52.0	48.6	44.9	40.9	36.6	32.2	27.6	22.8	17.9	13.0	40
50	55.0	51.9	48.5	44.8	40.8	36.5	32.0	27.4	22.6	17.8	12.8	50
00	38 54.9	43 51.8	48 48.4	53 44.6	58 40.6	63 36.4	68 31.9	73 27.2	78 22.5	83 17.6	88 12.7	00
10	54.8	51.7	48.2	44.5	40.5	36.2	31.7	27.1	22.3	17.4	12.5	10
20	54.7	51.6	48.1	44.4	40.3	36.1	31.6	26.9	22.1	17.3	12.3	20
30	54.6	51.5	48.0	44.2	40.2	35.9	31.4	26.8	22.0	17.1	12.2	30
40	54.5	51.4	47.9	44.1	40.1	35.8	31.3	26.6	21.8	16.9	12.0	40
50	54.4	51.2	47.8	44.0	39.9	35.6	31.1	26.5	21.7	16.8	11.8	50
00	39 54.3	44 51.1	49 47.6	54 43.9	59 39.8	64 35.5	69 31.0	74 26.3	79 21.5	84 16.6	89 11.7	00
10	54.2	51.0	47.5	43.7	39.6	35.3	30.8	26.1	21.3	16.5	11.5	10
20	54.1	50.9	47.4	43.6	39.5	35.2	30.7	26.0	21.2	16.3	11.4	20
30	54.0	50.8	47.3	43.5	39.4	35.0	30.5	25.8	21.0	16.1	11.2	30
40	53.9	50.7	47.2	43.3	39.2	34.9	30.4	25.7	20.9	16.0	11.0	40
50	53.8	50.6	47.0	43.2	39.1	34.7	30.2	25.5	20.7	15.8	10.9	50

H.P.	L	U	L	U	L	U	L	U	L	U	L	U	L	U	L	U	L	U	L	U	L	U	H.P.
′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′
54.0	1.1	1.7	1.3	1.9	1.5	2.1	1.7	2.4	2.0	2.6	2.3	2.9	2.6	3.2	2.9	3.5	3.2	3.8	3.5	4.1	3.8	4.5	54.0
54.3	1.4	1.8	1.6	2.0	1.8	2.2	2.0	2.5	2.3	2.7	2.5	3.0	2.8	3.2	3.0	3.5	3.3	3.8	3.6	4.1	3.9	4.4	54.3
54.6	1.7	2.0	1.9	2.2	2.1	2.4	2.3	2.6	2.5	2.8	2.7	3.0	3.0	3.3	3.2	3.5	3.5	3.8	3.7	4.1	4.0	4.3	54.6
54.9	2.0	2.2	2.2	2.3	2.3	2.5	2.5	2.7	2.7	2.9	2.9	3.1	3.2	3.3	3.4	3.5	3.6	3.8	3.9	4.0	4.1	4.3	54.9
55.2	2.3	2.3	2.5	2.4	2.6	2.6	2.8	2.8	3.0	2.9	3.2	3.1	3.4	3.3	3.6	3.5	3.8	3.7	4.0	4.0	4.2	4.2	55.2
55.5	2.7	2.5	2.8	2.6	2.9	2.7	3.1	2.9	3.2	3.0	3.4	3.2	3.6	3.4	3.7	3.5	3.9	3.7	4.1	3.9	4.3	4.1	55.5
55.8	3.0	2.6	3.1	2.7	3.2	2.8	3.3	3.0	3.5	3.1	3.6	3.3	3.8	3.4	3.9	3.6	4.1	3.7	4.2	3.9	4.4	4.0	55.8
56.1	3.3	2.8	3.4	2.9	3.5	3.0	3.6	3.1	3.7	3.2	3.8	3.3	4.0	3.4	4.1	3.6	4.2	3.7	4.4	3.8	4.5	4.0	56.1
56.4	3.6	2.9	3.7	3.0	3.8	3.1	3.9	3.2	3.9	3.3	4.0	3.4	4.1	3.5	4.3	3.6	4.4	3.7	4.5	3.8	4.6	3.9	56.4
56.7	3.9	3.1	4.0	3.1	4.1	3.2	4.1	3.3	4.2	3.3	4.3	3.4	4.3	3.5	4.4	3.6	4.5	3.7	4.6	3.8	4.7	3.8	56.7
57.0	4.3	3.2	4.3	3.3	4.3	3.3	4.4	3.4	4.4	3.4	4.5	3.5	4.5	3.5	4.6	3.6	4.7	3.6	4.7	3.7	4.8	3.8	57.0
57.3	4.6	3.4	4.6	3.4	4.6	3.4	4.6	3.5	4.7	3.5	4.7	3.5	4.7	3.6	4.8	3.6	4.8	3.6	4.8	3.7	4.9	3.7	57.3
57.6	4.9	3.6	4.9	3.6	4.9	3.6	4.9	3.6	4.9	3.6	4.9	3.6	4.9	3.6	4.9	3.6	5.0	3.6	5.0	3.6	5.0	3.6	57.6
57.9	5.2	3.7	5.2	3.7	5.2	3.7	5.2	3.7	5.2	3.7	5.1	3.6	5.1	3.6	5.1	3.6	5.1	3.6	5.1	3.6	5.1	3.6	57.9
58.2	5.5	3.9	5.5	3.8	5.5	3.8	5.4	3.8	5.4	3.7	5.4	3.7	5.3	3.7	5.3	3.6	5.2	3.6	5.2	3.5	5.2	3.5	58.2
58.5	5.9	4.0	5.8	4.0	5.8	3.9	5.7	3.9	5.6	3.8	5.6	3.8	5.5	3.7	5.5	3.6	5.4	3.6	5.3	3.5	5.3	3.4	58.5
58.8	6.2	4.2	6.1	4.1	6.0	4.1	6.0	4.0	5.9	3.9	5.8	3.8	5.7	3.7	5.6	3.6	5.5	3.5	5.4	3.5	5.3	3.4	58.8
59.1	6.5	4.3	6.4	4.3	6.3	4.2	6.2	4.1	6.1	4.0	6.0	3.9	5.9	3.8	5.8	3.6	5.7	3.5	5.6	3.4	5.4	3.3	59.1
59.4	6.8	4.5	6.7	4.4	6.6	4.3	6.5	4.2	6.4	4.1	6.2	3.9	6.1	3.8	6.0	3.7	5.8	3.5	5.7	3.4	5.5	3.2	59.4
59.7	7.1	4.6	7.0	4.5	6.9	4.4	6.8	4.3	6.6	4.1	6.5	4.0	6.3	3.8	6.2	3.7	6.0	3.5	5.8	3.3	5.6	3.2	59.7
60.0	7.5	4.8	7.3	4.7	7.2	4.5	7.0	4.4	6.9	4.2	6.7	4.0	6.5	3.9	6.3	3.7	6.1	3.5	5.9	3.3	5.7	3.1	60.0
60.3	7.8	5.0	7.6	4.8	7.5	4.7	7.3	4.5	7.1	4.3	6.9	4.1	6.7	3.9	6.5	3.7	6.3	3.5	6.0	3.2	5.8	3.0	60.3
60.6	8.1	5.1	7.9	5.0	7.7	4.8	7.6	4.6	7.3	4.4	7.1	4.2	6.9	3.9	6.7	3.7	6.4	3.4	6.2	3.2	5.9	2.9	60.6
60.9	8.4	5.3	8.2	5.1	8.0	4.9	7.8	4.7	7.6	4.5	7.3	4.2	7.1	4.0	6.8	3.7	6.6	3.4	6.3	3.2	6.0	2.9	60.9
61.2	8.7	5.4	8.5	5.2	8.3	5.0	8.1	4.8	7.8	4.5	7.6	4.3	7.3	4.0	7.0	3.7	6.7	3.4	6.4	3.1	6.1	2.8	61.2
61.5	9.1	5.6	8.8	5.4	8.6	5.1	8.3	4.9	8.1	4.6	7.8	4.3	7.5	4.0	7.2	3.7	6.9	3.4	6.5	3.1	6.2	2.7	61.5

Answers

Chapter 1

Answers for table on page 8

(a)	092^0	(M)	;	095^0	(C)	;	2^0	E
(b)	151^0	(T)	;	170^0	(C)	;	19^0	W
(c)	7^0	W	;	2^0	W	;	9^0	W
(d)	167^0	(T)	;	174^0	(M)	;	1^0	W
(e)	5^0	E	;	343^0	(C)	;	NIL	

Exercise I

1. (a) 7^0 5.7' N, 0^0 39.0' W
   (b) 10^0 20.6' S, 32^0 22.4' W
   (c) 26^0 25.6' S, 5^0 27.8' E
   (d) 04^0 20.4' N, 68^0 52.8' W
2. (a) 28^0 24' N
   (b) 5^0 32' S
3. 17^0 22.5' N, 170^0 34.8' W
4. 18^0 27.4' N, 23^0 50.8' E
5. 33^0 06.3' N, 26^0 23.7' W
6. 4^0 W
7. 8^0 W

Exercise (II)

1. 6^0 40'
2. 2892 M
3. 923 M
4. 79^0 12' E
5. 60^0 N or S

Exercise (II-A)

1. d'lat = 2^0 55' S dep = 303.1 M West
2. dist = 307.9 M course N 57^0 36' E
3. Course = S 80^0 32.3' E

Exercise (III)

1. 3.662 cms and 0.06104 cms
2. 60^0 N or S
3. 187.3
4. 1.667 cms

Exercise (IV)

1. N 48^0 26½' W, 2749 miles
2. N 69^0 12.4' E, 5972 miles
3. S 49^0 29.8' E, 2770 miles
4. 3^0 15.3' E

Exercise (IV A)

1. Course N 65^0 56' W
   between lats 44^0 56.5' and 45^0 40.5' N or S
2. 13.68 kts
3. 30^0 N or S
4. 51^0 24' N

Exercise (IV B)

1. d'lat 199.3 S
   dep 237.5 E
2. Course N 67° W = 293°
   dist = 359.5 M
3. dist 144 M
   dep 106.9 E
4. dep 108.6 E or W
   Course S45° E
5. Course N59.9° W
   dist 402.2 miles

Exercise V

1. 187°
2. Lat 22° S ; Long 118° E
3. Lat 0°
   (Since by definition Aries is on the Equinoctial)
   Long 92° W

Exercise VI

1. 22.8°

Exercise VII

1. Lunar Eclipse

Exercise VIII

1. 90° 15'
2. -5m 40 sec
3. -9m 09 sec
4. 157° 01.4', 160° 31.3'
5. 2d 03h 45m 09 sec
6. -14m 04s
7. 11h 19m 55s
8. 139° 47.5'
9. 14h 01m, 13h 00m
10. 1d 04h 02m 16 sec ; 15.87 knots
11. 8h 49m 54 sec
12. 14h 16m 02 sec
13. 74° 00.1' E
14. 5° 00.5' W
15. 64°

Exercise IX

1. 54° 18.4'
2. 40° 09.1'
3. 27° 13.3'
4. 31° 35.7'

Exercise XI

1. 146° 11' or 326° 11'
2. 25° 02' N, 114° 43.1' E
3. 30° 46.6' . S, 45 08' E
4. 5 m 13.4s fast
5. 13.1'
6. 8 miles

Exercise XII

1. 36° 31.5' N or S
2. 9° 47' N
3. 18h 58m 06s
4. 18h 58m 06s
5. (a) 75° S to 90° S
   (b) between 57°S to 75° S
   (c) 75° N to 90° N

Exercise XIII

1. Dist 4292.5 miles, initial course 102° 20.6'
   final course 059° 49.2',
   Postn of vertex 35° 45.8' S, 44° 39.4' E
   Lat in which G.C crosses 40°E = 35° 40.4' S
   Lat in which G.C crosses 70°E = 33° 03.7' S
2. Dist 3572.3 miles
   - initial course 329° 03.1'
   - final course 319° 04.8'
   - vertex 59° 37.0' N ; 05° 59.3' W
   - long in which G.C crosses equator
     = 90° from long of vertex = 84° 00.7'E
   - course on crossing the equator = co. lat of vertex
     = N 30° 23' W
3. - Distance travelled by A = 1063.5 miles
   - Distance travelled by B = 1308.9 miles
4. 1126 miles
5. 50° N to S
6. 1417 miles
7. 31° N ; 120° E or 60° W
8. Long 60° W, course S 43° 48' E

Exercise XIII A

1. Initial co. 061° 25'
   Final co. 114° 05.8'
   Distance (2060.5 + 797.5 + 1655.4) = 4513.4 M
2. Initial Course 113° 22.8'
   Final Course 061° 51.4'
   Distance (1276.2 + 422.2 + 1600.2) = 3298.6 M

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